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Contextual Areas

"Now or Later?" When to Deploy Qualification Screening in Open-Bid Auction for Re-Sourcing

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Abstract. This paper considers a resourcing setting in which a qualified supplier (the incumbent) and multiple suppliers that have not yet been qualified (the entrants) compete in an open-bid descending auction for a single-supplier contract. Because of the risk of supplier nonperformance, the buyer only awards the contract to a qualified supplier; meanwhile, the buyer can conduct supplier qualification screening at a cost to verify whether the entrant suppliers can perform the contract. Conventionally, the buyer would screen entrants before running an auction, that is, the prequalification strategy (PRE). We explore an alternative approach called postqualification strategy (POST), in which the buyer first runs an auction and then conducts qualification screenings based on the suppliers' auction bids. Our characterization of the dynamic structure of the suppliers' equilibrium bidding strategy enables the calculation of the buyer's expected cost under POST, which is computationally intractable without this characterization. We show analytically that POST is cheaper than PRE when the cost of conducting qualification screening is high, the number of entrant suppliers is large, or the entrants' chance of passing qualification screening is high. To quantify the benefit of POST, we conduct a comprehensive numerical study and find that using the cheaper option between PRE and POST provides significant cost savings over the conventional PRE-only approach. Furthermore, we leverage a revelation principle for multistage games to derive the optimal mechanism as a stronger benchmark for performance comparison. Although the optimal mechanism is theoretically optimal, we find that its complexity renders it difficult to implement in practice; but quite strikingly, the simple and practical approach of choosing the cheaper option between PRE and POST captures the majority of the benefit that the optimal mechanism can offer over PRE, highlighting the practical benefit of POST.

Supplemental Material: The online appendix is available at https://doi.org/10.1287/opre.2021.2111.

Keywords: procurement • supplier asymmetry • open-bid auction • qualification screening

1. Introduction

Procurement plays an important role in manufacturers' operations. Toyota spends more than 70% of its revenue on purchasing goods and services (Toyota 2016). According to the 2016 annual survey of manufacturers from the Bureau of the Census, U. S. manufacturers, on average, spend roughly 50% to 60% of their revenue on procurement (U.S. Department of Commerce 2016). To reduce procurement cost, buying firms often routinely re-evaluate how competitive their existing suppliers are on price. If the current contract price is not very competitive, the buyer may resort to a *resourcing* process in which it would invite other potential suppliers in the market (*entrant suppliers*) and the existing supplier (incumbent supplier) to compete for that contract in a procurement auction; this resourcing process allows the buyer to obtain a more competitive price by either switching to a less expensive entrant supplier or utilizing the competitive pressure from the entrant suppliers in the auction to reduce the incumbent supplier's price.

Although the use of procurement auctions can be effective in reducing prices, the buyer also needs to watch out for potential pitfalls of *supplier nonperformance*. In fact, supplier nonperformance can cause catastrophic damages to the buyer's operations, reputations, and profitability. For example, major automakers had to recall more than 39 million vehicles in the United States due to faulty air bags supplied by Takata (NHTSA 2018). More recently, the Samsung Note 7 Battery Explosion Event due to supplier nonperformance forced Samsung Electronics to recall 2.5 million Note 7 smartphones globally at an estimated total recall expense of around $5 billion (Lee 2016). Although supply risks cannot be eliminated, they can be managed, and managing such supplier nonperformance risks is particularly critical during a resourcing process. Although even the incumbent supplier may fail, as evidenced by the examples we mentioned above, nevertheless, it is reasonable to assume that the buyer may have a good knowledge of the incumbent's capability to deliver the contract, based on previous assessments and past experience. In contrast, the buyer is likely to have less information about the ability of entrant suppliers to successfully execute the contract.

One common approach to managing the supplier nonperformance risks in resourcing is to integrate an auction with *supplier qualification screening* on the entrant suppliers, an act of verifying the supplier's ability to perform within a reasonable degree of certainty; only the *qualified* supplier, that is, either the incumbent supplier (because he had already undergone the qualification screening prior to becoming the incumbent) or an entrant supplier who passes the qualification screening, with the most competitive price, wins the contract. This integration leads to an important strategic decision by the buyer in designing the resourcing process. Should the buyer deploy supplier qualification screenings before or after the bidding? The conventional approach in practice is the *prequalification* strategy (PRE), in which a buyer would first conduct qualification screenings of entrant suppliers and then only allow *qualified* suppliers to participate in the competitive bidding for the contract. However, qualification screening is a costly process for the buyer because it involves visits to the supplier's facility, purchasing sample products, testing products, etc. (Beil 2010). Therefore, PRE may introduce unnecessary qualification expenses on suppliers who turn out to be not competitive. Motivated by this consideration, an alternative proposed strategy is the so-called *postqualification strategy* (POST), in which qualification screenings are postponed until after the bidding process. Informed by the suppliers' pricing bids, the buyer could choose to vet the capabilities only of suppliers who submitted competitive prices. Hence, compared with PRE, POST may reduce the buyer's expense on qualification screening; but the trade-off is that the resulting equilibrium bid prices will be higher, because suppliers are less inclined to bid aggressively against competitors who have not been qualified (i.e., a supplier can win the contract without placing the lowest bid, as long as all of the lower-bid suppliers fail qualification screenings).

The aforementioned main trade-off between PRE and POST has already been identified in various auction settings (e.g., see Wan and Beil 2009, Wan et al. 2012), but there are some obstacles to applying this managerial insight in practice. For example, Wan and Beil (2009) provide an elegant characterization of the optimal mechanism in the procurement context with qualification screenings, which provides clean managerial insights on the trade-off involved in the timing of conducting qualification screenings. However, optimal mechanisms are complex and difficult to explain to practitioners and are rarely used in practice. In contrast, Wan et al. (2012) studied the similar trade-off in the context of open-descending price-based auction (open-bid auction), a type of auction that allows suppliers to see competitors' bids in real time and respond by adjusting their own bids dynamically. Our paper also focuses on this auction format due to its popularity in the private sector. For example, auto parts manufacturers use open-bid auctions for sourcing highly engineered commodities (Beil and Duenyas 2012); pharmaceutical companies such as GlaxoSmithKline leverage open-bid auctions to source input materials globally (GlaxoSmithKline 2015); open-bid auction is also a key toolkit provided in SAP Ariba, a procurement and sourcing solution that the leading Enterprise Resource Planning software provider SAP offers to help its clients manage procurement. Whereas Wan et al. (2012) focus on a stylized setting with a single incumbent supplier and a single entrant supplier, in practice, in order to increase competition in the resourcing process, buyers usually consider multiple entrant suppliers to compete for the contract, a setting that cannot be addressed by Wan et al. (2012). In other words, open-bid auctions are highly prevalent in practice and are often a part of procurement toolkits due to their transparency and simplicity in implementation, and yet their theoretical properties are not well-understood. The main focus of our paper, therefore, is to gain insights into the trade-offs involved in open-bid auctions with pre- and postbidding qualification screening. The specific setting we are considering is one with a single incumbent and multiple entrants, and our primary goal is to characterize the factors that affect which strategy (PRE or POST) is better for the buyer and, even more specifically, how much value adding the POST option to the procurement toolkits would provide for the buyer. It is worth pointing out that one nuance of this setting with multiple entrants is to determine the payment rule of POST in an open-bid format: How much does the contract winner get paid if he or she is not the one with the lowest bid (i.e., this may happen if all lower bid suppliers fail to pass qualification screenings). Theoretically, there are many different options for the payment rule of POST. However, based on several practical considerations, such as ease of implementation and transparency (see a more detailed discussion in Section 2.2), we choose the following payment rule: If the contract winner is indeed the last supplier who remains in the auction (i.e., if this supplier is an entrant, he or she needs to pass qualification screening to be the contract winner), he or she gets paid the auction price right after the most recent supplier dropout; otherwise, the contract winner gets paid his or her dropout bid (see Section 2.2 for more details). Henceforth, PRE/POST refer specifically to open-bid auction with prebidding/postbidding qualification screenings, whose procedures will be formally introduced in Section 2.

To assess the value of the POST option, the buyer first needs to develop quantitative models and tools to estimate the expected total procurement cost (qualification screening cost plus contract payment) of both PRE and POST. Although PRE is relatively straightforward, the main challenge is the analysis of POST. Specifically, because an entrant supplier may not need to be the lowest bidder to win the contract, it is no longer a dominant strategy for suppliers to drop out at their costs as they would in a standard open-bid auction. This feature makes the open-bid auction with postbidding qualification screenings a complex dynamic game where suppliers' equilibrium bidding strategies depend on a multidimensional state that dynamically evolves as the auction proceeds (e.g., the current auction price, the number of remaining suppliers and their qualification status, and the posterior on the remaining suppliers' cost, etc.). Thus, solving the equilibrium bidding strategy using a standard approach (which requires solving several high-dimensional stochastic dynamic programs iteratively until the optimal dynamic policies in all dynamic programs converge) and numerically computing the expected cost of POST is not computationally tractable. To overcome this challenge, we identify several structural properties of suppliers' equilibrium bidding strategies, and these properties allow us to speed up the numerical computation. Additionally, this equilibrium analysis also provides rich insights on the incentive implications of POST. Therefore, one of our main analytical results deals with determining equilibrium bidding strategies for the entrant suppliers and for the incumbent supplier in POST.

The first analytical contribution of this paper is a full characterization of the suppliers' equilibrium bidding strategy in POST (i.e., Lemma 2 and Theorems 1 and 2). This characterization is very useful because it highlights the different bidding incentives between the entrant suppliers and the incumbent supplier and provides insights on the economic drivers in POST. It also significantly simplifies the computation of the expected cost of POST, which allows a convenient and efficient comparison between the expected cost of PRE and POST. Hence, the buyer can choose the cheaper option between PRE and POST to implement in his or her resourcing process; this is our proposed approach to the buyer. The second contribution of our work is that we establish analytical conditions under which the buyer should choose POST over the conventional approach PRE: when entrants' probability of passing qualification screening is sufficiently high, when the qualification screening cost is sufficiently high, or when the number of available entrants is sufficiently large (i.e., Proposition 4). These conditions correspond to scenarios where either the benefit of POST's more informed qualification screenings is most pronounced or the drawback of suppliers' less aggressive bidding behavior in POST is most subdued. Furthermore, using an extensive numerical study, we provide evidence that including POST as a resourcing option can provide significant cost savings compared with the conventional approach of only using PRE and identify settings in which the benefit of POST is most salient. It is worth noting that our proposed approach (i.e., the cheaper option between PRE and POST) is easy to implement in practice since it uses existing features of procurement auctions that practitioners are already familiar with and combines them in an innovative way. Although appealing from a practical standpoint, it naturally raises the question of whether a more sophisticated approach can provide significant benefit over our proposed approach. To that end, we numerically compare our approach with the optimal mechanism (i.e., Proposition 5) that provides a lower bound on the expected cost of any feasible mechanism but is difficult to implement in practice. Our result shows that our proposed approach captures the majority of the cost savings the optimal mechanism offers compared with the conventional PRE, which highlights the effectiveness of our simple approach.

1.1. Related Literature

Our paper is related to the literature in procurement and strategic sourcing (see Elmaghraby 2000 for a comprehensive survey and Beil 2010 for a comprehensive introduction about the procurement process). As pointed out earlier, one of the main challenges of procurement management is to address supply risks in pricing and quality. To deal with these risks, prior research has explored different approaches such as multisourcing (e.g., see Chaturvedi et al. 2019), total-cost auctions (e.g., see Aral et al. 2020, Stoll and Zöttl 2017), new supplier recruitment (e.g., see Beil et al. 2018), etc. We focus on the use of supplier qualification screening as a way to control supplier nonperformance risks. There is an important stream of work that focuses on this approach. For example, Gillen et al. (2017) study, in a forward auction setting, whether the auctioneer should demand certification of bidders' qualification before or after a second-price sealed-bid auction. In their model, they assume all bidders will pass the qualification for sure at a cost. In our paper, however, the suppliers face the risk of failing the qualification screenings. The paper closest to ours is Wan et al. (2012), in which the authors consider a setting where a qualified incumbent supplier and a single not-yet-qualified entrant supplier compete in an open-bid auction for a single-supplier contract from the buyer, and the buyer faces the choice of conducting qualification screening of the entrant before or after the auction. They reveal an important managerial insight to the buyer when postqualification strategy is used in their setting. The incumbent supplier will inflate the bid, but the entrant supplier will bid down to cost. Our paper captures a more general setting where there is more than one entrant competing with the incumbent for the contract; this generalization captures a more complex scenario that could never happen in Wan et al. (2012). After the incumbent drops out, multiple entrants may remain in the auction. This complication results in richer insights on the impact of postqualification on suppliers' bidding incentives. For example, we show that the entrant suppliers' bidding strategy is much more nuanced than that characterized in Wan et al. (2012). The entrants will bid to their costs before the incumbent drops out, but after the incumbent drops out they will inflate their bids above their costs.

There is also another stream of work that employs the optimal mechanism design approach (Myerson 1981) to incorporate supplier qualification screening to the procurement process (e.g., see Chaturvedi et al. 2014, Chaturvedi and Martínez-de Albéniz 2011, Chen et al. 2018, Wan and Beil 2009, etc). Although these papers optimally integrate supplier qualification screening with supplier selection in different settings to minimize expected procurement cost, the drawback is that optimal mechanisms may be difficult to explain to practitioners and hard to implement in practice (Rothkopf 2007, Roughgarden and Talgam-Cohen 2019). We focus on the open-bid auction approach, an important and widely used auction format in practice and provide insights that are very relevant to practitioners. Having said that, an optimal mechanism analysis of the setting we study provides a lower bound of the procurement cost under any feasible mechanism. Thus, in our paper, we also conduct an optimal mechanism analysis to facilitate a numerical evaluation of the effectiveness of our proposed open-bid auction approach.

1.2. Organization of the Paper

In the remainder of the paper, we first introduce the notation and the operational details of PRE and POST and present the analysis of PRE in Section 2, whereas the full analysis of POST is postponed in Section 3. Then Section 4 investigates the buyer's optimal choice between PRE and POST. Finally, we assess the benefit of POST numerically in Section 5 and conclude in Section 6.

2. Model

Consider a buyer who would like to renew a single-supplier contract. The buyer has an incumbent supplier, denoted by 0, who currently charges the buyer $\overline{R}$ for the contract. The incumbent's true cost, denoted by x_0, is a random variable with a cumulative distribution function (c.d.f.) F_0, a probability density function (p.d.f.) f_0, and support $[\underline{R}, \overline{R}]$. The buyer has already identified N ($N \geq 2$) potential suppliers in the market, the *entrants*, and would like to invite them to join the contract competition. These entrants are indexed by $j = 1, 2, \cdots, N$. The entrants' costs x_j s are independently and identically distributed with a c.d.f. F_e, a p.d.f. f_e, and support $[\underline{R}, \overline{R}]$. All suppliers' costs are private knowledge, but their cost distributions are common knowledge. We make the following assumption on entrants' cost.

Assumption 1. *$\frac{F_e}{f_e}$ is nondecreasing and convex.*

Note that most of the log-concave distributions, such as uniform, normal, logistic, exponential, and Weibull distributions, satisfy this condition (see table 5 in Bagnoli and Bergstrom 2005 for a more detailed list of these log-concave distributions). The incumbent and an entrant are ex ante asymmetric not only due to their asymmetric cost distributions but also due to their asymmetric *qualification status*. The incumbent supplier is qualified because he or she has been supplying the part to the buyer and his or her technical capabilities have already been validated by the buyer. In contrast, an entrant has not worked with the buyer before and needs to be carefully vetted via a qualification screening process before he or she can be awarded the contract. The qualification screening process may be lengthy and costly, requiring the buyer to visit the supplier's facility, purchase sample products, test products, etc. Additionally, there is no guarantee that an entrant will always pass the qualification. Following the supplier qualification literature (e.g., Wan et al. 2012), we model qualification screening cost and uncertainty as follows. It costs the buyer K to conduct the qualification screening on each entrant; if an entrant undergoes qualification screening, he or she will pass the screening requirements and become qualified with probability $\beta \in (0, 1)$, where we assume β to be common knowledge. In other words, we consider a setting where the entrant suppliers are ex ante symmetric, which is mostly appropriate when the buyer's prior belief about those entrant suppliers are similar and the cost drivers for buyer's qualification screenings are similar. Focusing on this setting also allows us to provide a clean way to highlight the different natures of bidding incentives between the incumbent supplier and the entrant suppliers. We also assume that if an entrant undergoes qualification screening, both the buyer and this entrant observe whether this entrant passes the screening.

The buyer's goal is to select a qualified supplier (i.e., either an entrant who passes the qualification or the incumbent) to minimize his or her expected total procurement cost, that is, contract payment plus qualification cost.

2.1. Prequalification Strategy

In the prequalification strategy (PRE), the buyer first chooses $N_{pre} \in [0, N]$ and randomly selects N_{pre} out of the N entrants (note that since entrants are ex ante symmetric, it is equivalent to select the entrants 1 to N_{pre}) to conduct qualification screenings simultaneously. Then, only the entrants who pass the qualification screenings and the incumbent are allowed to compete for the contract in a standard open-bid descending auction that proceeds as follows. The auction price is initially set at the current contract price $\overline{R}$ and then falls continuously until the auction ends when all but one bidder drops out; the last remaining bidder wins the auction and is paid the auction ending price. (Auctions with a continuously falling price are also known as "reverse clock auctions"; see Ausubel and Cramton 2006 for discussions about clock auctions in practice.) Note that, if none of the invited entrants pass the qualification, the incumbent will win the auction directly at price $\overline{R}$; otherwise, because all participants are qualified suppliers, in equilibrium, all of them will stay in the auction until the auction price reaches their true costs (Krishna 2009). Hence, the expected total procurement cost (qualification cost plus contract payment) when the buyer conducts N_{pre} prebidding qualification screenings equals

$$\mathcal{PC}_{pre}(N_{pre}) = \sum_{i=0}^{N_{pre}} \mathbf{E}\left[\mathbf{Min\,2}(\overline{R}, x_0, \ldots, x_i)\right] \times \frac{N_{pre}!}{i!(N_{pre}-i)!}\beta^i(1-\beta)^{N_{pre}-i} + N_{pre}K, \qquad (1)$$

where $\mathbf{Min\,2}(\ldots)$ corresponds to the second-lowest value of its arguments. The buyer chooses the optimal number of entrants for qualification screening, N^*_{pre}, to minimize his or her expected cost under PRE. Hence, the buyer's optimal expected cost under PRE, denoted by PC_{pre}, equals

$$PC_{pre} = \min_{N_{pre}\in\{1,\ldots,N\}} \mathcal{PC}_{pre}(N_{pre}) = \mathcal{PC}_{pre}\left(N^*_{pre}\right). \qquad (2)$$

2.2. Postqualification Strategy

An alternative approach to structure the supplier qualification screening and supplier selection is the postqualification strategy (POST), where qualification screenings are conducted postbidding rather than prebidding. The buyer invites all suppliers to bid in a (slightly modified) open-bid descending auction (see more details below) before conducting qualification screenings and awarding the contract to one of the qualified suppliers. Specifically, after all suppliers decide whether to join the auction, the buyer announces to all participating suppliers which supplier is the incumbent and a fixed price decrement $\Delta := K/\beta$ when incumbent drops out (see more details of this decrement later). Then the auction begins as its descending price clock starts at the current contract price $\overline{R}$ and continuously descends. In this process, all suppliers choose to drop out of the auction or stay at the prevailing price. In contrast to a standard open-bid descending auction as in PRE, when the incumbent drops out, the auction price immediately jumps down by Δ before keeping on continuously descending. The auction ends when there is only one supplier left in the auction, and the *ending auction price* equals the last dropout price bid (dropout price for short) if the last dropout is an entrant, and equals the last dropout price minus Δ if the last dropout is the incumbent (this is because the auction price immediately jumps down by Δ after the incumbent drops out). During this bidding process, suppliers observe the following. Before the auction starts, the number of suppliers in the auction, which one is the incumbent, and Δ are all common knowledge; as the auction proceeds and suppliers drop out one after another, each supplier observes the auction price and when other suppliers drop out in real time until he or she himself or herself drops out (this means that he or she can perfectly infer the dropout prices of all suppliers who drop out earlier than him or her). Note that it is possible that when the auction concludes, the only remaining supplier is an entrant who has not yet been qualified. Hence, although this supplier becomes the *auction winner*, the supplier does not automatically become the contract winner. This is where the postbidding qualification screenings take place. After the auction ends, informed by the suppliers' dropout prices, the buyer decides which suppliers (if any) and in what sequence to conduct qualification screenings. At the conclusion of the qualification screenings, the contract is awarded to a qualified supplier (either the incumbent or an entrant who passes the qualification screening) who has the lowest dropout price. The contract payment equals the ending auction price if the contract winner is the auction winner; otherwise, the contract payment equals the contract winner's dropout price.

Given the format of POST, the buyer's (ex post) optimal qualification screening decisions are characterized in the following lemma. (All our proofs can be found in the Online Appendix.)

Lemma 1. *In POST, it is ex post optimal for the buyer to keep conducting qualification screening in the reverse order of when the suppliers dropped out until either the first entrant passes the qualification screening or all entrants who drop out later than the incumbent fail to pass qualification screenings.*

The buyer's optimal postauction qualification screening rule is very intuitive. The buyer starts with the supplier with the lowest bid and keeps on conducting qualification screening in the sequence from the low bid to high until he or she finds the first qualified supplier (either an entrant who passes the qualification screening or the incumbent). Note that the observation that this screening rule is ex post optimal is driven by the design that the auction price immediately jumps down by $\Delta = K/\beta$ when the incumbent drops out. Indeed, if the incumbent drops out at p, this design feature ensures that the dropout price bids of entrants who drop out later are at least K/β lower, which makes it optimal to conduct qualification screening of any of these entrants before awarding the contract directly to the incumbent since the expected benefit (i.e., the reduction in payment when an entrant passes qualification) equals $\beta \times (p - entrant's\ bid) \geq \beta K/\beta = K$, which is larger than the cost of qualification. Having explained above how POST is operationalized, we now explain an appealing feature of POST that motivates us to propose this format in the first place: practical applicability. First of all, POST is quite straightforward to deploy mechanically because, compared with the standard open-bid descending auction that is widely adopted in practice, operationally, POST only requires the buyer to ensure that the descending price clock jumps down by Δ immediately after the incumbent drops out (e.g., if the incumbent drops out at some auction price p, then the price clock immediately drops down to $p - \Delta$). Note that this feature is not an arcane concept because one can interpret $p - \Delta$ as a "reserve price" to account for the extra expenditure on verifying entrants' qualification status. Second, the payment in POST is very transparent. The final contract winner always pays what he or she bids, except in one scenario when final contract winner wins the auction and passes the qualification screening. But even in that scenario, the payment is still very transparent since the contract winner pays the auction ending price (note that this is the same payment rule as in a standard open-bid descending auction widely used in practice). Finally, note that the qualification screening rule the buyer uses in POST is ex post optimal. Thus, the buyer does not need to commit to a particular qualification screening rule to the suppliers ex ante that would cause implementation issues, as the qualification screenings are not directly observable to all suppliers.

Although POST is a practical sourcing strategy, the expected cost of POST may be either higher or lower than PRE, depending on the business context. In order to determine the best strategy to use, the buyer needs to compute the expected cost of both options and then choose the less expensive one. Although the expected procurement cost for PRE can easily be established in (1) and (2), calculating the expected procurement cost under POST is challenging. We are not aware of any existing literature that provides numerical methods to calculate the expected procurement cost under POST. What makes POST much more complex to analyze than PRE is that suppliers' equilibrium bidding behavior in POST is much more nuanced. The fact that not all auction participants are qualified suppliers means that a supplier does not necessarily need to win the auction in order to win the contract. For example, when an entrant supplier j drops out, if all later dropout suppliers are entrants and they all fail the qualification screening, then supplier j can still win the contract if he or she passes qualification screening. Therefore, supplier j may have the incentive to drop out at a price higher than his or her true cost, which is different from the equilibrium bidding behavior one would expect in a standard open-bid descending auction. As one can imagine, the extent to which supplier j inflates his or her dropout price above his or her cost may depend on all the historic information the supplier observes (e.g., the current auction price, the number of remaining suppliers, and the supplier's posterior on the remaining suppliers' cost distributions). Thus, POST induces a complex stochastic dynamic bidding game with ex ante asymmetric bidders. This raises several interesting managerial questions for POST. How should a supplier decide when to drop out based on the information he or she observes in the bidding process? Would the incumbent and the entrants' equilibrium dropout strategies be different, and if so, how? How would the nature of qualification screening (e.g., the cost of qualification screening and the entrants' probability to pass the qualification screening) affect suppliers' equilibrium strategies? Finally, returning to our main research question, how do we compute the expected procurement cost of POST so that the buyer can compare it with the expected cost of PRE and make a more informed decision on the choice between PRE and POST? We address these questions in the next two sections.

3. Equilibrium Analysis of POST

In this section, we characterize suppliers' equilibrium dropout strategies in POST and then use this characterization to derive the buyer's expected procurement cost under POST. Recall that POST induces a stochastic dynamic game with incomplete information and ex ante asymmetric players. Here, the suppliers have incomplete information because they only know their own true costs but not others'. We use Perfect Bayesian Equilibrium (PBE) as our solution concept. Specifically in the context of POST, all suppliers start with a *common prior* on other suppliers' cost distribution. As the auction proceeds, each supplier observes the descending auction price and when other suppliers drop out until he or she drops out; this forms a sequence of (weakly) increasing information sets as auction proceeds. A dropout strategy is a collection of functions that map the supplier's information sets to an auction price at which the supplier should drop out. To ensure consistency condition of PBE, for any dropout strategy profiles of all suppliers, we use the Bayes rule to construct the posterior beliefs given the common prior (before the bidding starts) and all suppliers' strategies. Then, we say a collection of dropout strategy profiles along with the beliefs constructed above for this collection of dropout strategy profiles form a PBE if the strategies are *sequentially rational*. Because the entrants are ex ante symmetric, we focus on the PBE where entrants' equilibrium strategies are symmetric. In the remainder of this section, we first characterize the entrants' and the incumbent's equilibrium dropout strategies in POST (Sections 3.1 and 3.2), which are then used to characterize the buyer's expected total cost under POST (Section 3.3).

3.1. Entrants' Dropout Strategy

Since the incumbent is a qualified supplier, an entrant faces very different competition before and after the incumbent drops out. Thus, we characterize entrants' equilibrium dropout strategy before and after the incumbent drops out separately.

3.1.1. Entrants' Bidding Before Incumbent Drops Out.

By Lemma 1, an entrant can win the contract only if he or she drops out later than the incumbent. However, in POST, the auction price immediately drops by $\Delta = K/\beta$ when the incumbent drops out; therefore, to avoid the possibility of winning the contract but losing money, as long as the incumbent is still in the auction, an entrant supplier j should drop out no later than when the auction price reaches his *effective cost*, defined as $x_j + K/\beta$. (We assume that $K/\beta < \overline{R} - \underline{R}$ to avoid the trivial situation that no entrant would like to join the auction because all *effective costs* are higher than the auction starting price.) By a similar argument as in the analysis for a standard open-bid descending auction, Lemma 2 establishes the entrants' dominant bidding strategy before the incumbent drops out.

Lemma 2. *Before the incumbent drops out, it is a weakly dominant strategy for the entrants to stay in the auction until the auction price clock reaches their effective costs.*

3.1.2. Entrants' Bidding After Incumbent Drops Out.

After the incumbent drops out, although the remaining bidding process only involves ex ante symmetric suppliers (i.e., the remaining entrants), their equilibrium dropout strategy is more complicated than a standard open-bid descending auction because any of the remaining entrant may lose an auction to other remaining entrants but still win the contract if all later dropout entrants fail to pass the qualification screening; thus, an entrant has an incentive to drop out at a price higher than his or her cost. For each of the remaining entrants, at what price to drop out depends on not only the entrant's true cost but also how many competing entrants are still in the auction and his or her posterior on these entrants' costs; of course, the price to drop out should also be dynamically updated as other competing entrants drop out over time. In the remainder of this subsection, given the incumbent's dropout price and the number of remaining entrants, we characterize the symmetric equilibrium (dynamic) bidding strategy for the remaining entrant suppliers after the incumbent drops out.

Formally, consider the scenario where there are $n \in \{1, \cdots, N\}$ remaining entrants after the incumbent drops out. Let $p^n_{(n)}$ denote the auction price right after the incumbent drops out: $p^n_{(n)}$ equals the incumbent's dropout price minus $\Delta = K/\beta$. Moreover, for all $k = 1, \cdots, n-1$, let $p^n_{(k)}$ denote the $(n-k)^{th}$ dropout price among the remaining n entrants and $c^n_{(k)}$ denote the cost of the $(n-k)^{th}$ dropout entrant. Suppose now that there are only k entrants in the auction, and entrant j is one of them. At this point, the price b_j at which entrant j should drop out should depend on all the payoff relevant information that he or she observes so far, which includes not only the entrant's cost x_j and the number of remaining entrants k but also the entrant's posterior of the remaining entrants' costs; apparently, this posterior is determined by the previous dropout prices and the current auction price, as well as entrant j's belief of other entrants' dropout strategies. Below, we first introduce a class of *Active-Bidder-Number Dependent Threshold strategy* (ABN) and then construct an ABN that, along with the remaining entrants' posteriors when the incumbent drops out, forms a symmetric perfect Bayesian Equilibrium in the bidding process after the incumbent drops out.

Definition 1. An ABN is determined by a sequence of $n-1$ functions $\{\tilde{s}_k(x_j;\bar{c},\bar{p})\}^n_{k=2}$, where $\tilde{s}_k$ is strictly increasing and differentiable with respect to x_j, and we denote by $\tilde{s}^{-1}_k(\cdot;\bar{c},\bar{p})$ its inverse function. For an entrant with cost x_j, the ABN specified by $\{\tilde{s}_k\}^n_{k=2}$ works as follows:

1: Set $k = n$, $\bar{p} = p^n_{(n)}$, and $\bar{c} = p^n_{(n)}$
2: **if** $k = 1$ **then** *terminate*
3: **else**
4: *compute* a threshold $b_j = \tilde{s}_k(x_j;\bar{c},\bar{p})$
5: **if** one of the other entrants drops out at an auction price $p^n_{(k-1)} > b_j$, **then**
6: *set* $\bar{c} = c^n_{(k-1)} := \tilde{s}^{-1}_k(p^n_{(k-1)};\bar{c},\bar{p})$ and *set* $\bar{p} = p^n_{(k-1)}$,
7: *set* $k = k-1$, and *go* to 2
8: **else if** no entrant drops out before the auction price hits b_j, **then**
9: *drop* out at b_j, and *terminate*
10: **end if**
11: **end if**

The function $\tilde{s}_k(x_j;\bar{c},\bar{p})$ maps an entrant's cost type x_j into a dropout threshold b_j, given the number of remaining entrants k, the most recent dropout price $\bar{p}$, and an estimate of the upper bound of other remaining entrants' costs $\bar{c}$. Note that under the supposition that all n entrant suppliers who drop out after the incumbent use the ABN characterized by the same set of functions $\{\tilde{s}_k\}_{k=2}^n$; $\bar{c}$ determined in ABN is consistent with entrants' posterior on other remaining entrants' cost distribution. To see that, recall that by definition, $\tilde{s}_k$ is increasing in x_j; this implies that entrants with higher cost types drop out first. Therefore, when the highest cost entrant among k (for any $2 \le k \le n$) remaining entrants drops out at $p^n_{(k-1)}$, all $k-1$ remaining entrants would correctly deduce that the cost of the entrant who just dropped out equals $c^n_{(k-1)} = \tilde{s}_k^{-1}(p^n_{(k-1)};\bar{c},\bar{p})$, and it coincides with the posterior of the upper bound of the support of the $k-1$ remaining entrants' cost distribution. Similarly, when the incumbent drops out with an auction price of $p^n_{(n)}$(recall that it equals the incumbent's dropout price minus Δ), the upper bound of the support of the n remaining entrants' cost distribution equals $p^n_{(n)}$, as is implied by Lemma 2. Having explained above that all n remaining entrants using the same ABN ensures the *consistency* condition for PBE, we now construct the ABN that ensures *sequential rationality* condition given entrants' beliefs. Toward that end, recall that we are considering a scenario where there are exactly n remaining entrants when the incumbent drops out, and the auction price (right after a price decrement of Δ from the incumbent's dropout price) is $p^n_{(n)}$. Our goal is to find $\{\tilde{s}_k\}_{k=2}^n$ such that, given that all other remaining entrants employ the ABN specified by $\{\tilde{s}_k\}_{k=2}^n$, entrant j's best response coincides with the ABN specified by $\{\tilde{s}_k\}_{k=2}^n$, thus ensuring sequential rationality. Note that entrant j's optimal dropout problem can be formulated as a stochastic dynamic program. Specifically, denote by $G_e^k(z;\bar{c}) := [F_e(z)/F_e(\bar{c})]^k$ and $g_e^k(z;\bar{c}) := k[F_e(z)]^{k-1}f_e(z)/[F_e(\bar{c})]^k$, respectively, the c.d.f. and p.d.f. of the highest cost out of the k remaining entrants' costs when their costs' upper bound is $\bar{c}$, and denote by $V_e^k(x_j,p;\bar{c},\bar{p})$ the maximum profit entrant j can make when there are k active entrants in the auction where the current auction price is p, the most recent dropout occurs when the auction price hits $\bar{p}$, and the latest drop-out supplier's cost is $\bar{c}$ (recall that assuming other entrants follow the ABN, entrant j can correctly infer that $\bar{c} = \tilde{s}_{k+1}^{-1}(p^n_{(k)};\bar{c},\bar{p})$ when $k \le n-1$ and $\bar{c} = p^n_{(n)}$ when $k = n$). Then, entrant j's optimal dropout strategy can be obtained by solving the Bellman equations below. For all $k = 2,\ldots,n$,

$$V_e^k(x_j,p;\bar{c},\bar{p}) = \max_{b_j\in[x_j,\,p]} \Bigg[\underbrace{\int_{\tilde{s}_k^{-1}(b_j;\bar{c},\bar{p})}^{\tilde{s}_k^{-1}(p;\bar{c},\bar{p})} V_e^{k-1}(x_j,\tilde{s}_k(z;\bar{c},\bar{p});z,\tilde{s}_k(z;\bar{c},\bar{p}))g_e^{k-1}(z;\tilde{s}_k^{-1}(p;\bar{c},\bar{p}))dz}_{\text{when an entrant drops out before entrant } j \text{ (i.e., before auction price reaches } b_j)} + \underbrace{\beta G_e^{k-1}(\tilde{s}_k^{-1}(b_j;\bar{c},\bar{p});\tilde{s}_k^{-1}(p;\bar{c},\bar{p}))(1-\beta)^{k-1}(b_j-x_j)}_{\text{when no other entrant drops out before entrant } j}\Bigg], \quad (3)$$

$$\text{and } V_e^1(x_j,p;\bar{c},\bar{p}) = \beta(p-x_j). \quad (4)$$

Theorem 1 below establishes, by construction, that an ABN forms a pure strategy symmetric PBE for the dynamic bidding game that ensues if the incumbent drops out at $p^n_{(n)}+\Delta$ and with n remaining entrants.

Theorem 1. *For any $p \in [\underline{R}+\Delta,\bar{R}]$ and any $n = 1,\ldots,N$, suppose that the incumbent drops out at price p with n remaining entrants still in the auction and that it is common knowledge that all remaining entrants' costs are no higher than $p^n_{(n)}$, where $p^n_{(n)} := p-\Delta$, then the following holds:*

i. *The ABN specified by a sequence of $n-1$ functions $\{\tilde{s}_k\}_{k=2}^n$, defined below along with the belief system that is consistent with ABN, and the prior that all n entrants' costs are below $p^n_{(n)}$ forms a symmetric PBE: For all $k = 2,\ldots,n$, $x_j \in [\underline{R},\bar{c}]$, $\underline{R} \le \bar{c} \le \bar{p} \le \bar{R}$, and*

$$\frac{\partial \tilde{s}_k(x_j;\bar{c},\bar{p})}{\partial x_j} = \frac{\beta(k-1)f_e(x_j)(\tilde{s}_k(x_j;\bar{c},\bar{p})-x_j)}{(1-\beta)F_e(x_j)}, \quad (5)$$

with the boundary condition

$$\tilde{s}_k(\bar{c};\bar{c},\bar{p}) = \bar{p}. \quad (6)$$

ii. *$\tilde{s}_k(x_j;\bar{c},\bar{p})$ decreases in k for all $x_j \in [\underline{R},\bar{c})$, decreases in $\bar{c}$, and increases in x_j and $\bar{p}$.*

iii. *$\tilde{s}_k(x_j;\bar{c},\bar{p}) > x_j$ for all $x_j \in [\underline{R},\bar{c})$.*

iv. *If $x_j < c^n_{(k-1)}$, then $\tilde{s}_k(x_j;c^n_{(k)},p^n_{(k)}) < \tilde{s}_{k-1}(x_j;c^n_{(k-1)},p^n_{(k-1)})$.*

Several remarks are in order. First, note that after the incumbent drops out, by Lemma 2 and the Bayes rule, all remaining entrants infer that their competitors' costs are no higher than $p^n_{(n)}$; thus, Lemma 2 and Theorem 1 jointly characterize entrant suppliers' equilibrium bidding strategy. It is worth noting that we do not need to explicitly characterize the incumbent's equilibrium bidding strategy in order to characterize the entrants' equilibrium bidding strategy. This is because entrants have dominant bidding strategies before the incumbent drops out; once the incumbent drops out, the remaining entrant's expected payoff depends on the incumbent's bidding strategy only through the incumbent's dropout price. Second, note that Theorem 1 part (iii) implies that after the incumbent drops out, in equilibrium all entrants will drop out at a price higher than their costs; this is because any of the remaining entrants are incentivized to bid less aggressively because they anticipate that their competing entrants who drop out later than them may not always pass qualification screening. Third, recall that $\tilde{s}_k(x_j; c^n_{(k)}, p^n_{(k)})$ is entrant j's dropout threshold when there are exactly k entrants in the auction. Thus, Theorem 1 part (iv) implies that after the incumbent drops out, every time an entrant drops out results in an increase of all remaining entrants' dropout thresholds. In other words, once the incumbent drops out, all remaining entrants start to increasingly hold back their aggressiveness on bidding as more entrants drop out. As a result, the buyer should not safely assume that intense competition would naturally occur in POST simply because more entrants joined the bidding process. Instead, the buyer should take into account the incentive implications of postbidding qualification screenings on supplier bidding behavior when evaluating what final contract price to expect from POST. Fourth, we would like to point out that although the entrant suppliers' optimal dropout problem is a high-dimensional stochastic dynamic program and is computationally difficult to solve in brute force, we show in Theorem 1 part (i) that instead of solving the dynamic program directly, one only needs to solve a set of one-dimensional first-order ordinary differential equations that can be easily computed numerically. This characterization also facilitates a set of comparative statics results summarized in Theorem 1 part (ii). These comparative statics results are not surprising. For example, when there are a higher number of remaining entrants k, there is more competition, so the entrants bid more aggressively by having lower dropout thresholds. Finally, if the entrants' cost distribution is a uniform distribution, the equilibrium dropout strategy can be derived in closed form.

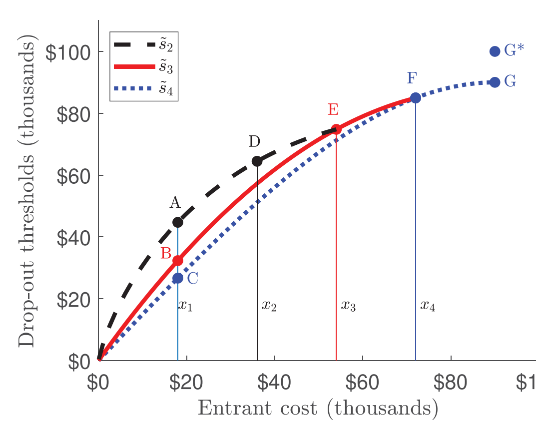
Proposition 1. *When $F_e \sim U[\underline{R}, \overline{R}]$, the ABN in Theorem 1 has a closed-form expression:*

$$\tilde{s}_k(x_j, \bar{c}, \bar{p}) = \begin{cases} \dfrac{\beta(k-1)}{\beta k - 1}(x_j - \underline{R}) + \left(\bar{p} - \underline{R} - \dfrac{\beta(k-1)}{\beta k-1}\right. \\ \qquad \left.(\bar{c} - \underline{R})\right)\left(\dfrac{x_j - \underline{R}}{\bar{c} - \underline{R}}\right)^{\frac{\beta(k-1)}{1-\beta}} + \underline{R}, \quad \text{if } \beta \neq \dfrac{1}{k}, \\ (\bar{p} - \underline{R})\left(\dfrac{x_j - \underline{R}}{\bar{c} - \underline{R}}\right)^{\frac{\beta(k-1)}{1-\beta}} - (x_j - \underline{R}) \\ \qquad \log\left(\dfrac{x_j - \underline{R}}{\bar{c} - \underline{R}}\right) + \underline{R}, \quad \text{if } \beta = \dfrac{1}{k}. \end{cases} \tag{7}$$

We illustrate below an entrant's equilibrium dropout strategy after the incumbent drops out.

Example 1. Suppose an incumbent competes with four entrants whose costs are uniformly distributed between \$0 and \$100,000. The qualification cost is $K = \$5{,}000$, and the passing probability is $\beta = 0.5$. In this setting, the buyer sets $\Delta = K/\beta = \$10{,}000$. Suppose the incumbent drops out first at $p = \$100{,}000$ and leaves behind four entrants in the auction. We now take the perspective of entrant 1 with cost $x_1 = \$18{,}000$ to illustrate the entrant's equilibrium dropout strategy (see Figure 1). As soon as the incumbent drops out (point $G*$ in Figure 1), the auction price immediately drops by Δ and becomes $\$100{,}000 - \$5{,}000/0.5 = \$9{,}000$ (point G in Figure 1). Then, entrant 1 infers that all other entrants' costs are no higher than $\bar{c} = \$90{,}000$ and then computes a dropout threshold according to (7): $\tilde{s}_4(\$18{,}000; \$90{,}000, \$90{,}000) = \$26{,}640$ (point C in Figure 1). Suppose that before the auction price reaches entrant 1's threshold, entrant 4 drops out first at \$84,960 (point F in Figure 1). Entrant 1 updates his belief about the remaining entrants' cost upper bound to $\bar{c} = \tilde{s}_4^{-1}(\$84{,}960;\ \$90{,}000, \$90{,}000) = \$72{,}000$ (note that this also equals entrant 4's cost) and then recalculates his or her dropout threshold as $\tilde{s}_3(\$18{,}000; \$72{,}000, \$84{,}960) = \$32{,}310$ (point B in Figure 1). Suppose that entrant 3 drops out at \$74,790 (point E in Figure 1) before the auction price reaches

Figure 1. (Color online) Illustration of the Dynamics of an Entrant's Equilibrium Strategy After the Incumbent Drops Out



Notes. In this example, F_e follows uniform distribution $U[\$0, \$100{,}000]$, $K = \$5{,}000$, $\beta = 0.5$. Moreover, the incumbent drops out at \$100,000, and the remaining four entrants' costs equal $x_1 = \$18{,}000$, $x_2 = \$36{,}000$, $x_3 = \$54{,}000$, and $x_4 = \$72{,}000$, respectively. Point D (resp., $E, F, G*$) corresponds to entrant 2's (resp., entrant 3's, entrant 4's, incumbent's) dropout price. Point G corresponds to the auction price after the incumbent drops out. Point A (resp., B, C) corresponds to entrant 1's dropout price when there are 2 (resp., 3, 4) entrants in the auction in this example.

entrant 1's updated dropout threshold, leaving behind only entrants 1 and 2. This time, the updated cost upper bound becomes $\bar{c} = \tilde{s}_3^{-1}(\$74,790; \$72,000, \$84,960) = \$54,000$, and entrant 1's dropout threshold becomes $\tilde{s}_2(\$18,000; \$54,000, \$84,960) = \$44,710$ (point A in Figure 1). Finally, the other remaining entrant drops out at \$64,460 (point D in Figure 1), making entrant 1 the auction winner.

Apparently, the incentive implications on suppliers' dropout prices also depend on the model parameters predicated on the business context. We provide insight on this in our next result.

Proposition 2. *Fix any $x_j \in [\underline{R}, \bar{c})$, $\bar{c}$, and $\bar{p}$, $\tilde{s}_k(x_j, \bar{c}, \bar{p})$ decreases in β and is independent of K.*

As the probability of passing qualification β increases, there is a higher chance that at least one of the entrants with a lower dropout price would pass qualification and win the contract; thus, all remaining entrants have more incentive to bid more aggressively by lowering their dropout thresholds. In contrast, because the qualification cost K is the same for all of the remaining competing entrants after the incumbent drops out, it does not affect the competition among these entrants or their bidding incentives after the incumbent drops out.

3.2. Incumbent's Dropout Strategy

Given the entrants' equilibrium dropout strategy characterized in the previous subsection, we now analyze the incumbent's equilibrium dropout strategy. Recall that by Lemma 2, before the incumbent drops out, it is a weakly dominant strategy for entrants to stay in the auction until the auction price reaches their effective costs. Therefore, at any auction price p, the incumbent infers that all remaining entrants' costs are no higher than $p - \Delta$. Note that once the incumbent drops out, his or her chance of getting the contract is fully determined by the number of remaining entrants; that is, the incumbent wins the contract if and only if all remaining entrants fail their qualifications. Therefore, in POST, as the auction price descends, the incumbent needs to decide when to drop out based on the number of the remaining entrants and the incumbent's belief about the remaining entrants' cost distribution.

Denote by $V_0^m(x_0, p)$ the incumbent's maximum expected profit and denote by $s_m^\star(x_0; p)$ the incumbent's best response (henceforth, optimal dropout price) when his or her cost is x_0, the current auction price is p, and there are exactly m remaining entrants who follow their equilibrium dropout strategy characterized in Lemma 2 and Theorem 1. Note that it is without loss of optimality to restrict the set of the incumbent's feasible dropout price to be $[\max\{\underline{R} + \Delta, x_0\}, p]$ because dropping out below $\underline{R} + \Delta$, which equals the lowest possible dropout price from entrants, is never optimal. Hence, given entrants' equilibrium dropout strategy, the incumbent's best response can be found by solving the profit optimization problem characterized by the following Bellman equations. For all $m = 1, \ldots, N$, $x_0 \in [\underline{R}, \bar{R}]$, $p \in [\max\{\underline{R} + \Delta, x_0\}, \infty)$,

$$V_0^m(x_0, p) = \max_{b \in [\max\{\underline{R}+\Delta, x_0\}, p]} \Pi_0^m(b; x_0, p) = \Pi_0^m(s_m^\star(x_0; p); x_0, p), \tag{8}$$

where for all $m = 1, \ldots, N$, $x_0 \in [\underline{R}, \bar{R}]$, $p \in [\max\{\underline{R} + \Delta, x_0\}, \infty)$, $b \in [\max\{\underline{R} + \Delta, x_0\}, \infty)$,

$$\Pi_0^m(b; x_0, p) := \int_b^p V_0^{m-1}(x_0, z) g_e^m(z - \Delta; p - \Delta)\, dz + G_e^m(b - \Delta; p - \Delta)(1 - \beta)^m (b - x_0), \tag{9}$$

and the boundary conditions are as follows. For all $x_0 \in [\underline{R}, \bar{R}]$, $p \in [\max\{\underline{R} + \Delta, x_0\}, \infty)$,

$$V_0^0(x_0, p) = p - x_0. \tag{10}$$

Theorem 2 below fully characterizes the incumbent's equilibrium dropout strategy.

Theorem 2. *For all $x_0 \in [\underline{R}, \bar{R}]$, define*

$$m^\star(x_0) := \left\lfloor \min_{b \in [\max\{x_0, \underline{R}+\Delta\}, \bar{R}+\Delta]} \frac{(1-\beta)F_e(b-\Delta)}{\beta(b - x_0) f_e(b - \Delta)} \right\rfloor;$$

moreover, for all $x_0 \in [\underline{R}, \bar{R}]$ and $m > m^\star(x_0)$, define

$$b_m^\star(x_0) := \begin{cases} \underline{R} + \Delta, & \text{if } x_0 \le \underline{R} + \Delta \\ \inf\left\{ b \in (x_0, \bar{R} + \Delta] : \dfrac{F_e(b-\Delta)}{f_e(b-\Delta)} - \dfrac{m\beta(b - x_0)}{1-\beta} < 0 \right\}, & \text{if } x_0 > \underline{R} + \Delta, \end{cases}$$

$$p_m^\star(x_0) := \min\{b \in (b_m^\star(x_0), \infty] : \Pi_0^m(b; x_0, \bar{R} + \Delta) - \Pi_0^m(b_m^\star(x_0); x_0, \bar{R} + \Delta) \ge 0\}.$$

Suppose that the entrants follow the dropout strategy characterized in Lemma 2 and Theorem 1. Then the following statements hold:

i. *$m^\star(x_0), b_m^\star(x_0), p_m^\star(x_0)$ are well-defined, $p_m^\star(x_0) > b_m^\star(x_0) \ge x_0$, $b_m^\star(x_0) > x_0$ when $x_0 \ne \underline{R} + \Delta$, $m^\star(x_0)$ is nondecreasing in x_0, $b_m^\star(x_0)$ is nondecreasing in x_0 and nonincreasing in m, and $p_m^\star(x_0)$ is nonincreasing in x_0 and nondecreasing in m*

ii. *Given the incumbent's cost x_0, the number of remaining entrants m, and the current auction price p, the incumbent's optimal drop-out price is*

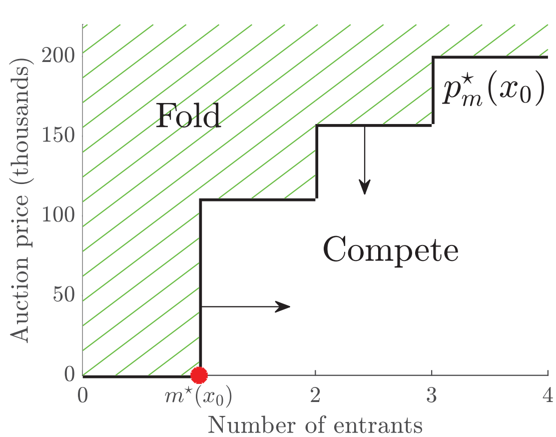
$$s_m^\star(x_0; p) = \begin{cases} b_m^\star(x_0) \wedge p, & \text{if } m > m^\star(x_0), \quad p \le p_m^\star(x_0) \\ p, & \text{otherwise} \end{cases};$$

moreover, $s_m^\star(x_0; p)$ is nondecreasing in x_0 and p and nonincreasing in m.

Before explaining the managerial insights revealed in Theorem 2, we would first like to point out that Lemma 2, Theorem 1, and Theorem 2 jointly establish the suppliers' equilibrium dropout strategies in POST. Indeed, if the entrants follow the symmetric strategy characterized in Lemma 2 and Theorem 1, Theorem 2 characterizes the incumbent supplier's best response. Conversely, if the incumbent follows the strategy characterized in Theorem 2, Lemma 2 and Theorem 1 characterize the entrants best response. It is a weakly dominant strategy for entrants to follow Lemma 2 before the incumbent drops out; after the incumbent drops out, by Lemma 2 and the Bayes rule, all remaining entrants infer the common posterior that all the remaining entrants' costs are Δ lower than the incumbent's dropout price, so the conditions in Theorem 1 are satisfied.

Theorem 2 reveals that when there are multiple entrants, the incumbent's equilibrium dropout strategy is a state-dependent dynamic strategy characterized by a threshold function $b_m^\star(x_0)$ and two switching curves, $p_m^\star(x_0)$ and $m^\star(x_0)$. Specifically, Theorem 2 part (ii) shows that, given the current state of the auction, that is, the number of remaining entrants m and the current auction price p, an incumbent with cost x_0 should follow either of the two options: the "compete" option where the incumbent remains in the auction until the first time the auction price drops below a threshold $b_m^\star(x_0)$ or the "fold" option where the incumbent immediately drops out at current auction price p and forfeits the chance of winning the auction. Figure 2 illustrates the incumbent's equilibrium dropout strategy. The threshold-type strategy in the compete option is not unusual in open-bid auctions; however, when would giving up winning the auction, the fold option, be helpful to the incumbent? Note that with the fold option, the incumbent can still win the contract if the remaining entrants all fail supplier qualification screenings. Thus, the fold option can be optimal when there is a sufficient chance that all m entrants fail qualifications (i.e., when the number of remaining entrants is small, $m \le m^\star(x_0)$). Moreover, the fold option is optimal when there are sufficient numbers of remaining entrant suppliers and the current auction price is high, that is, $m > m^\star(x_0)$ and $p \ge p_m^\star(x_0)$. In this case, dropping out at a slightly lower price than p does not significantly reduce the number of entrants that remain in the auction after the incumbent drops out, so the incumbent's chance of ultimately winning the contract does not increase much; in the meantime, a lower dropout price would definitely decrease the incumbent's profit upon winning the contract. Hence, it is optimal to fold by dropping out at p immediately. Finally, the comparative statics result of $b_m^\star(x_0)$ and $p_m^\star(x_0)$ with respect to x_0 established in Theorem 2 part (i) shows that as x_0 increases, the region where fold option is better expands (see the arrows in Figure 2 for an illustration). This is because when the incumbent's cost is high, dropping out at a lower price than p is even less appealing since the same absolute reduction in the incumbent's bid results in a larger percentage decrease in profit margin upon winning.

Figure 2. (Color online) The "Fold" and "Compete" Regions of the Incumbent's Equilibrium Dropout Strategy

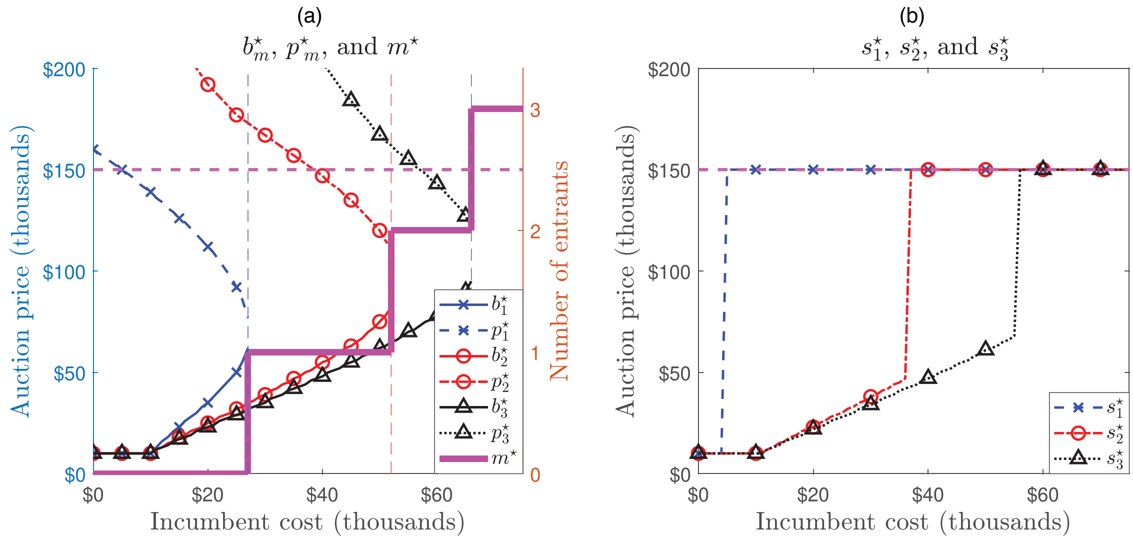


Notes. In this example, $x_0 = \$52{,}000$, F_e follows normal distribution $N(\$30{,}000, \$50{,}000)$ truncated at $\underline{R} = \$0$ and $\overline{R} = \$150{,}000$, $K = \$6{,}700$, and $\beta = 0.61$. The arrows show how the boundary of the two regions changes as x_0 increases.

Our full characterization of the structure of the incumbent's dropout strategy is very useful for two main reasons. First, from the managerial perspective, the incumbent's *state-dependent dynamic* equilibrium strategy in Theorem 2 is a nontrivial generalization of the *static* equilibrium strategy characterized in Wan et al. (2012). By generalizing Wan et al. (2012)'s model to a more realistic setting with multiple entrants, we can tease out how the dynamically evolving competition landscape (e.g., the number of remaining entrants m) affects the incumbent's bidding incentives. Specifically, Theorem 2 part (ii) (resp. (i)) shows that the incumbent's optimal dropout price (resp. threshold $b_m^\star(x_0)$) is nonincreasing in m (see Figure 3 for an illustration of how the equilibrium bidding function for all incumbent cost types changes as m varies). Intuitively, as m increases, the incumbent is faced with more competition from the entrants, so the incumbent should bid more aggressively. Note that such reduction in incumbent's dropout price can be quite significant for some cost types. For example, as illustrated in Figure 3, when the incumbent's cost is \$30,000, as the number of entrants increases from 1 to 2, the incumbent's dropout price reduces from \$150,000 to \$38,000, a 76.67% reduction in dropout price! Recall that from the buyer's perspective, the main drawback of POST is that inviting entrants to an auction will not always necessarily result in aggressive biddings from the incumbent because the incumbent may take the advantage of his or her qualified status to behave opportunistically by inflating his or her bid (the fold option discussed previously). Our observation seems to suggest that such opportunistic behavior may get mitigated significantly as more entrants participate in the auction. This motivates us to numerically explore how the number of entrant suppliers affects the buyer's choice between PRE and POST in more depth in Section 4.

Second, from a technical implementation perspective, Theorem 2 can greatly reduce the computational

Figure 3. (Color online) Illustration of the Incumbent's Equilibrium Dropout Strategy



Notes. In this example, F_e follows normal distribution $N(\$30{,}000,\ \$50{,}000)$ truncated at $\underline{R} = \$0$ and $\overline{R} = \$200{,}000$, $K = \$6{,}700$, and $\beta = 0.61$. The plot on the left illustrates the switching curves that determines incumbent's equilibrium dropout strategies when there are $m = 1, 2, 3$ entrants, respectively. The plot on the right illustrates the incumbent's equilibrium dropout price curves when the current auction price is \$150,000 and there are $m = 1, 2, 3$ entrants, respectively.

complexity of the incumbent's strategy. We would like to point out that the dynamic program in (8)–(10) is very difficult to solve computationally because the objective function $\Pi_0^m(b; x_0, p)$ in (8) is not necessarily unimodal in the decision variable b! In fact, the lack of unimodal structure is the main reason that the incumbent's equilibrium dropout strategy has the two separate options compete and fold. Fortunately, Theorem 2 part (ii) implies that the optimal solution to (8) can only take two possible values, $b_m^\star(x_0)$ and p, both of which can be easily computed. (Mathematically, this result is based on our observation that when Π_0^m has a local maximum, that is, $b_m^\star(x_0)$, the objective is increasing for all $b < b_m^\star(x_0)$ and for all $b > p_m^\star(x_0)$, where $p_m^\star(x_0)$ is the smallest point on the real line to the right of $b_m^\star(x_0)$ such that $\Pi_0^m(b_m^\star(x_0); x_0, p) = \Pi_0^m(p_m^\star(x_0); x_0, p)$.) What is nice about this structure of the optimal solution is that when solving the dynamic program, instead of optimizing a nonconvex optimization for each state, we now only need to compare the objective value of two easily computable points. This greatly speeds up the computation of the incumbent's equilibrium dropout strategy, which makes our numerical study in Section 5 feasible.

Next, we investigate how the incumbent's optimal dropout decision depends on the model parameters (i.e., K and β). We first study the impact of qualification cost K on the incumbent's optimal dropout price. Recall that Lemma 2 states that before the incumbent drops out, all entrants will drop out at their effective costs $x_j + \Delta$. As K decreases, all entrants' effective costs decrease. This means that the incumbent is effectively faced with entrants with more competitive costs; as a result, one may expect the incumbent to bid more aggressively by decreasing his or her dropout prices. Interestingly, our next example shows a nonmonotonic change of the incumbent's optimal dropout price as K varies.

Example 2. We consider that an example with $\underline{R} = \$0$, $\overline{R} = \$100{,}000$, F_e, and F_0 follows uniform distribution $U[\$0, \$100{,}000]$, and $\beta = 0.7$. By abuse of notation, we use $s_m^\star(x_0; p; K)$ to capture the dependence of $s_m^\star(x_0; p)$ on K. Suppose there are two entrants and the current auction price is $p = \$100{,}000$. We can compute an incumbent's optimal dropout prices under two different qualification costs, $K = \$5{,}000$ and $K = \$25{,}000$, as follows:

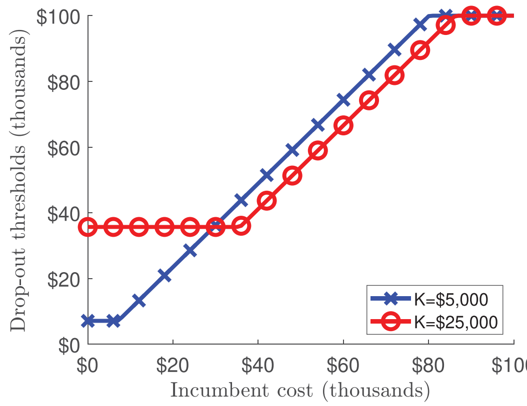
$$s_2^\star(x_0; \$100{,}000; \$5{,}000) = \begin{cases} \$7{,}143, & \text{if } x_0 \in [\$0, \$7{,}143], \\ 1.273x_0 - \$1{,}950, & \text{if } x_0 \in [\$7{,}143, \$80{,}086], \\ \$100{,}000, & \text{if } x_0 \in [\$80{,}086, \$100{,}000]; \end{cases}$$

and

$$s_2^\star(x_0; \$100{,}000; \$25{,}000) = \begin{cases} \$35{,}714, & \text{if } x_0 \in [\$0, \$35{,}714], \\ 1.273x_0 - \$9{,}750, & \text{if } x_0 \in [\$35{,}714, \$86{,}213], \\ \$100{,}000, & \text{if } x_0 \in [\$86{,}213, \$100{,}000]. \end{cases}$$

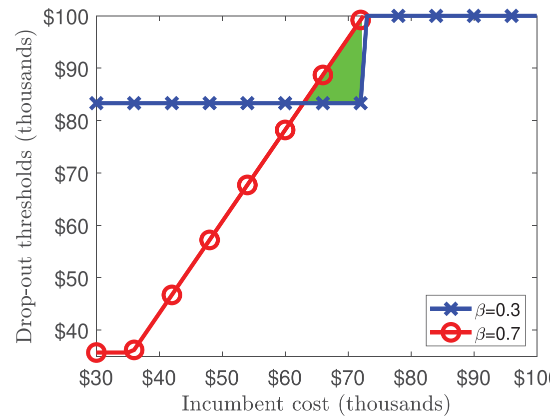
Figure 4 illustrates the optimal dropout thresholds for all incumbent cost types at $K = \$5{,}000$ and $K = \$25{,}000$, respectively. It shows that as the qualification cost

Figure 4. (Color online) The Impact of Qualification Cost K on the Incumbent's Optimal Dropout Prices



Note. In this example, F_e follows uniform distribution $U[\$0, \$100{,}000]$, $\beta = 0.7$, and $m = 2$.

Figure 5. (Color online) The Impact of Qualification Passing Probability β on the Incumbent's Optimal Dropout Prices



Note. In this example, F_e follows uniform distribution $U[\$0, \$100{,}000]$, $K = \$25{,}000$, and $m = 1$.

increases from \$5,000 to \$25,000, the incumbent will increase his or her optimal dropout price if his or her cost is $x_0 < \$29{,}586$ but will decrease or keep the same dropout price if his or her cost is $x_0 \geq \$29{,}586$.

The reason behind this nonmonotonic effect of K is due to the tradeoff between the compete and fold options identified in Theorem 2. Although the entrants become effectively more competitive in costs when K decreases, the incumbent may have different types of reactions, depending on his or her cost type. For an incumbent with a low cost type, he or she responds by bidding more aggressively. But for an incumbent with a high cost type, lowering his or her dropout price will result in a big percentage reduction in his or her profit margin upon winning but would not significantly reduce the number of entrants who drop out later than him or her (i.e., the incumbent's winning probability does not improve significantly). Hence, the incumbent will shy away from intense competition by dropping out at a high price when K decreases.

Next, we study how the entrants' qualification passing probability β affects the incumbent's optimal dropout prices. As β increases, the entrants are more likely to pass the qualification screening, so the incumbent is faced with more risk of losing the contract. As a result, intuitively, one would expect the incumbent to lower his or her bid; that is, the incumbent's optimal dropout price should decrease in β. Somewhat surprisingly, our next example shows that this intuition is not always true. For some incumbent cost types, the optimal dropout prices actually increase in β.

Example 3. We consider that an example with $\underline{R} = \$0$, $\overline{R} = \$100{,}000$, F_e and F_0 follows uniform distribution $U[\$0, \$100{,}000]$, and $K = \$25{,}000$. By abuse of notation, we use $s^\star_m(x_0; p; \beta)$ to capture the dependence of $s^\star_m(x_0; p)$ on β. Suppose there is only one entrant and the current auction price is $p = \$100{,}000$. We can compute an incumbent's optimal dropout prices under two different passing probabilities, $\beta = 0.3$ and $\beta = 0.7$, as follows:

$$s^\star_1(x_0; \$100{,}000; 0.3) = \begin{cases} \$83{,}333, & \text{if } x_0 \leq \$72{,}222 \\ \$100{,}000, & \text{if } x_0 > \$72{,}222 \end{cases},$$

and

$$s^\star_1(x_0; \$100{,}000; 0.7) = \begin{cases} \$35{,}714, & \text{if } x_0 \in [\$0, \$35{,}714] \\ 1.75x_0 - \$26{,}785, & \text{if } x_0 \in [\$35{,}714, \$72{,}449] \\ \$100{,}000, & \text{if } x_0 \in [\$72{,}449, \$100{,}000] \end{cases}.$$

Figure 5 illustrates the optimal dropout prices for all incumbent cost types under $\beta = 0.3$ and $\beta = 0.7$. It shows that when the incumbent's cost is $x_0 \in (\$62{,}925, \$72{,}222)$, his or her optimal dropout price increases as the passing probability increases from 0.3 to 0.7.

The reason behind this nonmonotonic effect of β is similar to that of K. As β increases, there is a higher risk of losing the contract if the incumbent drops out at a high price, so the incumbent has the incentive to decrease his or her dropout price. However, there is also another competing effect. As β increases, all entrants' effective costs decrease. Similar to the rationale behind the fold option, this intensified cost competition incentivizes the incumbent with moderate or high cost to bid less aggressively. When this latter effect dominates the former effect, the incumbent's optimal dropout price increases in β.

3.3. Buyer's Expected Procurement Cost

Having established the suppliers' equilibrium dropout strategies, we now derive the buyer's expected procurement cost under POST. Denote by $\mathcal{PC}_{post}(x_0)$

the buyer's procurement cost under POST when the incumbent's cost is x_0. The buyer's expected procurement cost under POST equals

$$PC_{post} = \int_{\underline{R}}^{\overline{R}} \mathrm{E}\big[\mathcal{PC}_{post}(x_0)\big]\, dF_0(x_0). \tag{11}$$

To derive a closed-form expression for $\mathbf{E}[\mathcal{PC}_{post}(x_0)]$, we utilize the structural properties of the incumbent's equilibrium dropout strategy characterized in Theorem 2. Recall that given x_0, the incumbent's dropout strategy is fully characterized by a set of known constants, $m^\star(x_0)$, $\{b_m^\star(x_0)\}_{m=m^\star(x_0)+1}^N$, and $\{p_m^\star(x_0)\}_{m=m^\star(x_0)+1}^N$. Moreover, based on the incumbent's dropout price $\overline{p}$, we can use the incumbent's equilibrium dropout strategy to infer how many entrant suppliers remain in the auction when the incumbent drops out. For example, if $\overline{p} = b_m^\star(x_0)$, then it means that, with probability one, there are exactly m entrants in the auction when the incumbent drops out. This observation motivates us to divide all possible incumbent dropout prices into different regions. To that end, we define some intervals as follows (see Figure 6 for an illustration):

$$\begin{aligned} B_m &:= (b_m^\star(x_0), b_{m-1}^\star(x_0)) \text{ for } m = m^\star(x_0)+2, \cdots, N, \\ &\quad \text{and } B_m := \left(b_m^\star(x_0), \overline{R} \wedge p_m^\star(x_0)\right] \text{ for} \\ &\quad m = m^\star(x_0)+1, \\ P_m &:= (p_m^\star(x_0), p_{m+1}^\star(x_0)] \text{ for} \\ &\quad m = m^\star(x_0)+1, \cdots, \tilde{m}(x_0)-1, \text{ and} \\ &\quad P_m := (p_m^\star(x_0), \overline{R}) \text{ for } m = \tilde{m}(x_0), \end{aligned}$$

where $\tilde{m}(x_0) := \max\{m : m^\star(x_0)+1 \le m \le N, p_m^\star(x_0) \le \overline{R}\}$ when $p_{m^\star(x_0)+1}^\star(x_0) \le \overline{R}$, and $\tilde{m}(x_0) := \min\{m^\star(x_0), N\}$ otherwise.

Then, based on the incumbent's dropout price $\overline{p}$, there are four types of scenarios: (a) $\overline{p} = \overline{R}$, (b) $\overline{p} \in P_m$ for $m = m^\star(x_0)+1, \dots, \tilde{m}(x_0)$, (c) $\overline{p} \in B_m$ for $m = m^\star(x_0)+1, \dots, N$, and (d) $\overline{p} = b_m^\star(x_0)$ for $m = m^\star(x_0)+1, \dots, N$. Based on the incumbent's equilibrium dropout strategy, one can verify that the following relationship between $\overline{p}$ and m is true. Scenario (a) implies that there are fewer than $\tilde{m}(x_0)$ entrants who participate in the auction, so the incumbent leaves behind fewer than $\tilde{m}(x_0)$ entrants in the auction; scenario (b) (resp. (c)) implies that the $m+1^{st}$ (resp. m^{th}) lowest entrant drops out at $\overline{p}$, so the incumbent leaves behind m (resp. $m-1$) entrants in the auction; scenario (d) implies that the incumbent drops out at the threshold $b_m^\star(x_0)$, leaving behind m entrants in the auction. By conditioning on the price at which the incumbent drops out, Proposition 3 below provides a closed-form expression for $\mathbf{E}[\mathcal{PC}_{post}(x_0)]$.

Proposition 3. *For all $m = 1, \dots, N$, denote by $X_{(m:N)}$ the m^{th} smallest cost among all N entrants' costs, and let $X_{(N+1:N)} := \overline{R}$. Then, when the incumbent's cost is x_0, the buyer's expected cost is*

$$\begin{aligned} \mathbf{E}\big[\mathcal{PC}_{post}(x_0)\big] &= \sum_{i=0}^{\tilde{m}} \frac{N!}{i!(N-i)!} [F_e(\overline{R}-\Delta)]^i \\ &\quad \times [1 - F_e(\overline{R}-\Delta)]^{N-i} h(i, \overline{R}) \\ &\quad + \sum_{m=m^\star+1}^{\tilde{m}} \int_{\overline{p} \in P_m} h(m, \overline{p}) \Phi'_{m+1}(\overline{p}) d\overline{p} \\ &\quad + \sum_{m=m^\star+1}^{N} \int_{\overline{p} \in B_m} h(m-1, \overline{p}) \Phi'_m(\overline{p}) d\overline{p} \\ &\quad + \sum_{m=m^\star+1}^{N} \phi_m h(m, b_m^\star), \end{aligned} \tag{12}$$

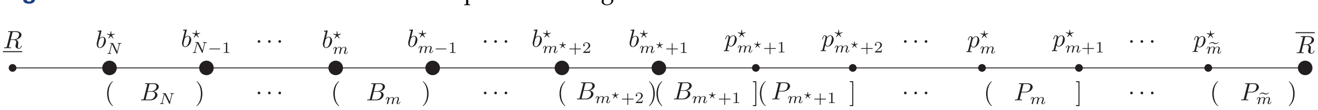
where, for all $m = m^\star(x_0)+1, \dots, N$ and $\overline{p} \in [\underline{R}, \overline{R}]$, we define $h(m, \overline{p}) := \mathbf{E}[\mathcal{PC}_{post}(x_0) | m, \overline{p}]$, $\Phi_m(\overline{p}) := \mathbf{P}(X_{(m:N)} \le \overline{p} - \Delta$, and $X_{(j+1:N)} \le p_j^\star(x_0) - \Delta\ \forall j = m, \dots, \tilde{m}(x_0))$, and $\phi_m := \mathbf{P}(X_{(m:N)} \le b_m^\star(x_0) - \Delta < X_{(m+1:N)}$, and $X_{(j+1:N)} \le p_j^\star x_0 - \Delta\ \forall j = m, \dots, \tilde{m}(x_0))$. (For expositional clarity, the closed-form expressions of $\Phi_m(\overline{p})$, $\phi_m(\overline{p})$, and $h(m, \overline{p})$ are derived in the proof.)

We would like to point out that the distribution of $\overline{p}$ is partly continuous and partly discrete; that is, $\Phi'_m(\overline{p})$ corresponds to the p.d.f of the incumbent's dropout price $\overline{p}$ when $\overline{p} \in B_m$ (or when $\overline{p} \in P_{m-1}$), and ϕ_m corresponds to the probability that the incumbent's dropout price $\overline{p}$ equals $b_m^\star(x_0)$. The derivation of $h(m, \overline{p})$ uses the properties of the entrants' equilibrium droout strategy and the Revenue Equivalence Theorem (Myerson 1981). Finally, Proposition 3 combined with (11) establishes a closed-form expression of PC_{post}.

4. Buyer's Choice Between PRE and POST

The expressions for the buyer's expected cost under PRE and POST, which are characterized in Section 2.1 and Section 3.3, respectively, allow the buyer to evaluate, ex ante, which qualification strategy is cheaper. In this section, we turn to the buyer's strategic choice between PRE and POST and identify conditions under which the buyer is better off using the POST we propose.

Figure 6. Illustration of the Incumbent's Dropout Price Regions



Note. The dependency of $b_m^\star(x_0), p_m^\star(x_0), m^\star(x_0), \tilde{m}(x_0)$ on x_0 is suppressed for notational simplicity.

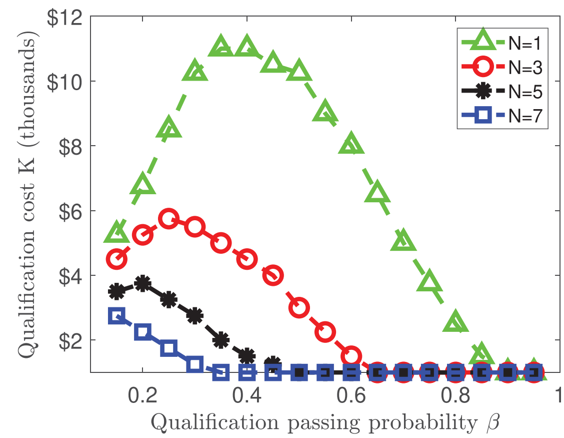
Our analysis and discussions have identified the main cost drivers for the tradeoff between the two qualification strategies. On the one hand, POST reduces the number of qualification screenings needed (and hence, the qualification cost) because the buyer would only select entrants with low bids for qualification screenings; on the other hand, although there are more suppliers competing in bidding in POST, the fact that not all of them have been qualified yet poses incentives for suppliers to bid less aggressively in the hope that lower bids may fail qualification, which may result in higher contract payments than PRE. The strengths of both driving forces depend on the model primitives, such as how well the entrants are positioned to pass the qualification screening (β), the economic burden of conducting qualification screenings (K), and the level of competition (N). Our next result characterizes conditions under which the buyer should choose our proposed POST.

Proposition 4. *Holding all other parameters constant, the buyer should choose POST if*

a. *the likelihood for an entrant to pass qualification screening, β, is sufficiently high;*
b. *the cost of qualification screening, K, is sufficiently high;*
c. *the number of entrants, N, is sufficiently high.*

The intuitions of the results above are closely related to the cost drivers of the tradeoff between PRE and POST identified previously. When entrants have a high chance of passing the qualification screening, β, suppliers face a higher risk of losing the contract by opportunistically holding back their aggressiveness in bidding; thus, the effect of less aggressive bidding in POST is dampened and makes POST more appealing. When each qualification screening costs a fortune for the buyer, the benefit of conducting more informed qualification screening based on suppliers' bids is more pronounced; thus, the buyer should choose POST. Finally, when N is large, POST has the advantage of tapping into a large pool of competing entrants in the bidding process (which effectively mitigate the magnitude of the incentive for less aggressive bidding in POST) without necessarily conducting a lot of qualification screenings. Figure 7 illustrates the buyer's optimal strategic decision on qualification strategy, where each curve corresponds to a given N and shows the optimal decision boundary between PRE and POST. For a given N, the buyer should choose POST if the point that corresponds to K and β lies in the region above the decision boundary. Quite strikingly, when the number of available entrants is more than 7 (i.e., $N > 7$), which is not uncommon in many manufacturing industries, POST outperforms PRE for a very high proportion of all of the combinations of K and β illustrated here. This seems to suggest that introducing POST in a buyer's

Figure 7. (Color online) The Buyer's Optimal Qualification Strategy



Notes. The curves correspond to the decision boundaries for different numbers of all entrant suppliers N. Given N, the buyer should use POST in the area above the corresponding curve and use PRE in the area below that curve.

sourcing tool can be quite powerful. This also motivates us to assess the magnitude of the benefit that POST offers via a comprehensive numerical study in the next section.

5. Assessing the Benefit of POST

Recall that our analysis makes it possible for the buyer to compute the expected costs of PRE and POST ex ante and choose the cheaper option to implement; thus, our proposed approach to the buyer is to use the cheaper option between PRE and POST. How effective is this approach? To assess the benefit of the POST option, we conduct an extensive numerical study, which consists of 19,635 problem scenarios with a wide range of different model parameters (see Table 1 for the factorial design of our numerical study) to answer the following two questions. (i) How much cost savings does our proposed approach provide compared with using PRE alone, and when does our proposed approach offer the most cost savings; and (ii) how much does our proposed approach help in bringing down the procurement cost closer to the optimal mechanism that achieves the lowest cost possible in theory but may not be implementable in practice?

Table 1. Factorial Design of the Numerical Study

Parameters	Values
Cost distribution	F_0 and F_e follow uniform distribution $U[\$0, \$100{,}000]$
Qualification cost K	$\$1{,}000, \$1{,}250, \$1{,}500, \cdots, \$20{,}000$
Qualification passing probability β	$0.15, 0.2, 0.25, \cdots, 0.95$
Number of entrants N	$1, 2, 3, \cdots, 15$
Number of the incumbent	1

Table 2. Summary Statistics for the Relative Cost Savings of the Proposed Strategy over PRE

	Cost-saving (%)
Mean	32.67
Std	21.03
90th percentile	60.14
75th percentile	51.43
50th percentile	34.35
25th percentile	13.03
10th percentile	2.04

5.1. Study 1: Comparing Proposed Approach with PRE

To assess the magnitude of cost savings that our proposed approach offers compared with the conventional approach PRE, we use the following cost-saving metric:

$$\frac{PC_{pre} - \min\{PC_{pre}, PC_{post}\}}{PC_{pre}}.$$

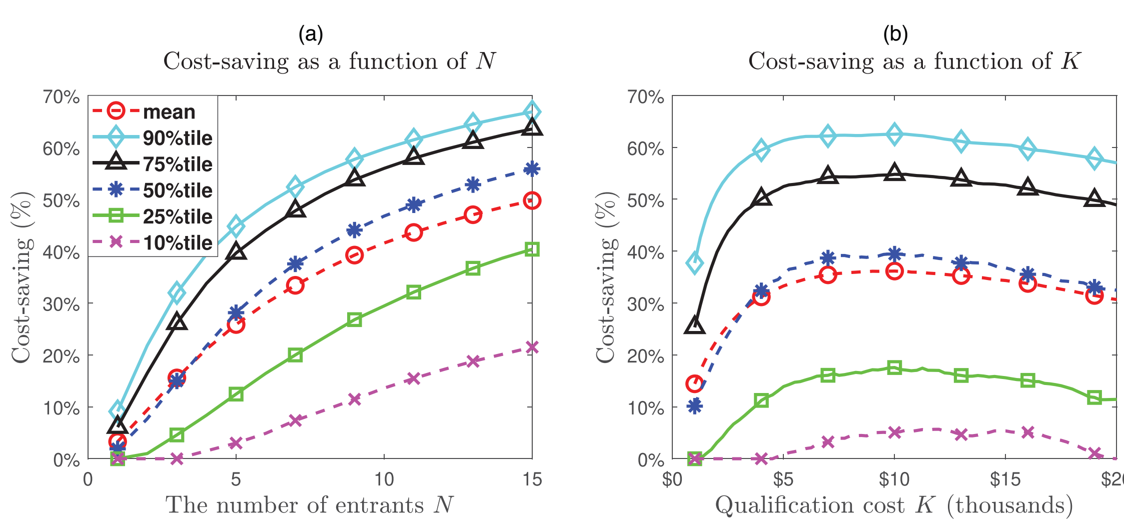
For each scenario, we compute the expected cost of PRE and POST and calculate the metric defined above (see Online Appendix for the detailed computation method and pseudocode); this results in a data set of the cost-saving metric with 19,635 samples, and we report key summary statistics of the cost-saving metrics in Table 2. Note that the cost savings range from 0% (this is when PRE is optimal) to more than 60%, which suggests that, depending on the model parameters, the magnitude of the benefit that our proposed strategy can offer over the conventional approach PRE can be very heterogeneous. Having said that, our proposed strategy can save the buyer 32.67% of the procurement cost under PRE on average. More strikingly, even at the 10% percentile, our proposed strategy still provides a cost saving of 2.04%, which is a quite significant cost improvement by industry standards. This suggests that in many practical scenarios, there is a significant cost improvement opportunity to incorporate POST into buyer's resourcing toolkit.

Motivated by the heterogeneity of cost savings across different scenarios, we next investigate when our proposed strategy offers the highest cost saving compared with PRE. To address this question, in the data set of 19,635 cost-saving metrics we have calculated, we take the subsamples that have the same N (resp. K) and plot their summary statistics as a function of N (resp. K) in Figure 8. Figure 8(a) shows that as the number of entrants N increases, the cost saving also increases, which is consistent with the observation in Figure 7 that when the number of entrants is sufficiently large, the buyer should use POST. Whereas the number of entrants N affects the cost saving monotonically, Figure 8(b) shows that the effect of qualification cost K is nonmonotone: The cost saving first increases and then decreases as K increases. When qualification cost is very small, PRE is already very effective, so our proposed strategy does not improve much; when qualification cost is very large, both PRE and POST will end up qualifying a small number of entrant suppliers, so the benefit of our proposed strategy is also limited.[1] In summary, our proposed strategy is most beneficial when the number of available entrants is large or the qualification cost is moderate.

5.2. Study 2: Comparing Proposed Approach with the Optimal Mechanism

As discussed earlier, both PRE and POST, and hence, our proposed approach, are easy to implement in

Figure 8. (Color online) Relative Cost Savings of the Proposed Strategy over PRE



practice with features that the practitioners are very familiar with. Although being simple is an appealing feature in practice, one may wonder how well our proposed approach compares with more complicated approaches that may be potentially more cost effective. To assess the additional cost savings a buyer could achieve in theory on top of our proposed approach, we now assume that the buyer has full commitment power and conducts an optimal mechanism analysis to the model we introduced in Section 2. In other words, instead of analyzing a particular mechanism such as POST in Section 3, we seek the optimal mechanism among a broader class of feasible mechanisms, where the only constraint we place is that the buyer can only award the contract to qualified suppliers. Note that this class of feasible mechanisms subsumes the mechanisms we have discussed in this paper, where each supplier is allowed to only place one single bid, which the buyer can choose to use (i.e., in POST) or ignore (i.e., in PRE) when conducting qualification screenings. However, it also includes mechanisms that are potentially very hard to implement in practice (e.g., after the qualification screenings of some of the entrant suppliers, the buyer allows the incumbent to reduce his or her bid if an entrant with a lower bid passes qualification screening.) Hence, the optimal mechanism serves as a lower bound of the lowest possible cost the buyer can ever achieve in practice.

To characterize the optimal mechanism, we define the virtual cost for the entrants and the incumbent as $\psi_e(x_i) := x_i + F_e(x_i)/f_e(x_i)$ and $\psi_0(x_0) := x_0 + F_0(x_0)/f_0(x_0)$, respectively.

Proposition 5. *Under the assumption that $\psi_i(x_i)$ is nondecreasing and Assumption 1, an optimal direct, individually rational, an incentive-compatible mechanism that minimizes the buyer's expected total cost is characterized as follows:*[2]

- ***Step 1.*** *The buyer asks all suppliers to report their true costs.*
- ***Step 2.*** *Let i_k be the entrant, with k^{th} lowest cost among all entrants, and suppose there are $L \le N$ entrants whose costs are lower than $\psi_e^{-1}(\psi_0(x_0) - \Delta)$. The buyer conducts qualification screenings among entrants according to the sequence $\{i_k\}_{k=1}^{L}$ until either the buyer finds a qualified entrant or the buyer exhausts the list of all L entrants.*
- ***Step 3.*** *At the conclusion of Step 2, if the incumbent is the only qualified supplier, then the buyer awards the contract to the incumbent and pays him*

$$\beta \sum_{l=L+1}^{N} (1-\beta)^{l-L-1} \min\left\{\overline{R}, \psi_0^{-1}(\psi_e(x_{i_l}) + \Delta)\right\} + (1-\beta)^{N-L}\overline{R}; \tag{13}$$

otherwise, the buyer awards the contract to entrant x_{i_k} who passes the qualification screening and pays him or her

$$\beta \sum_{l=k+1}^{L} (1-\beta)^{l-k-1} x_{i_l} + (1-\beta)^{L-k} \psi_e^{-1}(\psi_0(x_0) - \Delta). \tag{14}$$

Note that in the optimal mechanism, it is optimal to conduct supplier qualification screenings postbidding. This is because, on the one hand, using the suppliers' initial bids helps the buyer make more informed supplier qualification screening decisions and save qualification cost (this is consistent with the finding in Study 1 in that POST performs well compared with PRE in most of the scenarios); on the other hand, the optimal payment rule, under which the buyer commits to ex ante, ensures that he or she provides the minimum incentive needed for the suppliers to not inflate their cost bids (in contrast, the simple payment rule in POST creates incentives for suppliers to inflate their bids compared with PRE). Although the optimal mechanism, by definition, results in lower expected total cost for the buyer compared with our proposed approach, we want to point out that the optimal mechanism is difficult to implement due to various practical concerns. For example, the optimal mechanism's payment rule in (13) and (14) is very complicated. The winner's payment not only depends on his own bid but also depends on his competitors' bids in a nontrivial way, which is difficult for the buyer to rationalize to suppliers in the first place in practice. Moreover, the optimal mechanism's qualification screening rule is not necessarily ex post optimal for the buyer because, in contrast to Step 2 in Proposition 5, ex post, the buyer may actually have an incentive to not screen an entrant with cost $x_j < \psi_e^{-1}(\psi_0(x_0) - \Delta)$ if all lower cost entrants fail qualification screenings.[3] Thus the buyer not only faces the challenge of explaining the rules of the mechanism to the suppliers but also needs to convince the suppliers that he or she will strictly follow the complicated rules, even though following these rules may not be in the buyer's best interest ex post. Another aspect of the implementation challenge of the optimal mechanism relates to the impracticality of truthful bidding. In practice, truthful bidding is hard to induce because both the incumbent and entrants would be hesitant to reveal their true costs for fear of, for example, revealing private cost information that would put them in a disadvantaged position for future business negotiations with the buyer. (We refer interested readers to Roughgarden and Talgam-Cohen (2019) for more discussions on the drawbacks of optimal mechanisms in practice and Rothkopf (2007) for several more general critiques about why mechanisms that induce truthful bidding may not be practical.) Note that these practical implementation issues do not arise in POST. In POST, the payment is essentially a simple pay-as-bid rule

Table 3. Summary Statistics for the Fraction of the Cost-Saving of the Optimal Mechanism That Is Recovered by the Proposed Strategy

	Cost-saving (%)
Mean	72.23
Std	28.10
90th percentile	96.11
75th percentile	92.99
50th percentile	84.45
25th percentile	59.95
10th percentile	26.07

(see Endnote 1), the qualification screening rule is ex post optimal for the buyer, and the suppliers do not need to reveal their true costs.

Although the optimal mechanism is hard to implement in practice, it does provide a lower bound of the best achievable cost for the buyer in practice. We can then use this lower bound to numerically evaluate how effective our proposed approach helps bring the cost down to the optimal cost from the conventional approach PRE. To that end, we adopt a metric defined as the fraction of the absolute potential savings the optimal mechanism offers over PRE that can be captured by our proposed approach. To compute this metric, for each scenario, we calculate the savings of our proposed approach over PRE across all scenarios and divide this by the potential savings using the optimal mechanism (see Online Appendix for the detailed computation method and pseudocode):

$$\frac{PC_{PRE} - \min\{PC_{PRE}, PC_{POST}\}}{PC_{PRE} - PC_{OPT}}, \tag{15}$$

where PC_{OPT} is the expected cost under the optimal mechanism. Next, we report key summary statistics in Table 3. Consistent with our previous observation, our proposed approach performs very well. Across all scenarios, it captures on average 72.23% of the potential savings from the optimal mechanism that, as we mentioned, would be hard to implement in practice. Thus, we believe that our proposed approach is a powerful procurement toolkit for the buyer.

6. Conclusions

In this paper, we study the buyer's optimal choice between prequalification (PRE) and postqualification (POST) strategies in a resourcing setting where a qualified incumbent and multiple not-yet-qualified entrant suppliers compete for a single-supplier contract in an open-bid auction. Although this is an important problem in the academic literature, there is little guidance on how to quantitatively choose between PRE and POST in practice. This is primarily because POST in the context of open-bid auction is a very complex dynamic game, and it is deemed computationally intractable to compute the buyer's expected cost under POST when multiple entrant suppliers participate in the auction. By providing a full characterization of suppliers' equilibrium bidding strategies in POST, we uncover managerial insights on the entrants' and the incumbent's different bidding incentives in this setting. More importantly, by leveraging the structural properties of suppliers' bidding strategies, we provide the first computationally tractable approach to evaluate the expected cost of POST, which enables practitioners to quantitatively compare the expected costs of PRE and POST ex ante. Hence, we propose the buyer to use the cheaper option between PRE and POST to minimize the expected procurement cost.

Although PRE is predominantly used in practice, we derive analytical conditions under which POST is cheaper than PRE and show via an extensive numerical study that our proposed approach (i.e., the cheaper option between PRE and POST) can significantly reduce the buyer's procurement cost compared with only using PRE for many practical settings. We show that even though our proposed approach is simple and only involves combining features of existing auction methods in a novel way, it captures most of the benefit that a theoretically optimal mechanism (which is quite difficult to implement in practice) offers over PRE. These results provide evidence that POST can be a powerful tool to manage resourcing processes in practice. Note that POST is one possible arrangement to incorporate post-bidding qualification screening into open-bid auctions, and there may be other practical alternative mechanisms that could provide more cost savings to the buyer. We leave this as an open research direction.

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Endnotes

[1] For example, consider the case where $K > \overline{R}\beta$. This implies that $\Delta = K/\beta > \overline{R} - \underline{R}$, so the buyer should not conduct qualification screening in either PRE or POST. As a result, PRE and POST result in the same expected cost.

[2] Note that a similar optimal mechanism design setting has been investigated in Wan and Beil (2009) and Chen et al. (2018). It is worth noting that the notion of prequalification screening in Wan and Beil (2009) refers to buyer's endogenous effort on identifying suppliers with certain qualification probabilities and hence, is different from our setting. Having said that, our optimal mechanism analysis is different from the model in Wan and Beil (2009) in the following way. We consider the case where the suppliers have ex ante asymmetric cost distributions, and the "prequalification probabilities" (per Wan and Beil (2009) terminology) are exogenous (i.e., it equals 1 for the incumbent and β for the entrants). Our mechanism design problem is not a special case of the model considered in Chen et al. (2018) because suppliers' cost distributions are asymmetric in our setting.

[3] Here is an example of when the optimal mechanism's qualification screening rule is not ex post optimal. Suppose there is one entrant and one incumbent and both suppliers' cost distributions follow uniform $U[0,\overline{R}]$. Moreover, the entrant's cost is $x_1 < \psi_e^{-1}(\psi_0(x_0) - \Delta)$ (namely, $N = L = 1$) and the incumbent's cost $x_0 > \overline{R} - \Delta/2$. By Proposition 5, (13) becomes $\overline{R}$ since $N = L$, and (14) becomes $\psi_e^{-1}(\psi_0(x_0) - \Delta)$. Then, after seeing both suppliers' bids, x_0 *and* x_1 (i.e., ex post), the buyer's total expected cost under the optimal mechanism is $K + \beta\psi_e^{-1}(\psi_0(x_0) - \Delta) + (1-\beta)\overline{R} = \beta(x_0 + \Delta/2) + (1-\beta)\overline{R} > \overline{R}$. Thus, ex post, he or she has an incentive to deviate from his or her committed qualification screening rule by not screening the entrant but awarding the contract to the incumbent directly with the payment $\overline{R}$, as in (13).

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