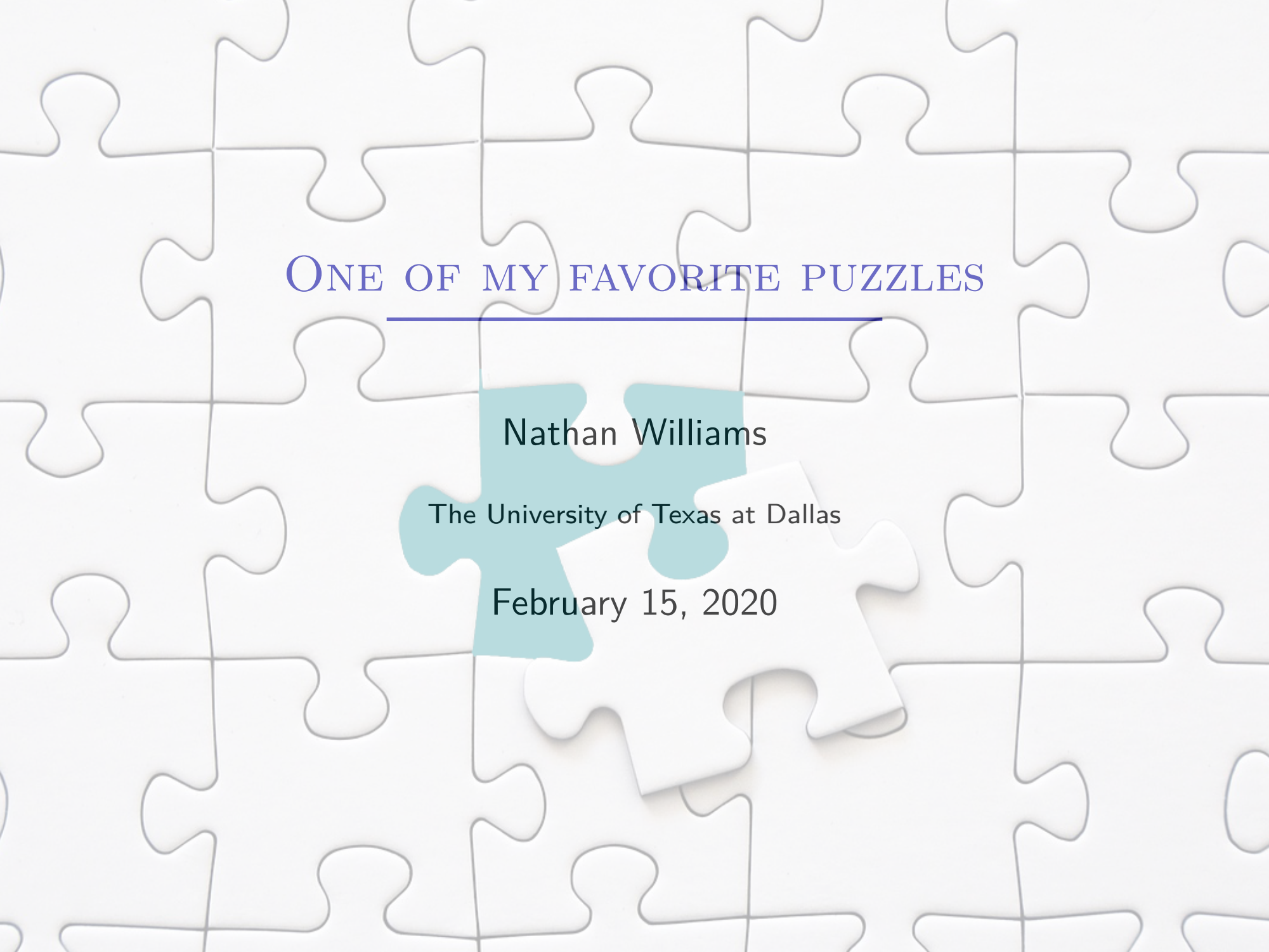


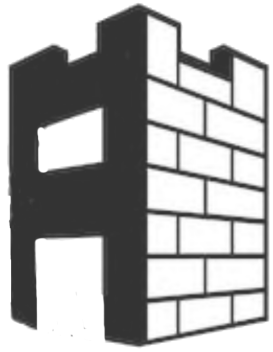
One of my favorite puzzles

Nathan Williams

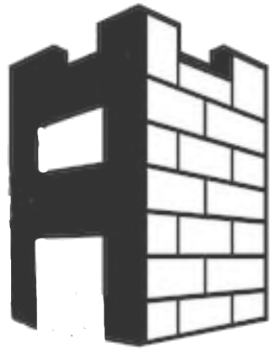
The University of Texas at Dallas

February 15, 2020

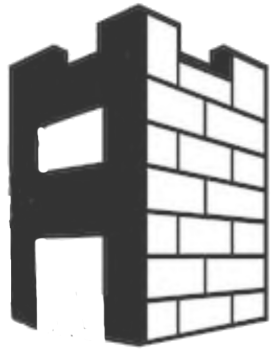
Two Kingdoms: Kingdom A & Kingdom B



Each Kingdom has a king and each Kingdom has a thief.



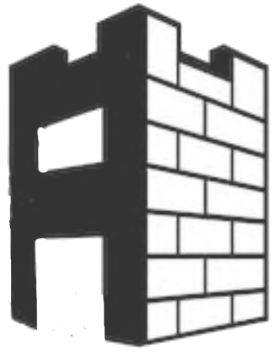
At the start of each day, the king places **two** coins on top of his stack of coins (which starts out empty).



At the start of each day, the king places **two** coins on top of his stack of coins (which starts out empty).

At the end of each day, the thief steals **one** coin from the stack:

- In Kingdom A (for **A**bove), the thief steals the top coin.
- In Kingdom B (for **B**elow), the thief steals the bottom coin.



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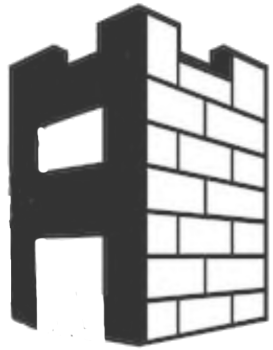

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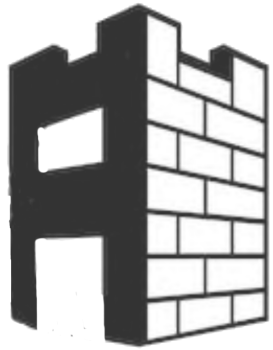

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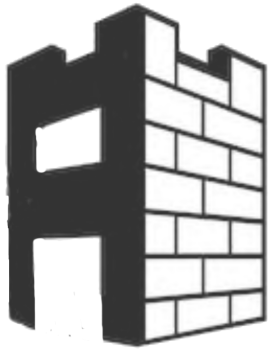
Question

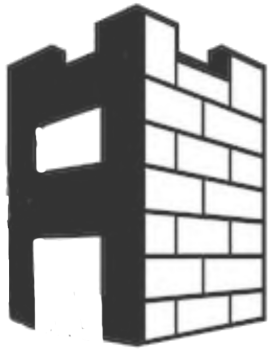


After an infinite number of days, how many coins are left?

"Turtles all the way down..."

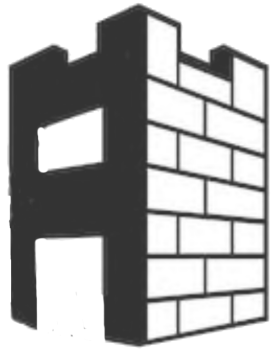
- The set of all sets that don't contain themselves. *(Russell)*
- There are different **kinds** of infinity! *(cardinals and ordinals)*
- There are different **sizes** of infinity! *($|\mathbb{R}| > |\mathbb{N}|$)*
- You can choose one shoe from each pair of an infinite collection of pairs of shoes (take the left one from each pair), but the story is different for socks! (Axiom of Choice)
- There are true arithmetical statements that "can't be proven with arithmetic."
- ...

Answer?



Every day, the king puts down two coins. Every day, the thief steals one. A net gain of one coin. Therefore, an infinite number of coins. . . right?

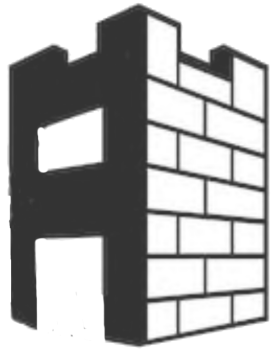
Kingdom A (for **A**bove): Label Coins



2.

1.

Kingdom A (for **A**bove): Label Coins



2.

1.

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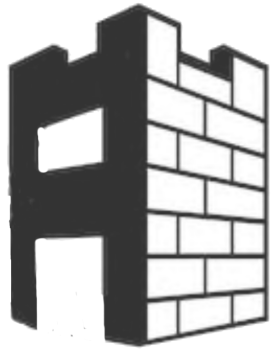
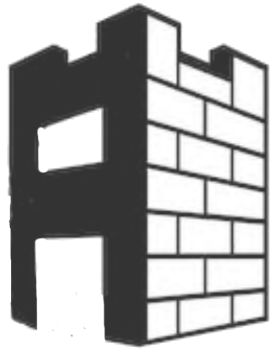


4.
3.
2.
1.

Kingdom A (for **A**bove): Label Coins



All odd numbered coins are left.

∞ odd numbers, hence ∞ coins left in A.

Kingdom B (for **B**elow): Label Coins

2.
1.

Kingdom B (for **B**elow): Label Coins

2.

1.

Kingdom B (for **B**elow): Label Coins

4.
3.
2.
1.

Kingdom B (for **B**elow): Label Coins

4.
3.
2.
1.

If you think there are ∞ coins left, name *a single one*!

Kingdom B (for **B**elow): Label Coins

4.
3.
2.
1.

If you think there are ∞ coins left, name *a single one*!
You can't—*because there are none.*