Getting Out of Your Own Way: Introducing Autonomous Vehicles on a Ride-Hailing Platform

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After autonomous vehicles (AVs) are deployed for ride-hailing platforms but before their costs decrease enough to push human drivers off the road entirely, human drivers will compete for rides with AVs. We consider a ride-hailing platform's strategy to recruit human drivers while also operating a private fleet of AVs. We formulate and solve a game-theoretic model of a ride-hailing platform with a private AV fleet that also recruits self-interested human drivers. The platform sets the human-driver wage and its AV fleet size, and human drivers make strategic joining decisions based on a rational anticipation of their expected earnings. We show that having the option to augment its AV fleet after observing human participation levels can, counterintuitively, *hurt* the platform's bottom line. Human drivers anticipate that the platform may reactively acquire more AVs; this reduces their expected earnings, so the platform must offer higher wages to attract a given number of drivers. The higher wage implies that the platform should acquire even more AVs, leading to a feedback loop or "race to the top" of increasing wages and increasing AV acquisition. The race to the top hurts the platform's profits by effectively preventing it from attracting more than a limited number of human drivers and increasing the cost of attracting a given number. Our findings reveal the importance of the platform's credibility with its drivers. The ride-hailing platform can succeed in avoiding the race to the top only to the extent that it can convince human drivers that it will not reactively acquire a large number of AVs. Otherwise, significant profit losses await.

Key words: innovation, gig economy, game theory, ride-hailing, autonomous vehicles, market design

1 Introduction

As the ride-hailing market has grown and self-driving technology has advanced rapidly in recent years, speculation has mounted about the future of autonomous vehicles (AVs) in this market. Indeed, such a future looms closer than ever. For example, Waymo One,[1] a pioneering AV-operated ride-hailing service in Phoenix and San Francisco with planned expansion to Los Angeles and Austin, has even dispensed with human safety drivers (although as of early 2024, Waymo's AVs are just beginning to be tested on highways, and not yet with paying customers: see Hawkins 2024). However, while multiple firms are testing AVs, the race to full autonomy for ride-hailing and automobility in general has not been without challenges. Several fatal accidents have been reported, including one on GM's autonomous ride-hailing service Cruise[2] in late 2023 that led it to lose its permit to test AVs in California. The incident created turmoil at the company, prompting leadership changes and a "pause" of Cruise's AV operations throughout the U.S. (Kolodny and Wayland 2023) that was ongoing as of January 2024. "Level 5" autonomy (when humans will not be needed

[1] https://waymo.com/waymo-one/

[2] https://getcruise.com/

for any driving tasks) has been called "one of the hardest problems we have."[3] Indeed, high-profile early proponents of AV-operated ride-hailing services, including Uber[4] and Ford,[5] have abandoned their attempts to commercialize the technology due to out-of-control costs. These challenges raise the question of whether and how ride-hailing platforms can benefit from AV technology during the long journey toward a future with no human-driven vehicles.

Although it is widely accepted that eventually all cars will be driven by software, human-driven vehicles still dominate the road at present. Moreover, the transition between these extremes will not happen overnight. A recent New York Times article described the small-scale fully-driverless ride-hailing experiment operated by GM's Cruise in San Francisco (Metz 2022). The article also describes the currently high costs of AVs: "The development costs, back-end computing infrastructure and technicians needed to support these cars increase costs by hundreds of millions of dollars—at least for now." Accordingly, although the long-run value proposition for AVs revolves around lower costs achieved by removing humans from the equation, this promise is unlikely to be realized in the near-term future, not even once the AV roll-out starts to scale up (see Nunes and Hernandez 2019, 2020).

In addition, platforms must plan for a new form of competition on the supply side of their marketplaces: man vs. machine. It is not clear how gig-economy workers will react to AVs joining the market, or how ride-hailing platforms should best balance the two sources of supply. We consider the problem of how a platform should manage its ride-hailing marketplace to simultaneously leverage a private AV fleet and successfully recruit human-driven vehicles. Since AV technology is still expensive, the question may reasonably be asked why AVs are yet needed, if rides could be served more cheaply with human drivers. First, as evidenced by their heavy investments in AV technology and initial deployments of it discussed above, ride-hailing firms indeed see value in serving rides with AVs, even though costs are still quite high as the technology is in active development. Second, as usually happens with emerging technology (e.g., the original "horseless carriage," personal computers, etc.), AVs will need to be perfected over time in order for the technology to decrease in cost and achieve full market penetration. During this process, platforms can and should aim for the best mix of AV- and human-served rides. We find that even with AVs that are relatively expensive, an AV fleet can benefit a platform if the fleet is appropriately sized. However, we also identify some critical problems that can arise due to the interaction between a platform's AV acquisition decisions and its ability to recruit human drivers.

A platform with a mixed fleet must manage several tradeoffs. One benefit of a company-owned or leased AV is that its cost is known and fixed. By contrast, a human driver must be recruited to join the platform, and hence, the wages and earning rate offered must satisfy an individual rationality constraint. The cost of a human-served ride is thus endogenous, and increasing human-driver participation may require higher

[3] https://www.thedrive.com/tech/31816/key-volkswagen-exec-admits-level-5-autonomous-cars-may-never-happen

[4] https://www.wsj.com/articles/uber-sells-self-driving-car-unit-to-autonomous-driving-startup-11607380167

[5] https://www.theverge.com/2022/10/26/23423998/argo-ai-shut-down-ford-vw-av-self-driving

wages, since the more drivers join the platform, the greater the competition among them for rides (they also must compete with AVs, as discussed below). On the other hand, human drivers only earn money from the platform while with a passenger, so they offer a hedge against demand variability, protecting the platform against both underage and overage costs. However, knowing that the platform may prefer to serve demand with AVs, human drivers may not join the platform because they cannot rely on being matched with fares. As noted by Jiang and Tian (2018) and Guda and Subramanian (2019), the two-sided marketplace of ride-hailing involves "customers" on both sides of the market in the sense that both passengers and drivers freely choose whether to use the platform's service based on its value. As such, the new element of AVs competing for rides necessarily affects the platform's strategy for recruiting human drivers. This novel supply-side competition motivates our two main research questions: (i) how should a ride-hailing platform set the human-driver wages and AV quantity to exploit its AV fleet while simultaneously recruiting enough human drivers (at an affordable wage) to create an effective demand hedge, and (ii) in the presence of AVs, what effect do the self-interested joining decisions of human drivers have on the platform's bottom line?

To address these questions, we consider a ride-hailing platform with two supply sources: (i) a platform-operated fleet of AVs, and (ii) a population of human drivers that make strategic joining decisions. The platform first chooses an initial AV acquisition quantity and sets the wage rate that it will pay human drivers for each ride. Then, human drivers decide whether to join the platform in equilibrium, determining the available pool of human labor. Finally, knowing the wage and the human labor pool size, the platform has the chance to make an additional acquisition of AVs (at a possibly higher unit cost due to the shorter notice), and then demand is realized. In equilibrium, the human drivers' expected earnings must match or exceed the outside option, and these earnings depend on the platform's initial AV acquisition, the labor pool size, the wage rate, and a rational anticipation of the second AV acquisition decision.

Contributions. We first solve the platform's second-stage AV acquisition problem in quasi-closed form. The solution reveals the interaction between the human-driver wage and participation level and the optimal AV quantity. Specifically, the optimal AV quantity increases in the human-driver wage for a fixed human participation level; if the platform must pay more for human labor, then this labor becomes less desirable, inducing it to acquire more AVs. Additionally, and underscoring the complex nature of the relationship, the optimal AV quantity moves in different directions with the human participation level depending on the wage: for low (high) wages, an increase in human participation causes the platform to decrease (increase) the AV quantity. The preceding discussion relates to the optimal second-stage AV quantity with the wage and human participation level fixed, which they are when the platform makes this decision. However, like the AV quantity, the wage and human participation level are also endogenously determined (at an earlier stage of the game), and the nuanced interaction between these three quantities turns out to be a key driver of equilibrium outcomes.

We then establish structural results regarding the platform's use of the second-stage AV acquisition option and its impact on the optimal profit. First, we show that optimally the platform does not exercise the second-stage AV acquisition option at all, acquiring AVs only in the first stage. Because AVs are either cheaper or the same cost in the first stage, the platform prefers to do all of its acquisition upfront. We might expect, then, that the second-stage acquisition option is irrelevant and that outcomes are the same with or without it. Strikingly, however, this intuition turns out to be incorrect. Our second structural result reveals that the platform's optimal profit is *worse* when it has access to a second acquisition opportunity, and that the apparent flexibility provided by such an opportunity counterintuitively restricts the platform from achieving the ideal AV/human driver mix.

To resolve this apparent conflict requires a deeper understanding of human drivers' equilibrium behavior and its consequences. Since the platform's optimal second-stage AV quantity is increasing in the human-driver wage, the positive impact of a wage increase on human drivers' expected earnings is dampened (possibly even negated) by the increased competition from AVs. This reflects the endogenous relationship described above that is governed by the equilibrium participation condition for human drivers and the optimality condition for the platform's second-stage AV quantity. Perhaps counterintuitively, the equilibrium wage can actually *decrease* in the human joining fraction because at low joining fractions, the wage is low enough that increasing this fraction reduces the optimal AV quantity and thus increases human drivers' matching rate.[6] However, we also show that human participation may be intrinsically limited. As more humans join and push out AVs, eventually not many AVs are left, and the human-driver matching rate begins to decrease. The platform then must increase the wage to satisfy the equilibrium participation condition, but this makes human drivers less attractive, so it will acquire more AVs, which again increases competition and decreases the matching rate, so the wage must increase even more. The result is a feedback loop of increasing wages and increasing AV acquisition. This *race to the top* effectively prevents the platform from attracting more than a limited number of human drivers, and it increases the cost of attracting a given number. Interestingly, the feedback loop emerges despite the platform not actually exercising the second-stage acquisition option. Essentially, the optimality condition in the second stage determines the number of AVs that the platform will acquire, but rather than waiting until the second stage, it is cheaper to acquire these AVs upfront, which in turn reduces the optimal second-stage quantity to zero.

Hence, because its second-stage AV quantity is a best response to the wage and human joining fraction, the platform cannot credibly commit to a quantity that ensures human drivers a high matching rate. This leads to the feedback loop mentioned above and keeps the platform from achieving the ideal AV/human-driver mix. Indeed, the platform's profit can be as much as 38% less than if it did not have the ability to acquire more AVs reactively after observing the human-driver participation level.

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[6] This is the measure of the expected number of ride requests that each human driver will serve.

To deepen our understanding of the race to the top and its consequences, we conduct comparative statics and also study different extensions of our model. First, we consider the impact of different values of the second-stage AV cost. Because the race to the top is stronger when AVs are more desirable for the platform, its optimal profit can actually be higher if the second-stage AV cost *increases* because a higher AV cost makes AVs less attractive. We also extend our model to allow for several variations that may be relevant to practice. First, we incorporate finite supply constraints for AVs, such that the platform may have a tight bound on the number of AVs that it is able to acquire in each stage. We observe an interesting non-monotonicity of the optimal quantities as the limits change for each stage, and we find that the profit is decreasing as the second-stage AV bound increases. That is, as the platform is able to buy more AVs in the second stage, the feedback loop becomes more severe (the wage is increasing in the AV bound), harming the profit. In short, the race to the top continues to harm the platform. Second, we allow for customers who may have preferences between AVs and human-driven vehicles. In this setting also, we observe the now-familiar feedback loop of increasing wages and increasing AV acquisition. Interestingly, the race to the top is strong enough that the platform may actually prefer to have some customers *not* be willing to accept an AV-served ride, since human-driver-only demand partially dampens the feedback loop. Finally, we allow for a random second-stage AV acquisition cost, and we obtain qualitatively similar findings to those in our base model. Thus, we find that the feedback loop of increasing wages and increasing AV acquisition is robust to different variations in the setting, and the consequences for the platform are similar in each case.

Overall, our findings demonstrate the importance to ride-hailing platforms of appropriately managing the introduction of AV technology. The threat that the platform may obtain a large fleet of AVs can scare away human drivers while they remain an important labor source. A profitable balance of AVs and human drivers can outperform both a human-only and an AV-only fleet. However, during the transition between only humans on the road and only AVs, platforms must carefully manage their AV fleets and the expectations of human drivers: too many AVs will trigger the race to the top and push humans out of the labor market, which in the near-term future will hurt profitability. Too few, on the other hand, will also leave money on the table, as well as put the platform behind in AV adoption, which is of strategic importance in the long term. In their strategy to introduce AVs to the marketplace, it is crucial for platforms to properly manage the endogenous interaction between wages, human participation, and AV acquisition; significant profit losses await otherwise.

2 Related Literature

Only a few very recent papers in the operations management literature incorporate AVs in the context of ride-hailing. In the setting of Siddiq and Taylor (2022) with competition between platforms, they assume linear demand and supply functions. By contrast, we model human drivers as strategic agents who make participation decisions based on their expected earnings given the posted wage and a rational anticipation of their matching rate. This creates a non-linear, endogenous relationship in our model between wage and human driver supply through the impact of both on the optimal AV acquisition that is not seen in Siddiq and Taylor (2022).

Lian and van Ryzin (2022) treat platforms as market-clearers and study the decisions of market participants (human drivers and AV owners). They consider two platform scenarios—common platform for AVs and human drivers or independent platforms for each—and two AV ownership scenarios—AVs owned by individuals or AVs owned by a monopolist. The combinations yield a total of four different market designs, and they analyze the resulting prices, wages, and utilization in each. The setting in Lian and van Ryzin (2022) differs from ours along several dimensions. First, in contrast to their model of ride-hailing platforms as market-clearers, we consider a profit-maximizing platform that sources rides from both humans and its own AV fleet, and we provide important insights about the platform's human-driver recruiting strategy. Second, they assume an unlimited human labor pool; we treat this pool as finite at the outset, which is important in our setting because the competition both among human drivers and between AVs and humans depends on the labor pool size. In contrast to both Siddiq and Taylor (2022) and Lian and van Ryzin (2022), in the present work, human drivers' rational anticipation of the platform's best-response AV acquisition entails a unique endogenous relationship between wages, available human drivers, and AV acquisition. Our findings also highlight the impact of commitment power on the platform's ability to recruit human drivers. Freund et al. (2022) analyze a supply chain model involving a platform, an external AV supplier, and human drivers, and conclude that the need to maintain driver engagement may result in underutilization of AVs; however, they propose that this issue can be resolved through usage contracts or prioritization contracts. Unlike a supply chain framework, our focus is on a platform that owns its own AV fleet, allowing for more control as the platform is not required to provide a guaranteed payment to secure a certain level of AVs. In Castro et al. (2023), the authors employ a queuing-theoretical model to demonstrate that the introduction of autonomous vehicles to a platform can lead to a decrease in service level—stemming from a reduction in human drivers' earnings due to AV prioritization. Furthermore, the authors establish that this decrease may be different across regions as lower-demand areas may experience a greater decline in service levels. Although the setting is similar to ours, our focus is not on prioritization and service levels but rather on how the interplay between wages set by the platform (which are determined in equilibrium in our model, unlike their exogenous fixed commission) and its AV acquisition decisions affects its ability to recruit human drivers. For another study that looks into the granular spatial incentives that AV deployment can produce on human vehicle repositioning, we refer the reader to Benjaafar et al. (2021), and for a study that considers a centralized firm that can be viewed as a taxi service or an AV ride-hailing firm (but does not model AVs and human drivers on a shared platform), see Noh et al. (2021). For other recent studies involving AVs, we refer the reader to Liu (2018), Mirzaeian et al. (2021), and Baron et al. (2022).

Another related stream of work concerns blended workforces composed of full-time employees (similar to our AVs) and flexible agents (similar to our human drivers). Early works on this front consider how to optimally source contingent workers from an external labor supply agency, see e.g., Milner and Pinker (2001) and Pinker and Larson (2003). More recent studies, however, consider a related problem in which flexible agents are independent. For example, Hu et al. (2022) analyze the welfare implications of uniform and hybrid worker classification in on-demand platforms. This work establishes that having full-time employees and contractors can be more beneficial for some workers, consumers and the platform than uniform classification. Lobel et al. (2023) use a similar modeling framework to ours to study the impact of wages on the labor composition; in contrast, we focus on the endogenous competition in a blended workforce and its implications for the platform's bottom line. Dong and Ibrahim (2020), on the other hand, consider the multi-period staffing problem faced by an on-demand platform that employs a mix of full-time employees and randomly determined flexible agents. A key difference between our setting and that of Dong and Ibrahim (2020) is that the endogeneity in their flexible agents' joining decision is stochastic and not strategic; additionally, their endogeneity is related to the number of other flexible workers while ours is related to both the human drivers that join and the autonomous vehicles that the firm deploys. Chakravarty (2021) studies the viability of a blended workforce and the pricing implications of preferential rationing (employees are matched before independent drivers) in a two-stage model. While our study and that of Chakravarty (2021) are related, our modeling approach is different and our results focus on the analysis of potential equilibrium outcomes and not solely on the viability of a blended workforce.

Finally, this paper is also related to works that study how to incentivize independent, strategic supply units in on-demand platforms, see, e.g., Cachon et al. (2017), Daniels (2017), Bimpikis et al. (2019), Guda and Subramanian (2019), Hu and Zhou (2020), and Besbes et al. (2021). In our model, supply units make their joining decision strategically given wages; however, they also consider the impact of AVs on their earnings. Also related to our paper are studies that compare contractors and full-time employees in on-demand platforms. Taylor (2018) studies how prices and wages are impacted by customers' delay sensitivity and agents' independence, and by uncertainty in customers' valuations and agents' opportunity cost. A key observation in this work is the *agent participation externality* which implies that equilibria with low wages and many agents can be sustained because an improved service (due to more agents) leads to larger demand, and thus better agent utilization. Interestingly, the latter effect dominates the competition effect which would lead to low utilization and higher wages. In our setting, as the platform increases the induced fraction of agents in the system, wages may decrease due not to an improved service but rather to a substitution effect. More agents in the system implies that the platform can rely less on autonomous vehicles, which can improve agent utilization despite generating more competition among humans, leading to lower wages. Gurvich et al. (2019) consider a related newsvendor setting without a mixed fleet, in which supply units are self-scheduling, and they establish that higher wages are needed to induce a higher number of self-interested supply units. The latter effect occurs because demand is not affected by service but also because, in contrast to our paper, the platform does not have access to its own supply. In Nikzad (2020), the author considers on-demand

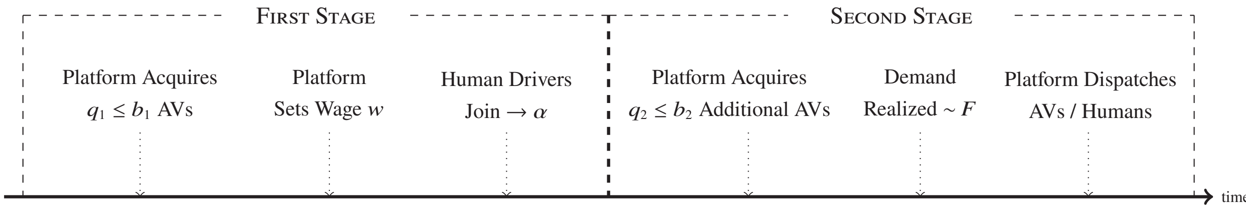


Figure 1 Sequence of Events

service platforms and establishes that in thin markets (small labor pool), the platform can sustain high wages because the marginal improvement in service level can outweigh the higher cost associated with those higher wages. In parallel work to Nikzad (2020), Benjaafar et al. (2022) study how different policies affect labor welfare in on-demand service platforms. They establish that nominal wages (without utilization) always decrease, albeit at a diminishing rate, in the size of the labor pool because more supply stimulates demand by decreasing delay. The latter leads to a non-monotone effective wage (nominal wage times utilization) which first increases and then decreases. In the present paper, we observe the reverse due to the substitution effect between human drivers and autonomous vehicles. The nominal wage can decrease in the induced fraction of supply at small values of this fraction. Then, for larger fractions of induced supply, the nominal wages can be arbitrarily large due to a feedback loop of increasing wages and increasing AV acquisition.

To summarize, our work takes a fresh perspective on the already fresh problem of operating a ride-hailing service supported by human drivers in the presence of AVs, developing an optimal strategy to manage human-driver incentives through wages and matching rates. In what follows, we reveal pitfalls that ride-hailing platforms may succumb to as they grow their AV fleets. We uncover a surprising "race to the top" of increasing wages and increasing AV acquisition that fundamentally limits human driver recruitment.

3 Model

We study a ride-hailing platform that connects passengers requesting rides with AVs or human drivers. The players in our game are (i) the ride-hailing platform and (ii) a nonatomic population (of size M) of self-interested human drivers.

We suppose that the platform's average revenue per ride is r for either type of vehicle. Our model is a sequential game with two stages. In the first, the platform makes an initial AV acquisition decision, sets wages, and human drivers make joining decisions. In the second, the platform learns the human-driver participation level and has an opportunity to augment its AV fleet (at a possibly higher unit cost), and finally demand is realized. The first stage captures the long-term interaction of drivers with the platform, while the second stage models the shorter-term decisions of the platform. The detailed sequence of events and the relevant notation follow and are also summarized in Figure 1.

First stage. At the beginning of the first stage, the ride-hailing platform makes its initial AV acquisition decision, acquiring $q_1 \leq b_1$ AVs at unit cost $0 < c_1 < r$. The upper bound b_1 can depend on supplier availability constraints, the platform's budget constraints and/or its strategic deployment plan. Then, it sets the wage w that human drivers will receive per ride. Next, the human drivers decide whether to join the platform. For simplicity, we assume that all drivers who join the platform are available to serve passengers in the second stage, but our model could easily incorporate a fraction of drivers who join the platform but in the end are not available in the second stage. The drivers are homogeneous and have an outside option valued at $v > 0$, where $v < c_1$ due to the currently high cost of acquiring AVs. Drivers join the platform if their expected payoff from doing so (weakly) exceeds their outside option. The population of nonatomic human drivers has mass M. Denoting by α the fraction of human drivers who choose to join the platform (we will call this the joining fraction), the supply of human drivers in the second stage is αM. We defer the formal definition of driver equilibrium to Section 5.

Second stage. At the beginning of the second stage, the platform observes the fraction α of human drivers who have joined the platform. The platform already has access to a fleet of $q_1 \geq 0$ AVs. After observing the human joining fraction α, it can augment its AV fleet if needed by sourcing an additional $q_2 \leq b_2$ AVs at a premium, e.g., from a fast supplier, at unit cost c_2, where $c_1 \leq c_2 < r$.[7] Because AVs do not require a paid human driver and moreover are likely to be electric, their operating costs are likely to be lower than for human-operated vehicles that are for the time being predominantly gas-powered.[8] So, for simplicity and to capture the main feature that AVs are cheaper to operate than human-operated vehicles, we normalize the AV operating cost to zero.

Finally, demand is realized. The number of requested rides D is a random variable with cumulative distribution function (CDF) $F(\cdot)$ with $F(0) = 0$, corresponding probability density function (PDF) $f(\cdot)$ and mean μ. We use $\bar{F}(\cdot)$ to denote $1 - F(\cdot)$. Each unit of AV can serve one unit of demand, and the same holds for human drivers. The platform allocates demand first to AVs, and human drivers serve excess demand. Note that allocating demand to AVs first is optimal for the platform due to their negligible operating cost. The total available supply is $q_1 + q_2 + \alpha M$, and any demand exceeding this quantity is lost.

Platform's problem. Suppose that in the first stage, the platform initially acquired q_1 AVs, it has set the wage at w, and a fraction α of human drivers have joined the platform. If the platform acquires an additional q_2 AVs in the second stage and the realized demand is d, then the platform's profit is

$$\pi(q_1, w, \alpha, q_2; d) \triangleq r \min\{d, q_1 + q_2 + \alpha M\} - c_1 q_1 - c_2 q_2 - w \min\{[d - (q_1 + q_2)]^+, \alpha M\}. \tag{1}$$

The number of passengers served is the minimum of the available supply $q_1 + q_2 + \alpha M$ and the realized demand d, and multiplying by the revenue per ride r gives the total revenue. There are two components of the cost: the AV acquisition cost and the human driver wages. The AV acquisition cost is $c_1 q_1 + c_2 q_2$. The wage

[7] We assume $c_1 \leq c_2$ because obtaining AVs on short notice may be more expensive. However, our main insights carry over to the case with $c_1 > c_2$.

[8] Electric vehicles represented only 14% of global new car sales in 2022, and only 8% in the U.S (International Energy Agency 2023).

cost is based on the *realized* number of human-served rides, which is the minimum of the excess demand not handled by AVs and the available human supply. Human drivers receive no compensation when not with a fare. In what follows we use $\Pi(q_1, w, \alpha, q_2)$ to denote the expected value of $\pi(q_1, w, \alpha, q_2; d)$, where the expectation is taken over the demand D. Accordingly, we have

$$\begin{aligned}\Pi(q_1, w, \alpha, q_2) = r &\left(\int_0^{q_1+q_2+\alpha M} u f(u)\mathrm{d}u + (q_1+q_2+\alpha M)\bar{F}(q_1+q_2+\alpha M)\right) - c_1 q_1 - c_2 q_2 \\ &- w\left(\int_{q_1+q_2}^{q_1+q_2+\alpha M} (u-(q_1+q_2)) f(u)\mathrm{d}u + \alpha M \bar{F}(q_1+q_2+\alpha M)\right),\end{aligned} \tag{2}$$

and $\Pi(q_1, w, \alpha, q_2)$ serves as the platform's utility function.

Letting (EQ) be the human drivers' equilibrium condition (see Section 5), the platform's optimization problem is then

$$\begin{aligned}\Pi^\star \triangleq \max_{(q_1,w,\alpha,q_2)} \quad & \Pi(q_1, w, \alpha, q_2) \\ \text{s.t.} \quad & \alpha \textit{ satisfies } \text{(EQ)} \\ & 0 \le q_1 \le b_1 \\ & q_2 \in \arg\max_{0\le q_2\le b_2} \Pi(q_1, w, \alpha, q_2)\end{aligned} \tag{$\mathcal{P}$}$$

We will use $(q_1^\star, w^\star, \alpha^\star, q_2^\star)$ to denote the optimal solution of $\mathcal{P}$. Observe that the platform must solve a bi-level optimization problem because the second-stage acquisition quantity is set after the wages and the joining fraction are determined. Note also that the human joining fraction α is treated as a decision variable for the platform, subject to the equilibrium condition (EQ).

4 Second-Stage AV Acquisition

We first study the second-stage AV acquisition decision, denoted by $q_2(q_1, w, \alpha)$ as a function of the first-stage acquisition quantity q_1, the wage w, and the human-driver joining fraction α.

With no human drivers (i.e., for $\alpha = 0$), the platform's second-stage AV acquisition problem reduces to a standard newsvendor problem with overage (over-acquisition) cost c_2, underage (lost-sales) cost $r - c_2$, and a starting stock of q_1 AVs. The optimal second-stage acquisition quantity q_2 uniquely solves

$$F(q_1 + q_2) = \frac{r - c_2}{r}. \tag{3}$$

The exceptions are if the solution above exceeds b_2, in which case the optimal acquisition quantity is b_2, or if it is negative, in which case the optimal quantity is 0.

For $\alpha > 0$, the problem departs from the newsvendor model in that we have two tiers of underage cost. Demand between $q_1 + q_2$ and $q_1 + q_2 + \alpha M$ is served by human drivers, with a cost difference of $w - c_2$ compared to an AV-served ride.[9] But demand above $q_1 + q_2 + \alpha M$ is lost entirely, incurring a lost-sales cost

[9] We use c_2 because this is the marginal AV cost in the second stage, whose acquisition decision we are studying here.

of $r - c_2$. Due to this tiered cost structure, the platform's first-order condition (FOC) does not reduce to a simple critical ratio as in (3).

The following result characterizes the optimal second-stage AV acquisition quantity $q_2(q_1, w, \alpha)$ in quasi-closed form. Let $x^+ = \max\{x, 0\}$, and note that all proofs are relegated to the e-companion.

Proposition 1 **(Optimal Second-Stage AV Acquisition)**. *Let $\hat{q}_2(q_1, w, \alpha)$ be a solution in q_2 to*

$$(r - w)F(q_1 + q_2 + \alpha M) + wF(q_1 + q_2) = r - c_2. \tag{4}$$

Then:

(i) If $w > r$, then either $q_2(q_1, w, \alpha)$ coincides with a solution to (4) *such that $0 < \hat{q}_2(q_1, w, \alpha) \le b_2$, or we have $q_2(q_1, w, \alpha) = b_2$, or $q_2(q_1, w, \alpha) = 0$. Moreover, $q_2(q_1, w, \alpha)$ is non-decreasing in α.*

(ii) If $w \le r$, then $\hat{q}_2(q_1, w, \alpha)$ is unique and we have

$$q_2(q_1, w, \alpha) = \min\{\hat{q}_2(q_1, w, \alpha)^+, b_2\},$$

and $q_2(q_1, w, \alpha)$ is non-increasing in α.

Moreover, $q_2(q_1, w, \alpha)$ is non-decreasing in w.

Condition (4) is the FOC obtained from the platform's expected profit (2). We note that in *(i)* there could be multiple solutions because when $w > r$, the left-hand side in (4) is not necessarily monotone. For $w \le r$, the left-hand side in (4) is monotone in q_2 and, therefore, there is a unique solution. Additionally, the proposition establishes the monotonicity of $q_2(q_1, w, \alpha)$ with respect to α and w. For a given wage w, $q_2(q_1, w, \alpha)$ optimally balances the profit coming from AVs and human-driven vehicles. As the joining fraction α increases, the platform can satisfy a larger fraction of the demand with humans which leads to a (weakly) larger human-driver profit if and only if $r \ge w$. In turn, to compensate, it is optimal for the platform to (weakly) decrease q_2 if and only if $r \ge w$. Now, for a given joining fraction α, the platform is always better off increasing q_2 as the wage increases because human-served rides become more expensive and they can be substituted by AVs. Interestingly, in the next section, we will see that when we consider the equilibrium wage associated with each joining fraction α, the optimal q_2 can increase in α even when $w \le r$.

In what follows, we will rely on the implicit characterization of $q_2(q_1, w, \alpha)$ given by Proposition 1 to analyze the optimal wage and joining fraction. Nevertheless, in the special case of exponential demand, we obtain a closed-form expression. We provide this expression in the following corollary, and we will later return to the exponential distribution as a running example to make our results more concrete and to build intuition. For exponentially distributed demand, the cumulative distribution function F is given by $F(x) = 1 - e^{-x/\mu}$ (recall that μ is the mean demand).

Corollary 1 **(Second-Stage AV Acquisition with Exponential Demand)**. *If demand is exponentially distributed, then the optimal q_2 is*

$$q_2(q_1, w, \alpha) = \min\left\{\left(\mu \log\left(\frac{(r-w)e^{-\alpha M/\mu} + w}{c_2}\right) - q_1\right)^+, b_2\right\}. \tag{5}$$

The larger expression on the RHS of (5) is obtained by substituting the exponential CDF into the FOC (4) and isolating the unique solution. Note that for exponential demand, the FOC has a unique solution even when $w > r$. For ease of exposition, we will henceforth assume that $b_1 = b_2 = \infty$, except where otherwise specified. Later, in Section 7.1, we will return to the case of finite b_1 and b_2.

5 Human Driver Recruitment

Human drivers join the platform if and only if their expected payoff exceeds their outside option. Let $\gamma(\cdot)$ be the *matching rate* of a driver to passengers when a fraction α of humans joins the platform and the platform's AV acquisition decisions are q_1 and q_2; that is,

$$\gamma(\alpha, q_1 + q_2) \triangleq \frac{\mathsf{E}\left[\min\{[D - (q_1 + q_2)]^+, \alpha M\}\right]}{\alpha M}. \tag{6}$$

Note that γ represents the anticipated number of requests per unit of time (up to a constant that we normalize to one) that each driver receives in the horizon considered in their joining decision. We also point out that the contribution of q_1 and q_2 to γ is fully specified by their sum $q_1 + q_2$. The expression for γ captures the matching decision of the platform. If $D < q_1 + q_2$, the platform only matches AVs; if $D \in [q_1 + q_2, q_1 + q_2 + \alpha M]$, the platform randomizes the excess demand among human drivers; and finally, if $D > q_1 + q_2 + \alpha M$, then all drivers are matched. For a human driver who joins the platform, her utility function is her expected earnings, i.e., $w\gamma(\alpha, q_1 + q_2)$. For a human driver who does not join the platform, her utility is simply the outside option v. A human driver joining fraction α is a *driver equilibrium* if, given q_1, w, and the anticipated q_2, no joining human driver has incentive to switch to not joining and vice versa (see Appendix F for a formal definition). Recalling that $q_2(q_1, w, \alpha)$ is the optimal second-stage acquisition quantity, we can write our driver equilibrium condition as

$$w\gamma(\alpha, q_1 + q_2(q_1, w, \alpha)) = v.^{10} \tag{EQ}$$

Later, we will discuss in detail the endogenous relationship between w, α, and q_2 in equilibrium. For now, it suffices to point out this endogeneity and note that the equilibrium condition (EQ) plays a critical role in our analysis, as well as precipitating some counterintuitive findings.

In what follows, it will be useful to define the wage w that can induce a certain joining fraction α. That is, we fix α in (EQ) and use $w(\alpha)$ to denote a value of w that induces a fraction α of drivers to join the platform

[10] In some cases, it is possible to satisfy the equilibrium condition strictly (driver expected earnings strictly larger than the outside option); Lemma EC.7 in Appendix F shows that it is always optimal to satisfy it at equality.

(as needed, this notation can be augmented as $w(q_1, \alpha)$ to incorporate the dependence on q_1). In general, $w(\alpha)$ may not be unique, but our results either do not need uniqueness or verify it for the setting under consideration.

The following proposition demonstrates that while AVs are still relatively expensive to acquire (recall that $v < c_1$ in our model), there is indeed a place on the road for human drivers.

Proposition 2 **(Need for Human Drivers)**. *It is optimal for the platform to induce some human drivers to participate in equilibrium; that is,* $\alpha^\star > 0$.

Having now established that the platform prefers to include human drivers in its sourcing for rides, we next examine in detail the platform's recruitment strategy.

6 Solving the Platform's Problem and the Race to the Top

Here, we study in detail the endogenous relationship between the wage, the human driver joining fraction, and the second-stage AV quantity, revealing a surprising phenomenon that has negative implications for the platform's human-driver recruiting efforts and, by extension, its profits.

6.1 Structural Results

We start by showing that in the platform's optimal solution, it does *not* exercise the second-stage AV acquisition option. That is, the platform does not find it beneficial to acquire additional AVs after observing the human driver participation level, and this is true even if the AV acquisition cost is the same in both stages.

Proposition 3 **(Second-Stage AV Acquisition Option Not Used)**. *If* $c_1 < c_2$*, then in the optimal solution, the platform does not use the second-stage acquisition option (i.e.,* $q_2^\star = 0$*). Moreover, if* $c_1 = c_2$*, then an optimal solution exists where the platform does not use the second-stage acquisition option.*

Thus, despite the second-stage AV acquisition opportunity enabling the platform to react to the realized human-driver joining fraction α by acquiring additional AVs if needed, Proposition 3 reveals that in the optimal solution, the platform does not avail itself of this opportunity. The proof relies on showing that for any feasible solution where $q_2 > 0$, another feasible solution exists with $q_2 = 0$ and equal (if $c_1 = c_2$) or higher (if $c_1 < c_2$) revenue.

Intuitively, then, we might naturally expect that the second-stage acquisition opportunity is completely irrelevant, and that the equilibrium outcome is the same with or without it (i.e., whether $b_2 > 0$ or $b_2 = 0$). That is, since the constraint $q_2 \leq b_2$ is not binding, it should not impact the platform's profit. Strikingly, however, this turns out not to be the case at all: as our next structural result shows, the platform's profit is in fact *lower* when b_2 is positive than when it is zero.

Let $\mathcal{P}_{b_2}$ denote the platform's problem $\mathcal{P}$ parameterized by given b_2. Similarly, let $\Pi^\star(b_2)$ denote the optimal profit for $\mathcal{P}_{b_2}$.

Proposition 4 **(Flexibility Hurts the Platform)**. *Consider fixed $b_2 > 0$ (not necessarily infinite). We have:*

(i) $\Pi^\star(b_2) \le \Pi^\star(0)$*, i.e., access to the second-stage AV acquisition option* ***decreases*** *the optimal profit;*

(ii) For any feasible solution to $\mathcal{P}_{b_2}$, there exists a feasible solution to $\mathcal{P}_0$ with equal or higher profit (and the same wage and human joining fraction);

(iii) There exists (x, α) such that a feasible solution $(q_1, w, \alpha, 0)$ exists to $\mathcal{P}_0$ with $q_1 + q_2 = q_1 + 0 = x$, but no feasible solution (q_1, w, α, q_2) exists to $\mathcal{P}_{b_2}$ with $q_1 + q_2 = x$.

Although part (i) of Proposition 4 has a weak inequality ($\Pi^\star(b_2) \le \Pi^\star(0)$), we will see later that a strict ordering is the norm, not the exception. That is, even when $c_1 = c_2$, the profit when $b_2 = 0$ is often substantially more than when b_2 is positive. Part (ii) provides an even stronger finding: not only does the ordering exist at the optimal solutions, but in fact for *any* solution to $\mathcal{P}_{b_2}$, we can find a corresponding feasible solution to $\mathcal{P}_0$ with the same wage and human joining fraction that achieves the same or higher profit. Finally, part (iii) reveals that having access to the second-stage acquisition option ($b_2 > 0$) counterintuitively *limits* the platform's options. Let x be the total number of AVs that the platform acquires, in aggregate across both stages. Because q_2 must be a best response to the other quantities in the game, it turns out that there are some values of x that are feasible in $\mathcal{P}_0$ but not in $\mathcal{P}_{b_2}$.

There is an apparent conflict between Propositions 3 and 4. Proposition 3 shows that the second-stage AV acquisition option is not used, but Proposition 4 shows that the optimal profit is higher when $b_2 = 0$ than when $b_2 > 0$. This raises an important question: why does having the flexibility to acquire more AVs in the second stage ($b_2 > 0$) *hurt* the platform's profit? As it turns out, the answer relates to human driver recruitment and the platform's commitment power (really, lack thereof) for its second-stage AV acquisition, as foreshadowed by part (iii) of Proposition 4. That is, the threat that the platform might set $q_2 > 0$ affects the required wage to recruit a given number of human drivers, harming the platform's profit in the process. To better understand this phenomenon, we next study human driver recruitment and the equilibrium wage in more detail.

6.2 Human Driver Wages and the Race to the Top

To reveal the drivers of the platform's human-driver recruitment decisions, we use the human joining fraction α as our reference quantity and study its impact on the equilibrium values of the other quantities. Important insights emerge, both for small and for large values of α, due to the platform's and the human drivers' equilibrium decisions and their interactions.

We start with small α and uncover the interplay between the substitution of AVs with human drivers and the competition among human drivers. When humans join, the platform reduces its AV acquisition (when $w \le r$—see Proposition 1) because humans cover some of the demand, entailing two opposing effects on the matching rate. As human participation increases, competition from AVs initially attenuates, while competition among human drivers intensifies; the net effect is ambiguous. With the next proposition, we characterize the relationship for small human joining fractions.

Proposition 5 **(Substitution vs. Competition)**. *The wage $w(0,\alpha) \leq r$ is decreasing in α in a neighborhood of zero if $c_2 > 2v$ and increasing in a neighborhood of zero if $c_2 \in (v, 2v)$.*

The proposition establishes that when c_2 is sufficiently high, if the platform does not acquire any AVs in the first stage, the required wage to induce a small fraction of human drivers to join the platform is decreasing in such fraction because the substitution of AVs with humans dominates the added competition among humans. In fact, because c_2 is relatively large, the platform chooses to reduce its AV acquisition quantity enough so that the matching rate γ increases despite more humans joining the platform. The equilibrium constraint (EQ) then forces the wages to decrease.

Wages that locally decrease in the labor supply have been observed in the literature (e.g., in Taylor 2018 and Benjaafar et al. 2022) but with a fundamentally different origin. Typically, the reason for decreasing wages in thicker markets is an increased service level. In our case, increased supply from one source causes the platform to decrease supply from the other source, and the net effect on wages can be negative.

We now study the impact of the endogenous relationship between wages, human participation decisions, and AV acquisition at higher human joining fractions α, where the behavior is of a decidedly different nature. Strikingly, we will see that the platform can get in its own way in its efforts to recruit human drivers.

In choosing the wage w, the platform must consider its impact on the human participation level and, by extension, on the optimal second-stage AV acquisition quantity. The next proposition characterizes a uniform bound for the impact of wages on the human participation.

Proposition 6 **(Limited Human Driver Recruitment)**. *If F has decreasing mean residual life*[11] *and $w \leq r$, then the human driver equilibrium joining fraction α is bounded above by*

$$\alpha \leq \frac{c_2 \mu}{M v}. \tag{7}$$

Proposition 6 establishes a cap on human driver recruitment, which depends on the problem parameters. For a low outside option v, it is easier for the platform to attract more human drivers. When the average demand μ is high, the platform may also be able to attract more humans simply because there are more potential passengers. However, as the second-stage acquisition cost c_2 of AVs decreases, fewer humans can be convinced to enter the market because they will anticipate more AVs.

When the upper bound in Equation (7) is less than 1, the platform cannot possibly induce all human drivers to participate and break even on human-served rides. Intuitively, to induce a very high level of human participation, the platform will have to offer higher wages to compensate for the increased competition among human drivers. However, for very high wages, the platform strongly prefers to meet demand with AVs, and the optimal q_2 will increase, which reduces the human matching rate γ. So, increasing the wage increases the

[11] The mean residual life (MRL) of a random variable D is $\mathsf{E}[D - x | D > x]$. Decreasing MRL represents a broad class of distributions, including all distributions with increasing hazard rate.

earnings conditional on matching, but it may reduce the equilibrium matching rate. The platform can attempt to counter this effect by increasing the wage even more, but that would increase the optimal q_2 still further and decrease the human matching rate in a feedback loop. The next proposition makes this intuition precise.

Note that $\alpha M\gamma(\alpha, q_1 + q_2(q_1, w, \alpha))$ corresponds to the expected aggregate demand served by human drivers. We define the maximum of this quantity for a given wage, w, and first-stage AV quantity, q_1, by

$$R(q_1, w) \triangleq \max_{\alpha M} \ \alpha M \cdot \gamma(\alpha, q_1 + q_2(q_1, w, \alpha)),$$

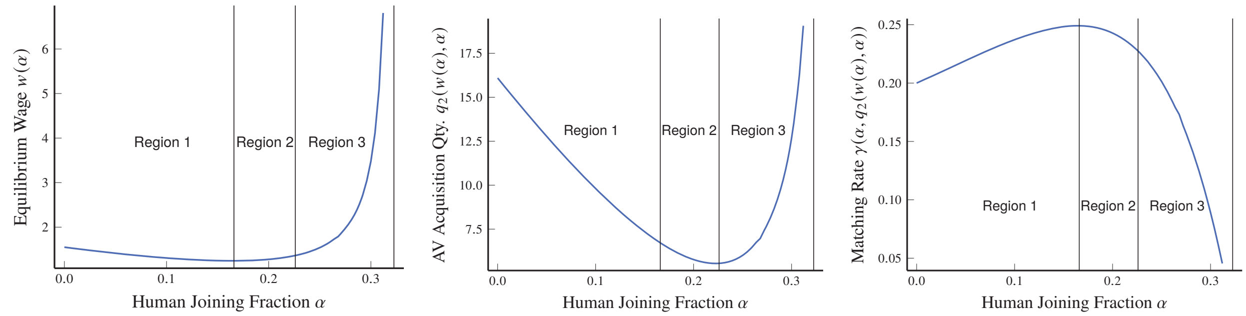
and thus $wR(q_1, w)$ is the maximum possible aggregate human driver earnings at wage w and first-stage AV quantity q_1.

Proposition 7 **(Race to the Top: General Case)**. *If $c_2/\bar{F}(q_1) \leq w \leq r$ and F has decreasing mean residual life, the maximum possible aggregate human driver earnings are non-increasing in the wage w.*

Proposition 7 reveals the surprising fact that (an upper bound on) aggregate human driver earnings is actually *decreasing* in the wage offered to human drivers (weakly or strictly so depending on the distribution, as we will see). This result has implications for the ability of the platform to recruit human drivers. By (EQ), in equilibrium we must have $\alpha M v = w\alpha M\gamma(\alpha, q_1 + q_2(q_1, w, \alpha)) \leq wR(q_1, w)$. By showing that the rightmost quantity in this relation is decreasing in w (and recalling that M and v are constants), Proposition 7 thus establishes that the largest human joining fraction α that could possibly be achieved in equilibrium is *decreasing* in the wage. In other words, and counterintuitively, increasing the human-driver wage deters human drivers! We proceed to investigate this phenomenon further.

For larger human joining fractions, the platform must raise the wage to increase the joining fraction. However, more human drivers at a higher wage increases the optimal second-stage AV acquisition quantity because AVs become relatively more preferable to the platform. At high wages, human drivers anticipate intense competition from AVs and a low matching rate. So, to increase human participation, the platform must raise wages even higher, creating a *race to the top with itself* that thwarts recruitment as shown by Propositions 6 and 7: the feedback loop of increasing wages and increased AV acquisition means that increasing wages can actually *reduce* human participation, effectively capping the amount of human drivers that can be incentivized into the system. Put simply, because q_2 must be a best response to the wage and human-driver joining fraction, the platform cannot credibly commit to a low quantity while also setting wages high enough to induce very high human participation. We note that the assumption on the range of w in Proposition 7 is always satisfied when the threat of acquiring AVs in the second period is strongest, i.e., $q_1 = 0$, and it is also satisfied at the optimal q_1 in our numerical simulations in Section 7.1.

As is evident from their proofs, Propositions 6 and 7 require careful analysis to establish key properties of the interaction between the platform and its human drivers. That these findings hold for a broad class of demand distributions places their structural implications on firm ground. With the structural results as a



(a) Wage (b) AV Quantity q_2 (c) Human Matching Rate

Figure 2 Equilibrium choices vs. α ($M = 100$, $r = 5$, $c_2 = 1$, $v = .31$, $b_1 = 0$, exponential demand with $\mu = 10$).

guide, for the remainder of this section we temporarily specialize to the exponential distribution, for which we can characterize all the main quantities in closed form. The precise expressions illuminate the driving forces behind the race to the top.

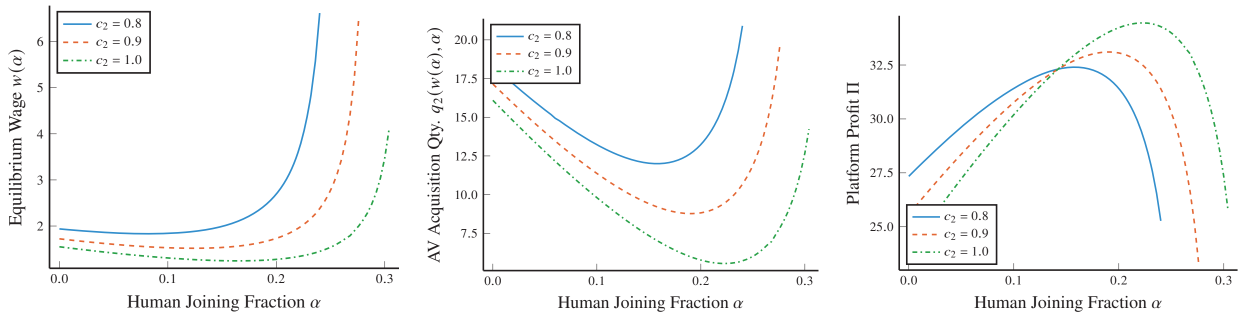
Proposition 8 **(Race to the Top: Closed Form)**. *If demand is exponential and $F(M) \le (r - c_2)/r$, then the equilibrium joining fraction α is bounded above by $c_2\mu/(Mv)$. Moreover, for $b_1 = 0$ (and thus dropping q_1 from the notation), inducing an equilibrium human joining fraction $0 < \alpha < c_2\mu/(Mv)$ is consistent with a wage w and AV quantity q_2 of*

$$w(\alpha) = \frac{\alpha M r v}{(e^{\alpha M/\mu} - 1)(c_2\mu - \alpha M v)}, \qquad q_2(w(\alpha), \alpha) = \mu \log\left(\frac{\mu e^{-\alpha\lambda M} r}{c_2\mu - \alpha M v}\right), \tag{8}$$

and a matching rate given by

$$\gamma(\alpha, q_2(w(\alpha), \alpha)) = \frac{(c_2\mu - \alpha M v)}{r}\left(\frac{1 - e^{-\alpha M/\mu}}{\alpha M}\right). \tag{9}$$

Proposition 8 reveals the inner workings of the race to the top, and we plot each of the relevant quantities in Figure 2. As α increases from zero, at first, the platform decreases its AV quantity q_2. Reduced competition from AVs dominates the increased competition among human drivers, and thus the required wage actually decreases as α increases (see Proposition 5). However, this trend eventually reverses as competition among humans intensifies. Moreover, eventually (near $c_2\mu/(Mv) \approx 0.32$, indicated by the rightmost vertical line in each plot), the platform must set the wage *higher* than its revenue per ride ($w(\alpha)$ increases without bound as α approaches $c_2\mu/(Mv)$, which can be seen by inspecting the denominator of $w(\alpha)$ in Eq. 8), so that it would rather lose a unit of demand entirely than serve it with a human driver. The optimal AV quantity q_2 thus increases ($q_2(w(\alpha), \alpha)$ can also be arbitrarily large), so that as α increases, eventually the matching rate decreases fast enough that the platform cannot further increase human participation—hence the first part of Proposition 8. Note that this extends Proposition 6 for the exponential case by showing that the upper bound on human driver recruitment holds for arbitrarily high wages; that is, the platform could not escape the race to the top even by paying wages higher than the revenue per ride (which, naturally, would never be optimal anyway). Additional details about the case of exponential demand can be found in Appendix A in the e-companion.



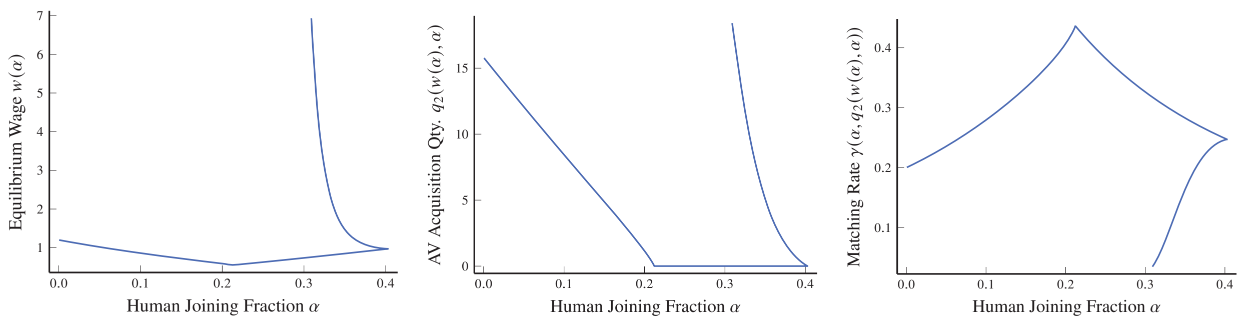
(a) Wage (b) AV Quantity q_2 (c) Profit $\Pi(w(\alpha), \alpha, q_2(w(\alpha), \alpha))$

Figure 3 Equilibrium choices vs. α for different c_2 ($M = 100$, $r = 5$, $v = .31$, exponential demand with $\mu = 10$).

Returning our attention to Figure 2, we observe three distinct regions of α. In the first region, as α increases from zero, the optimal AV quantity q_2 (Figure 2b) decreases because human drivers can cover some demand, outweighing the increased competition among human drivers and yielding a net increase in γ, the human matching rate (see Figure 2c). With a higher matching rate, the platform can pay a lower wage (compare Figure 2a and 2c). So, the firm gets more human drivers at a lower cost, unambiguously positive. Eventually, q_2 slows its descent (Figure 2b), so the additional competition among humans is no longer offset by the reduction in AVs. At this point, we enter the second region. The matching rate γ now decreases in α, so the wage must increase. However, the wage is still low enough that more human drivers benefits the platform, so the optimal q_2 still decreases in α. Finally, there is a critical point at which the increased competition causes the matching rate γ to decrease more rapidly, so the wage must increase more rapidly to counter, bringing us into the third region. But when the required wage becomes too high, the firm begins to increase q_2, triggering the runaway race to the top of increasing wages and increasing AV quantity. Note that, at this critical point, the only potential way for the platform to increase human participation would be to increase wages; however, the wages are already so high that the platform must counter with a further elevated acquisition of AVs. The net recruitment effect is negative in that it fundamentally limits the amount of human drivers that join the platform and, as a consequence, the platform's ability to garner additional revenue.

The race to the top phenomenon answers the question raised in Section 6.1 about why the flexibility of the second-stage AV acquisition option hurts the platform's profit. Namely, the platform cannot commit to acquiring a suboptimal number of AVs in the second stage. Thus, human drivers' anticipation of the platform's optimal quantity leads to a feedback loop that raises the required wage to recruit a given number of human drivers, which decreases the optimal profit. We can further validate this intuition by considering the role of the second-stage AV cost c_2, as well as more general demand distributions, to see whether this phenomenon continues to prevail.

We now briefly compare outcomes for different values of the AV acquisition cost c_2, whose effect turns out to be rather nuanced: see Figure 3. At small α, as we might expect, the profit is highest when c_2 is smallest, simply because AVs are cheaper. However, for larger α when more human drivers join the platform, the effect

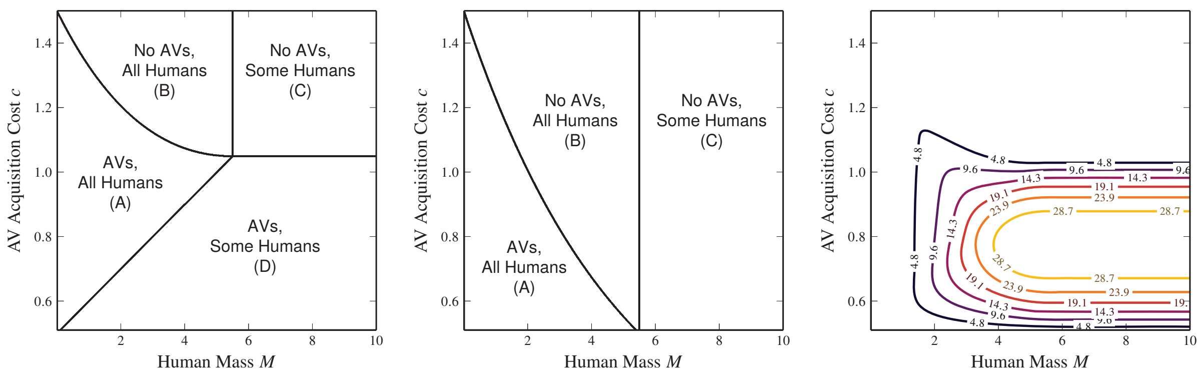


(a) Wage (b) AV Quantity q_2 (c) Human Matching Rate

Figure 4 Equilibrium choices vs. α ($M = 100$, $r = 5$, $c_2 = 1$, $v = .24$, Weibull demand: $\mu = 10$, $k = 1.2$).

of the AV cost on the relationship between the wage and q_2 yields a more complicated picture. When c_2 is larger, the platform acquires fewer AVs for given α, and the transition point where q_2 starts to increase is at a higher α (Figure 3b). The lower q_2 implies that the platform can achieve a given α with a lower wage (Figure 3a), and moreover the race to the top does not take hold until a higher human participation level (the bound of Propositions 6 and 8 is increasing in c_2). Perhaps surprisingly, in Figure 3c we see that for larger α, the indirect wage reduction effect of higher c_2 is strong enough to outweigh the direct AV cost increase. This leads to higher profit for higher c_2 at fixed α, and furthermore, we see that *higher c_2 leads to a higher optimal profit*. In other words, the race to the top phenomenon is strong enough that the platform would actually *prefer* to pay more for its second-stage AVs because mitigating the race to the top is worth more to it than cheaper AVs.

Finally, as mentioned, the exponential distribution yields closed-form results that paint a clear picture of the race to the top and the resulting limits on human-driver recruitment. Interestingly, for other distributions, recruitment can be even more severely hindered, and next we provide an example to illustrate this additional richness. We have just seen that for exponential demand, the human joining fraction has an upper bound but does continue to increase (albeit at a declining rate) with the wage asymptotically toward this bound. This is a consequence of the constant failure rate. By contrast, for distributions with strictly increasing failure rate, the human joining fraction eventually begins to “curl back” on itself as the wage increases, reflecting an even more severe version of the race to the top. This behavior is reflected in Figure 4, for a Weibull distribution with an increasing failure rate. As with exponential demand, the wage is decreasing for small α, then increasing for larger α. The two main differences are that (i) the optimal AV quantity is zero for a range of α and (ii) near the bound of Proposition 6, instead of the wage increasing without bound as α increases to a limit, here the joining fraction “curls back” and starts to *decrease* as the wage increases further. In this case, at the critical point where higher wages are the only option for the platform to recruit more human drivers, the joining fraction is not merely fundamentally limited but actually strictly decreases with the wages. Below, we study the effect of the race to the top on the platform’s profit.



(a) Both Acquisition Options (b) Early Acquisition Only ($b_2 = 0$) (c) Pct. Profit Difference

Figure 5 Equilibrium comparison and profit loss for $r = 1.5$, $v = .5$, exponential demand with $\mu = 5$.

6.3 Magnitude of Profit Loss Due to the Race to the Top

Figure 5a characterizes the equilibrium when $b_2 = \infty$,[12] over ranges of the AV acquisition cost c (to illuminate the role of the relative timing of AV acquisition rather than that of any cost advantage in the first stage, in the figure $c_1 = c_2 = c$) and human driver population size M, and Figure 5b depicts the equilibrium for $b_2 = 0$ (see Appendices A and B in the e-companion for closed-form expressions). We will refer to the setting with $b_2 = 0$ as the "early-only" case and with $b_2 = \infty$ as the "early-late" case. The area in which all human drivers are active is larger in the early-only case, due primarily to the race to the top in the early-late case. There is a sizable overlap between region D (AVs, Some Humans) in the early-late case with region A (AVs, All Humans) in the early-only case. In region D (AVs, Some Humans) of Figure 5a, the AV acquisition cost is intermediate, so human drivers are valuable, but if the wage increases then the platform will quickly switch to AVs, hence the race to the top. In this region, the platform employs some but not all humans in the early-late case. By contrast, in the early-only case, the AV quantity does not respond to changes in the wage, so the platform can profitably convince all humans to join the platform over a wider range of parameters, allowing it to achieve the ideal AV/human-driver mix and the corresponding higher profit implied by Proposition 4.

We now demonstrate that the second-stage AV acquisition option can lead to a substantial difference in optimal profits. In Figure 5c, the contour bands reflect the percentage decrease in the optimal profit between the early-only and early-late cases. In regions with similar solutions in both cases, the loss is minimal. However, for other parameters, the difference in profit is nearly 30%. Particularly large profit losses occur with large M and intermediate c. This regime implies plenty of potential human drivers and an AV cost high enough that overage cost is significant, so the hedge offered by humans is indeed valuable, but not so high that AVs are not viable. In the early-only case, the platform can acquire only a few AVs in the first stage; in this case, knowing that the platform does not have access to the late acquisition option, human drivers expect higher earnings and are willing to join in larger numbers. However, in the early-late case, the race to the top

[12] Note that panels (a) and (c) would be the same for any $b_1 \geq 0$: for $b_2 = \infty$, the optimal profit is the same for any $b_1 \geq 0$ by Lemma EC.6 in Appendix E in the e-companion, and the optimal solution for $b_1 = 0$ is feasible and thus optimal for any $b_1 > 0$.

hinders recruitment; if the wage rises, then because the AV cost is not too high, the best-response second-stage AV acquisition quantity will increase substantially. This increase reduces the human-driver matching rate and offsets any increase in their expected earnings, and either the bound on human drivers of Proposition 6 is tight, or the required wages to induce full human participation are too high to be optimal. The platform must then rely more on AVs than it would like. The next proposition complements these observations by establishing that there are problem instances for which the profit loss is even larger than that identified in the figure.

Proposition 9 (**Magnitude of Harm from Second-Stage AV Acquisition**). *There exist instances of our problem for which the optimal profit* $\Pi^\star(b_2)$ *(for the case where the platform has access to both the first- and second-stage AV acquisition options) is more than* ***38% lower*** *than the optimal profit* $\Pi^\star(0)$ *(for the case where the platform has only the first-stage AV acquisition option).*

Importantly, the instance identified in the proof has $c_1 = c_2$, so the large profit difference that it identifies is not due to a cost advantage of the early acquisition opportunity over the late acquisition opportunity. Rather, the difference here stems entirely from the race to the top.

7 Extensions

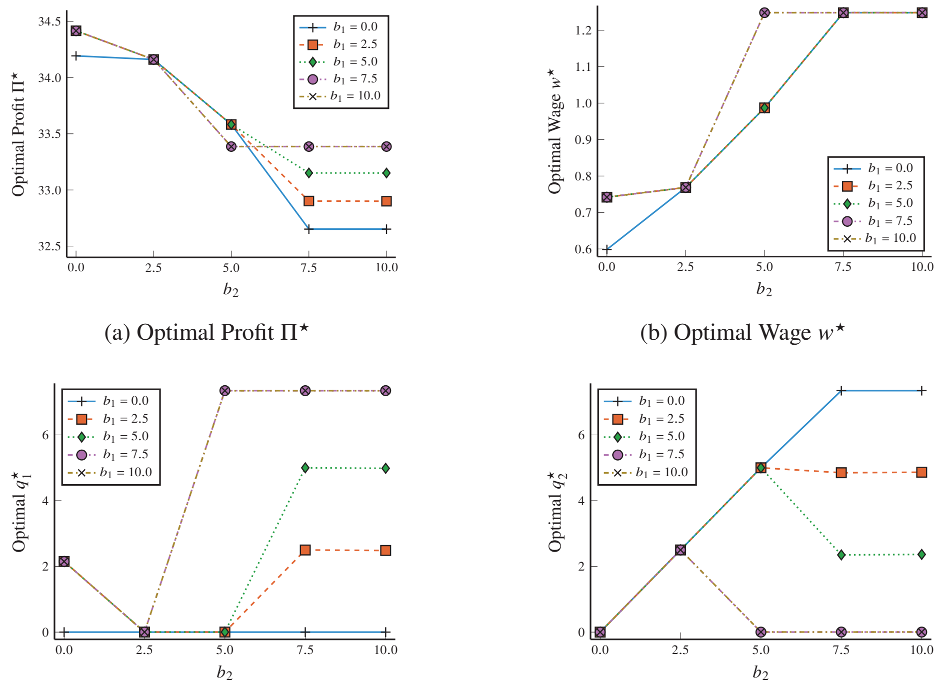
In this section, we extend our study in two directions: first, we investigate the role of finite but positive b_1 and b_2, and second, we allow for customers who may have preferences between AVs and human-driven vehicles.

7.1 Capacity-Constrained AV Acquisition

Especially as AV technology continues to develop, it is possible that AV suppliers may be constrained, limiting the number of AVs that a ride-hailing platform can acquire. Accordingly, we now consider the case where the first- or second-stage AV acquisition opportunity is available but limited, i.e., such that $0 < b_1 < \infty$ and/or $0 < b_2 < \infty$.

Figure 6 plots the optimal profit (panel a) and wage (b) and the optimal q_1 (c) and q_2 (d) against b_2 for different values of b_1. For all values of b_1 that we consider, the optimal profit is decreasing in b_2. Thus, our main managerial insight—that the second-stage acquisition option hurts the platform due to the race to the top—continues to hold qualitatively for finite b_1 and b_2. Indeed, the finding is more granular in this case. Our earlier results showed that the platform's profit was lower when $b_2 > 0$ than when $b_2 = 0$ and revealed why this was so; here, we find furthermore that the profit is decreasing in b_2. Additionally, aligning with our earlier discussion about the race to the top in wages, we see that the wage is increasing as b_2 increases (even though the optimal α—not plotted—stays the same).

Interestingly, the individual-stage AV quantities are non-monotonic in b_2: see panels (c) and (d) in the figure. For larger values of b_2, the platform purchases AVs in the first stage ($q_1^\star > 0$), and for b_1 high enough, it does all of its purchasing in the first stage ($q_2^\star = 0$), in line with Proposition 3 for the non-capacity-constrained



(a) Optimal Profit $\Pi^\star$

(b) Optimal Wage $w^\star$

(c) Optimal $q_1^\star$

(d) Optimal $q_2^\star$

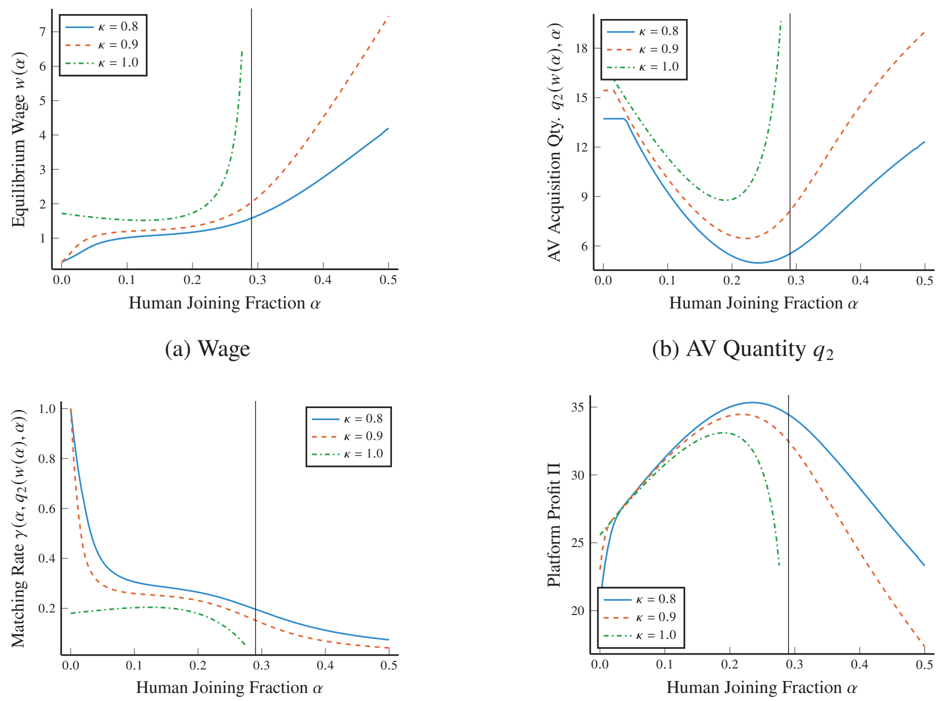
Figure 6 Equilibrium outcomes for finite b_1 and b_2 ($M = 15$, $r = 5$, $c_1 = .9$, $c_2 = 1$, $v = .31$, exponential demand with $\mu = 10$).

case. Similarly, for $b_2 = 0$ and b_1 high enough, the platform acquires some AVs in the first stage ($q_1^\star > 0$ and $q_2^\star = 0$). However, for $b_2 = 2.5$, for all values of b_1, we have $q_1^\star = 0$ and $q_2^\star = 2.5$. If the platform anticipates buying in the second stage at a higher price and attempts to buy early to avoid this, it will still end up buying in the second stage anyway because of the high unconstrained optimal quantity due to the race to the top. In other words, buying in the first stage will not "substitute" for the second stage in this regime, but will instead merely increase the total number of AVs acquired. Hence, for this value of b_2, it is better for the platform not to acquire any AVs at all in the first stage when they are cheaper ($q_1^\star = 0$, as mentioned), even knowing that this means it will pay a higher price when it buys them in the second stage.

Overall, when there are capacity constraints on the acquisition of AVs, the race to the top continues to drive the outcome, fundamentally influencing the mix of human drivers and AVs that supply the platform.

7.2 Rider Preferences

To capture the possibility that some passengers may not be willing to accept rides from AVs, we now extend our model to the case where a fraction $0 < \kappa \leq 1$ of requested rides can be served with either an AV or a human-operated vehicles and that the remaining $1 - \kappa$ fraction must be served with a human-operated vehicle. The platform allocates AV-ready demand first to AVs, and human drivers serve excess AV-ready demand and all human-only demand, up to the total available supply.



(a) Wage

(b) AV Quantity q_2

(c) Human Matching Rate

(d) Profit $\Pi(w(\alpha), \alpha, q_2(w(\alpha), \alpha))$

Figure 7 Equilibrium vs. α for different κ ($M = 100$, $r = 5$, $c_2 = .9$, $v = .31$, exponential demand with $\mu = 10$).

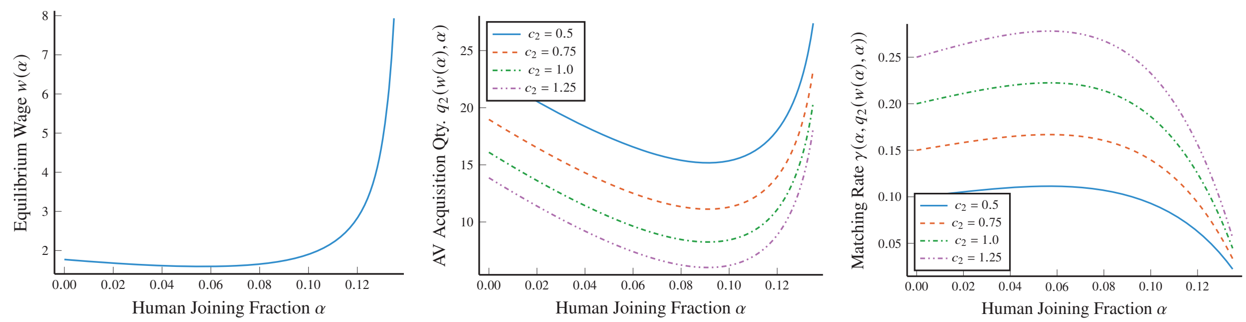
Suppose that in the first stage, the platform initially acquired q_1 AVs, it has set the wage at w, and a fraction α of human drivers have joined the platform. If the platform acquires an additional q_2 AVs in the second stage and the realized demand is d, then the platform's profit with rider preferences is

$$\pi(q_1, w, \alpha, q_2; d) \triangleq r \min\{\kappa d, q_1 + q_2\} - c_1 q_1 - c_2 q_2 \\ + (r - w)\left(\min\{\alpha M, (1-\kappa)d\} + \min\{\alpha M - \min\{\alpha M, (1-\kappa)d\}, [\kappa d - q_1 - q_2]^+\}\right),$$

which can be rewritten as

$$\pi(q_1, w, \alpha, q_2; d) \triangleq r \min\{\kappa d, q_1 + q_2\} - c_1 q_1 - c_2 q_2 \\ + (r - w)\left(\min\{\alpha M, (1-\kappa)d\} + \min\{[\alpha M - (1-\kappa)d]^+, [\kappa d - q_1 - q_2]^+\}\right). \tag{10}$$

As before, the platform seeks to maximize its expected profit, calculated by integrating the RHS of Equation (10) over the demand PDF f. In Equation (10), observe that there are now two contributions to the volume of human-served rides (the quantity on the second line that is multiplied by $r - w$). The first component comes from "human-only" passengers, i.e., those not willing to take an AV-served ride, and this is the minimum of the total number of human drivers available (αM) and the amount of human-only demand ($(1-\kappa)d$). The second component comes from left-over demand from passengers who are willing to take an AV but for whom there are not enough AVs to serve; this is the minimum of (i) the excess AV-ready demand not served by human drivers and (ii) the number of human drivers available after serving all of the human-only demand. To focus on the interaction between the AV-readiness of the customer population on the race to the top, for the rest of the subsection we assume $b_1 = 0$ (we also drop q_1 from our notation).



(a) Equilibrium Wage $w(\alpha)$ (b) AV Quantity q_2 (c) Human Matching Rate

Figure 8 Equilibrium outcomes (c_2 disc. uniform on $[.75, 1, 1.25, 1.5]$, $r = 5$, $v = .31$, $M = 200$, expon. dem. with $\mu = 5$).

Figure 7 depicts the equilibrium outcomes over a range of human joining fractions α for three different values of the AV-ready fraction κ. For reference, a vertical line is plotted in each panel at the upper bound of Proposition 6 (which holds for $\kappa = 1$). Intuitively, the smaller κ is, the better off human drivers are in that they will face less competition from AVs: for small κ, independent of how many AVs the platform acquires, a large fraction of the demand can only be served by human drivers. This increases human drivers' matching rate (see Figure 7c) and simultaneously reduces the pressure on the platform to acquire AVs (Figure 7b), mitigating the competition between AVs and human drivers and thus making it easier for the platform to recruit human drivers. So, the race to the top (the feedback loop of increasing wages and increasing q_2) becomes less severe. Indeed, Figure 7a validates this intuition: as κ decreases, the required wage to recruit a given human joining fraction α decreases accordingly, and joining fractions that were not even achievable at higher κ become achievable. Interestingly, at lower α, we do not observe the decreasing wage for $\kappa < 1$ as we do for $\kappa = 1$. Because there is built-in demand for human drivers for $\kappa < 1$, the interaction between q_2 and the human-driver matching rate γ is weaker; as α increases from zero, the platform does not initially remove any AVs because the few human drivers will essentially be dedicated to human-only demand. The AV quantity eventually begins to decrease in α, but not by enough to make the wage decrease. Then, at higher α, we observe qualitatively similar behavior to the $\kappa = 1$ case of our base model: increasing wage and increasing AV quantity q_2, leading to a familiar (albeit less severe) feedback loop. Intuitively, for even smaller κ the feedback loop will continue to diminish in severity. Nevertheless, in this extended setting where some customers are only willing to accept human-served rides, we clearly see the same driving force—the feedback loop of increasing wages and increasing AV acquisition—that precipitates the race to the top rearing its head once again.

Interestingly, the end result of the above phenomena is that the optimal profit (Figure 7d) *increases* as the AV-ready fraction κ *decreases*. That is, because of its mitigating effect on the race to the top, some customers' reticence to accept AV technology can actually benefit the platform.

7.3 Random AV Cost c_2

In our base model, both c_1 and c_2 are common knowledge from the start of the game. To account for possible uncertainty in the second-stage AV cost c_2, due, e.g., to frequently changing market conditions, we now incorporate a random c_2.[13] Specifically, we suppose that c_2 is realized from a (known) probability distribution and revealed to the players at the start of the second stage (i.e., after human drivers make their joining decisions but before the platform chooses q_2). To focus on the race to the top, we here assume $b_1 = 0$.

Figure 8a depicts the equilibrium wage as a function of α, and Figure 8b plots the corresponding AV quantity q_2 for each possible realization of the random c_2. In this setting, human drivers make their joining decisions based on the *expected value* of the matching rate $\gamma(\alpha, q_2(w(\alpha), \alpha))$, where the expectation is with respect to the AV cost c_2. Encouragingly, we observe a very similar phenomenon to our base model. The AV quantity is different at given α for different realizations of c_2 (matching our intuition, it is higher for lower values of c_2), but its curvature in α is similar across realizations and to that observed earlier in, e.g., Figure 2b. Hence, the effect on the human drivers' matching rate (and thus its expected value) is also similar across realizations and to that observed earlier (Figure 2c); additionally, as we might expect, the matching rate is higher for higher values of c_2, since q_2 is lower for such values as just described. Because for each c_2, the optimal q_2 and human matching rate follow trajectories of similar shapes to those observed throughout our study, the equilibrium wage also follows a similar trajectory, even though the second-stage AV cost is random. In short, the underlying driving force behind the race to the top is still present under a random second-stage AV acquisition cost.

8 Conclusion

We have studied a ride-hailing platform supported by both a fleet of autonomous vehicles (AVs) and self-interested human drivers with their own vehicles. The platform faces a tradeoff between these sources for serving rides. AVs are fully in the platform's control with known acquisition cost (they are also strategically valuable for the long term), but this is a fixed cost incurred before demand is realized. On the other hand, human drivers only represent a cost when matched, but the platform must manage incentives successfully to recruit them, which requires promising high enough expected earnings; indeed, the required wage is endogenous and is tightly bound to the platform's AV decisions.

Our key findings relate to the value of a shared road for ride-hailing platforms and the confounding race to the top. Human drivers can be a valuable hedge against demand risk because they incur no cost when not serving a passenger. However, when a ride-hailing platform serves rides with both human drivers and AVs, complicated dynamics govern the endogenous relationship between human driver wages, the human driver participation level, and the platform's AV acquisition decisions.

For relatively low levels of human participation, the equilibrium wage can actually be *decreasing* in the participation level: as human participation increases, the platform optimally acquires fewer AVs, reducing

[13] For clarity, we note that the setting of this section is different from the analysis and discussion surrounding Figure 3. There, we compared outcomes for different instances of our game, each with a different deterministic value of c_2; here, we are considering a single instance of our game with a random c_2.

the competition from AVs so that a lower wage is required to satisfy human drivers. At higher levels of human participation, eventually not many AVs remain, so the net effect on the matching rate of an increase in human participation switches from positive to negative, and hence the required wage switches from decreasing to increasing. Above this point, as human participation—and hence the required wage—increases, human-served rides become less profitable for the platform, which increases its AV quantity to compensate. Increased competition from AVs negates the benefit to human drivers of increased wages, necessitating still higher wages and creating a feedback loop that leads the platform into a race to the top with itself. The feedback loop arises because the platform cannot credibly commit to a second-stage AV quantity that will be sub-optimal given the equilibrium wage and human participation level. In fact, the platform's profit is always higher if the platform does not even have an opportunity to acquire AVs reactively after observing the human participation level, and we showed that the difference in profit can be more than 38%. So, in planning for the shared road, a ride-hailing platform must beware the consequences of human drivers' rational anticipation of its AV acquisition decisions; despite the promise of this exciting technology, its introduction may trigger the race to the top, pushing human drivers out of the market and harming profits along the way.

The success or failure of a ride-hailing platform in avoiding the race to the top may depend primarily on exogenous factors: as we have observed, higher AV costs or limited short-term AV supply can mitigate the feedback loop and thus, counterintuitively, lead to higher profits. Absent such factors, however, the race to the top is difficult to avoid due to the strong incentives that precipitate it. Interestingly, a platform with a history of being forthright with its drivers in other matters may have an easier time. A promise not to acquire a large fleet of AVs can be considered cheap talk, but if, for instance, a platform has consistently done right by its drivers in paying promised wages, giving accurate hourly earnings forecasts, etc., then such a verbal assurance about AVs may be believed by human drivers. If so, then these drivers will have less reason to fear a large reactive AV acquisition, the anticipation of which would otherwise create the feedback loop. On the other hand, a platform with a reputation for focusing on the bottom line at the expense of driver satisfaction may be especially vulnerable to the race to the top. While such messaging games and reputation effects fell outside the scope of our study, they could (among many other exciting questions in this area) form the basis for future research on AVs in ride-hailing, which we hope will continue apace.

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E-Companion

Appendix. Technical Details and Proofs

In this appendix, we first provide additional details for the case of exponentially distributed demand. Then, we give technical details for the special case of a single upfront acquisition opportunity (i.e., $b_2 = 0$). After that, we provide the proofs of the main results in the paper, and finally we give auxiliary results and proofs, including a separate section with some technical details for the human driver equilibrium.

A Details for Exponential Demand

We focus here on the case with $b_1 = 0$ and thus drop the dependence on q_1 from our notation. First, we note that the driver equilibrium condition (EQ) may have multiple solutions or no solution at all in α. If the equation has no solution, then in equilibrium either $\alpha = 0$ or $\alpha = 1$, while if there are multiple solutions to the equation, then all are equilibria. However, although there may be multiple equilibria for a given wage, with exponential demand, for any given equilibrium joining fraction α, there is a unique wage that can induce this fraction. In case of multiple equilibrium fractions for the same wage, we assume that the platform can induce the equilibrium fraction of its choosing. Because the platform prefers more human drivers all else being equal, it should always choose the highest fraction corresponding to a given wage. In summary, instead of describing how the joining fraction, the AV quantity, and the matching rate vary with the wage, we can express the latter two quantities and the wage in terms of the joining fraction.

The wage may exceed the unit revenue r, entailing a negative overage "cost" such that matching passengers with human drivers actually represents a net loss for the platform. This possibility may seem far-fetched because the platform would be foolish to facilitate rides on which it loses money (apart from special discounts to gain market share, which are beyond the scope of this work), but it is relevant because of the equilibrium condition (EQ). In some cases, to induce a particular α, the firm must pay a wage higher than the revenue per ride.

Proposition 8 in the main body provides closed-form solutions for all the quantities of interest. This result parallels Propositions 6 and 7 for the case of exponential demand. It first establishes that regardless of the wage that the platform sets (cf. Proposition 6, which requires $w \le r$), the joining fraction α cannot exceed $c_2\mu/(Mv)$. The platform can never provide enough incentive for drivers to induce them all to join (when this bound is smaller than 1). To see why this is true, first note that the matching rate defined in Equation (6) specialized to the exponential case is

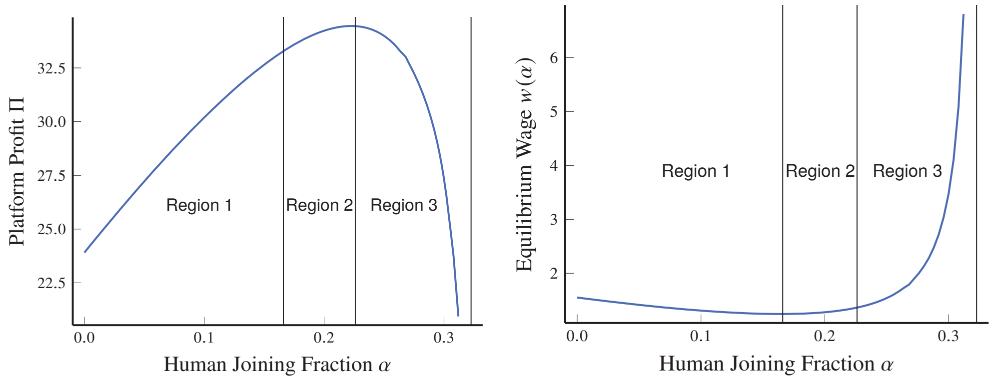
$$\gamma(\alpha, q_2) = \mu e^{-q_2/\mu}\left(\frac{1 - e^{-\alpha M/\mu}}{\alpha M}\right).$$

For a given wage w, combining the above with the FOC for the optimal AV quantity in Proposition 1 gives the following expression for a driver's expected earnings (the wage multiplied by the matching rate)

$$w\gamma(\alpha, q_2(w,\alpha)) = \frac{c_2\mu}{\alpha M}\left(1 - \frac{r}{r + w(e^{\alpha M/\mu} - 1)}\right). \tag{EC.1}$$

Note that this quantity is increasing in the wage w (this also implies that for given α, the wage $w(\alpha)$ is unique if it exists). However, because q_2 adjusts to changes in w, the platform is restricted in its ability to increase the expected earnings, even for arbitrarily high wages. Indeed,

$$\lim_{w\to\infty} \frac{c_2\mu}{\alpha M}\left(1 - \frac{r}{r + w(e^{\alpha M/\mu} - 1)}\right) = \frac{c_2\mu}{\alpha M}.$$



(a) Platform Profit $\Pi(w(\alpha), \alpha, q_2(w(\alpha), \alpha))$ (b) Equilibrium Wage

Figure EC.1 Relationship between wage, joining fraction, and profit ($M = 100$, $r = 5$, $c_2 = 1$, $v = .31$, $\mu = 10$).

Because the expected human driver earnings are increasing in the wage, the limiting value $c_2\mu/(\alpha M)$ is an upper bound on the human driver expected earnings for any wage. This bound on human driver expected earnings constrains the level of human participation that the platform can induce. In fact, when we account for the equilibrium outcome, i.e., we set Equation (EC.1) equal to the outside option v, we see that the joining fraction α can be no larger than $c_2\mu/(Mv)$.

We next provide in closed form the optimal wage, human joining fraction, and AV acquisition quantity q_2.

Proposition EC.1 **(Platform's Optimal Solution)**. *If $F(M) \le (r - c_2)/r$, $b_1 = 0$, and the demand is exponentially distributed with mean μ, then the platform's optimal solution is*

$$w^\star = \frac{r(c_2 - v)}{v(e^{\frac{c_2 - v}{v}} - 1)}, \quad \alpha^\star = \min\left\{\frac{\mu(c_2 - v)}{Mv}, 1\right\}, \quad \textit{and} \quad q_2^\star = \mu \log\left[\frac{(r - w^\star)e^{-\alpha^\star M/\mu} + w^\star}{c_2}\right]^+. \tag{EC.2}$$

The solution in Proposition EC.1 achieves a balance between the required wages and AV quantity. Consider Figure EC.1 and Figure 2 as the joining fraction increases from 0. First, both the required wage and the AV quantity decrease while the matching rate increases. This is profitable for the platform because it can get more human drivers at a smaller wage while saving by acquiring fewer AVs. But as more human drivers enter the market, the drops in wage diminish until they become zero. In fact, to further induce more human drivers to join, the platform must start increasing wages while reducing the AV quantity as the matching rate decreases due to competition. This is still profitable for the platform because it prefers to serve customers with human drivers and it can cover a larger portion of the demand. There is a tipping point α at which the required wage becomes large enough that it is no longer profitable for the platform to continue incentivizing human drivers into the market, at which point the platform would prefer to use AVs to serve demand. At this point, it is better for the firm to increase its AV quantity so that more of the demand is served by higher-margin AVs. In fact, inducing a finite fraction of human drivers could require an arbitrarily large wage. Interestingly, $\alpha^\star$ is different from the joining fraction at which the wage starts increasing and the matching rate starts decreasing. That is, there is a region in which it is yet profitable for the platform to induce more humans into the market despite having to pay them more because their wage is still sufficiently small.

Note that the induced equilibrium joining fraction $\alpha^\star$ is increasing in c_2. That is, the more expensive AVs are, the more the platform wishes to offload some of the demand (or at least, demand risk) to human drivers. Additionally, $\alpha^\star$ decreases with the outside option reflecting the fact that a higher outside option makes human drivers more costly for the platform. The optimal wage is increasing in $c_2/v > 1$. For larger AV cost or for a lower outside option, the optimal wage increases because the platform aims to induce into the market a higher fraction of humans which, in turn, requires a higher wage.

B Single Upfront AV Acquisition Opportunity

In this section, we consider the special case of $b_2 = 0$. This implies that the platform has a single opportunity to acquire AVs, and that this opportunity falls before human drivers make their joining decisions.

As $b_1 = \infty$ and $b_2 = 0$ (and thus also $q_2 = 0$), the platform's optimization problem in this case reduces to

$$\begin{aligned} \Pi^\star(0) \triangleq \max_{(q_1, w, \alpha)} \quad & \Pi(q_1, w, \alpha, 0) \\ \text{s.t.} \quad & w\gamma(\alpha, q_1) = v. \end{aligned} \tag{$\mathcal{P}_0$}$$

We will use $(\tilde{w}, \tilde{\alpha}, \tilde{q}_1)$ to refer to the optimal solution of $\mathcal{P}_0$.

We know from Proposition 4 that $\Pi^\star(b_2') \le \Pi^\star(0)$ for any $b_2' > 0$. To induce an equilibrium human joining fraction α for a given AV quantity q_1, the platform must choose a wage w satisfying the driver equilibrium condition

$$w\gamma(\alpha, q_1) = v.^{14} \tag{EC.3}$$

This driver equilibrium condition differs from (EQ) in that q_1 replaces $q_2(w(\alpha), \alpha)$. When $b_2 = 0$, we again find that it is optimal to induce some human drivers to participate.

Proposition EC.2 **(Need for Human Drivers)**. *It is optimal for the platform to induce some human drivers to participate in equilibrium i.e., $\tilde{\alpha} > 0$.*

In the setting of the main body with $b_2 = \infty$, a closed-form characterization of the optimal solution is not achievable for general distributions due to the equilibrium condition and the dependence of q_2 on the joining fraction; by contrast, when $b_2 = 0$, we can completely characterize the optimal solution as follows. First, let $\hat{w}(q_1, \alpha)$ be a wage that satisfies equation (EC.3) for a joining fraction α and AV quantity q_1 (note that a unique $\hat{w}(q_1, \alpha)$ exists for any $0 \le \alpha \le 1$ and AV quantity q_1 with $F(q_1) < 1$: see Lemma EC.8 in Appendix F).

Let $\Pi_c^{\mathtt{NV}}(q)$ and $q_c^{\mathtt{NV}}$ denote the expected profit and the optimal quantity in the standard newsvendor problem with overage cost c and underage cost $r - c$; we also use the shorthand $\Pi_c^{\mathtt{NV}}$ for $\Pi_c^{\mathtt{NV}}(q_c^{\mathtt{NV}})$.

For the drivers who join, the matching rate is $\gamma(\alpha, q_1)$. As a result, in expectation the platform pays w to a total of $\alpha M\gamma(\alpha, q_1)$ human drivers. The platform's objective can then be rewritten as

$$\Pi(q_1, w, \alpha, 0) = r\Big[\int_0^{q_1+\alpha M} uf(u)\mathrm{d}u + (q_1 + \alpha M)\bar{F}(q_1 + \alpha M)\Big] - w\alpha M\gamma(\alpha, q_1) - c_1 q_1,$$

Using (EQ) in the equation above, and since $\hat{w}(q_1, \alpha)$ exists by Lemma EC.8, we have

$$\begin{aligned} \Pi(q_1, \hat{w}(q_1, \alpha), \alpha, 0) &= r\Big[\int_0^{q_1+\alpha M} uf(u)\mathrm{d}u + (q_1 + \alpha M)\bar{F}(q_1 + \alpha M)\Big] - \alpha M v - c_1 q_1 \\ &= r\Big[\int_0^{q_1+\alpha M} uf(u)\mathrm{d}u + (q_1 + \alpha M)\bar{F}(q_1 + \alpha M)\Big] - c_1(q_1 + \alpha M) - \alpha M(v - c_1) \\ &= \Pi_{c_1}^{\mathtt{NV}}(q_1 + \alpha M) + \alpha M(c_1 - v). \end{aligned} \tag{EC.4}$$

For given q_1, we can optimize this function in α to reveal the optimal human joining fraction $\alpha(q_1)$ for the platform to induce. The optimality equation for $\alpha(q_1)$ takes the form of a newsvendor-like critical fractile (see Lemma EC.3 in Appendix E). After substituting $\alpha(q_1)$ and the corresponding wage, we can optimize the platform's profit in q_1.

[14] By Lemma EC.7 in Appendix F, it is optimal to satisfy this condition at equality.

Proposition EC.3 **(Platform's Optimal Solution to $\mathcal{P}_0$).** *We have $\tilde{w} = v/\gamma(\tilde{\alpha}, \tilde{q}_1)$. For the optimal AV quantity and induced human joining fraction:*

(i) if $F(M) \leq (r - c_1)/r$, then the platform optimally chooses a quantity $\tilde{q}_1$ and induces full human participation (i.e., it induces $\tilde{\alpha} = 1$), where $\tilde{q}_1$ is the unique solution to

$$F(\tilde{q}_1 + M) = \frac{r - c_1}{r} \quad \text{and} \quad \tilde{q}_1 = q_{c_1}^{\mathtt{NV}} - M; \tag{EC.5}$$

(ii) if $(r - c_1)/r < F(M) < (r - v)/r$, then the platform optimally acquires no AVs ($\tilde{q}_1 = 0$) and induces full participation from human drivers ($\tilde{\alpha} = 1$);

(iii) if $F(M) \geq (r - v)/r$, then the platform optimally deploys no AVs ($\tilde{q}_2 = 0$), and it induces a human joining fraction $\tilde{\alpha}$ such that

$$F(\tilde{\alpha} M) = \frac{r - v}{r}. \tag{EC.6}$$

We can understand Proposition EC.3 by probing the origins of the critical fractiles in Equations (EC.5) and (EC.6). If $F(M) \leq (r - c_1)/r$, then AVs are relatively inexpensive in the sense that the original newsvendor critical fractile is large relative to the number of human drivers (human drivers alone would not even be enough to cover the original newsvendor critical fractile). The solution in part *(i)* of the proposition implies that the platform employs all of the human drivers, but it covers the same fractile of the demand distribution as it would without them—compare to Equation (3). In other words, the platform achieves the same probability of shortage as it would with only AVs, but it replaces M AVs with human drivers by setting $\tilde{q}_1 = q_{c_1}^{\mathtt{NV}} - M$ and inducing $\tilde{\alpha} = 1$, paying the required wage to accomplish this. These human drivers provide the platform with a hedge against demand risk because it need pay wages only to the human drivers who are matched with a fare. Relative to the case with no human drivers, the platform reduces the probability that it will be on the hook for unused supply, without increasing the probability of shortage. This level of human participation ($\alpha = 1$) might not be feasible when b_2 is nonzero (as we saw for exponential demand in Appendix A), and, even if feasible, it might not be profitable: if $M \geq (c_2/v)\mu$, then by Proposition 6, it would not be optimal to induce all human drivers to participate because the equilibrium wage will be higher than the unit revenue.

Conversely, if $(r - c_1)/r < F(M)$, then AVs are expensive enough (and there are enough human drivers) that the platform actually prefers not to use AVs at all. This is a "conservative" outcome for the platform: because AVs are relatively expensive and it has access to human drivers who present no overage risk, it is better to fulfill demand only with human drivers. Whether to induce all or only some of the human drivers to participate depends on the number of human drivers and the value of their outside option v. Fixing the number of human drivers, with a low outside option human drivers are cheap and, therefore, inducing all of them to join is optimal. With a higher outside option, it is not optimal to induce all of them to join, but it is optimal to cover the demand fractile $(r - v)/r$ using only human drivers. Summarizing, for $b_2 = 0$, it is always optimal for the platform to cover a fractile of the demand between $(r - c_1)/r$ and $(r - v)/r$. The optimal point within this interval to choose, and what mix of human drivers and AVs to use to reach it, depends on the cost parameters, the demand distribution, and the number of human drivers.

C Proofs of Main Results

Proof of Proposition 1. Differentiating the expected profit (2) with respect to q_2, we get

$$\frac{\partial \Pi}{\partial q_2} = r\bar{F}(q_1+q_2+\alpha M) + w\Big[F(q_1+q_2+\alpha M) - F(q_1+q_2)\Big] - c_2. \tag{EC.7}$$

The first-order condition (FOC) (4) follows by rearranging equation (EC.7).

First, suppose that $w > r$. At $q_2 = 0$, from equation (EC.7) we have

$$\frac{\partial \Pi}{\partial q_2}\Big|_{q_2=0} = r - c_2 + (w-r)F(q_1+\alpha M) - wF(q_1), \tag{EC.8}$$

and we always have that

$$\lim_{q_2\to\infty} \frac{\partial \Pi}{\partial q_2} = -c_2.$$

If the RHS of (EC.8) is positive, then by the Intermediate Value Theorem there exists $q_2 \in (0,\infty)$ such that $\frac{\partial \Pi}{\partial q_2} = 0$ and such q_2 is optimal. Otherwise, either $q_2 = 0$ is optimal, or the optimal q_2 solves the FOC.

Next, suppose that $w \le r$. At $q_2 = -q_1 - \alpha M$ we have $\frac{\partial \Pi}{\partial q_2} > 0$. Hence, again, by the Intermediate Value Theorem there exists $q_2 \in [-q_1 - \alpha M, \infty)$ such that $\frac{\partial \Pi}{\partial q_2} = 0$. Differentiating the profit Π a second time, from equation (EC.7) we get

$$\begin{aligned}\frac{\partial^2 \Pi}{\partial q_2^2} &= -rf(q_1+q_2+\alpha M) + w\Big[f(q_1+q_2+\alpha M) - f(q_1+q_2)\Big]\\ &= (w-r)f(q_1+q_2+\alpha M) - wf(q_1+q_2)\\ &< 0,\end{aligned}$$

where the last inequality follows because $w \le r$. Thus, the function is concave, and the FOC (4) is sufficient for a global maximum.

If the global maximizer is negative then because of the concavity, the optimal feasible quantity is zero. If the global maximizer is positive and below b_2, then the optimal quantity coincides with $\hat{q}_2(w,\alpha)$; but if $\hat{q}_2(w,\alpha) > b_2$, then the optimal quantity is b_2. In summary, if $w \le r$ then the optimal quantity is given by $\min\{\hat{q}_2(w,\alpha)^+, b_2\}$.

Finally, to see the monotonicity of $q_2(w,\alpha)$ with respect to α and w in the two cases, it is enough to study the modularity of Π. We have

$$\frac{\partial^2 \Pi}{\partial q_2 \partial \alpha} = (w-r)Mf(q_1+q_2+\alpha M), \quad \text{and} \quad \frac{\partial^2 \Pi}{\partial q_2 \partial w} = F(q_1+q_2+\alpha M) - F(q_1+q_2),$$

that is, given w, Π is super-modular when $w > r$ and sub-modular when $w \le r$. Hence, by Topkis Theorem, in the former case $q_2(w,\alpha)$ is non-decreasing in α and in the latter case $q_2(w,\alpha)$ is non-increasing in α. Also, given α, Π is always super-modular. Thus, by Topkis Theorem $q_2(w,\alpha)$ is non-decreasing in w. This concludes the proof. ■

Proof of Corollary 1. If $c_2 < r \le w$, then at $q_2 = -q_1$, the LHS of the FOC (4) is equal to

$$(r-w)F(\alpha M) \le 0 < r - c_2.$$

Thus, the LHS is less than the RHS at $q_2 = -q_1$. Also, from equation (EC.7) we have

$$\frac{\partial \Pi}{\partial q_2}\Big|_{q_2=-q_1} = r\bar{F}(\alpha M) - c_2 + wF(\alpha M) = r - c_2 + (w-r)F(\alpha M) > 0. \tag{EC.9}$$

Because also $\lim_{q_2\to\infty} \partial\Pi/\partial q_2 = -c_2$, a local maximum must exist by the Intermediate Value Theorem. Differentiating the LHS of equation (4) in q_2 gives

$$r f(q_1+q_2+\alpha M) + w\,(f(q_1+q_2) - f(q_1+q_2+\alpha M)) \geq 0,$$

where the inequality holds because the exponential density function f is decreasing. Thus, the LHS is increasing in q_2 and the RHS is constant, so the solution to the FOC is unique. Together with the above arguments, this implies that the function Π is unimodal on the extended domain $[-q_1, \infty)$, with the solution to the FOC (4) its unique maximizer.

We conclude the following: (i) if the solution $\hat{q}_2$ to the FOC is negative, then the optimal q_2 is zero; (ii) if $0 \leq \hat{q}_2 \leq b_2$, then the optimal q_2 is $\hat{q}_2$; and (iii) if $\hat{q}_2 > b_2$, then the function Π is still increasing at b_2 and therefore the optimal q_2 is b_2. The desired result is thus proved for $r \leq w$.

For $w < r$, the result follows from using the exponential distribution in Proposition 1 part (ii). ■

Proof of Proposition 2. Using the notation from Appendix B, first note that $\hat{w}(q_{c_1}^{\mathtt{NV}}, 0) = v/\gamma(0, q_{c_1}^{\mathtt{NV}}) < r$ because $\gamma(0, q_{c_1}^{\mathtt{NV}}) = \bar{F}(q_{c_1}^{\mathtt{NV}}) = c_1/r$ (see Lemma EC.5). Thus, by Proposition 1, for the given first-stage quantity $q_{c_1}^{\mathtt{NV}}$, wage $\hat{w}(q_{c_1}^{\mathtt{NV}}, 0)$, and $\alpha = 0$, the FOC (4) has a unique solution $\hat{q}_2(q_{c_1}^{\mathtt{NV}}, \hat{w}(q_{c_1}^{\mathtt{NV}}, 0), 0)$.

Consider $c_1 < c_2$ and fixed but arbitrary $b_2 > 0$. In this case, since $F(q_{c_1}^{\mathtt{NV}}) = (r - c_1)/r$, we have $\hat{q}_2(q_{c_1}^{\mathtt{NV}}, \hat{w}(q_{c_1}^{\mathtt{NV}}, 0), 0) < 0$ by equation (4). Together with Proposition 1 (which since $w \leq r$ also implies the continuity of $\hat{q}_2(\cdot)$ and $q_2(\cdot)$), this implies that (i) $q_2(q_{c_1}^{\mathtt{NV}}, \hat{w}(q_{c_1}^{\mathtt{NV}}, 0), 0) = 0$, and (ii) there exists $\varepsilon > 0$ such that $\hat{q}_2(q_{c_1}^{\mathtt{NV}}, \hat{w}(q_{c_1}^{\mathtt{NV}}, \alpha), \alpha) < 0$ (and thus $q_2(q_{c_1}^{\mathtt{NV}}, \hat{w}(q_{c_1}^{\mathtt{NV}}, \alpha), \alpha) = 0$) for all $\alpha < \varepsilon$. We then also have

$$\hat{w}(q_{c_1}^{\mathtt{NV}}, \alpha)\gamma(\alpha, q_{c_1}^{\mathtt{NV}} + q_2(q_{c_1}^{\mathtt{NV}}, \hat{w}(q_{c_1}^{\mathtt{NV}}, \alpha), \alpha)) = \hat{w}(q_{c_1}^{\mathtt{NV}}, \alpha)\gamma(\alpha, q_{c_1}^{\mathtt{NV}}) = v,$$

where the first equality holds because $\hat{w}(q_{c_1}^{\mathtt{NV}}, \alpha)\gamma(\alpha, q_{c_1}^{\mathtt{NV}})$ and the second follows by the definition of $\hat{w}(\cdot)$ (the existence of which comes from Lemma EC.8). In turn, we conclude that the solution $(q_{c_1}^{\mathtt{NV}}, \hat{w}(q_{c_1}^{\mathtt{NV}}, \alpha), \alpha, 0)$ is feasible for $\mathcal{P}_{b_2}$ for all $\alpha < \varepsilon$.

By the proof of Proposition EC.2 in Appendix D, this implies that the platform can achieve a strictly higher profit than $\Pi_{c_1}^{\mathtt{NV}}$ by recruiting a fraction $\alpha > 0$ of human drivers. Since $\Pi_{c_1}^{\mathtt{NV}}$ is the highest profit that can be achieved with $\alpha = 0$, the result follows.

For $c_1 = c_2$, the result follows by a related but more tedious argument, which we omit for brevity. ■

Proof of Proposition 3. By Proposition 2, it is optimal to have strictly positive α. Suppose that $\alpha' > 0$ is optimal, and note that this also implies that $w \leq r$, as otherwise the platform would be strictly better off setting a wage low enough that no human drivers join, which would contradict the optimality.

Now, assume by way of contradiction that a candidate solution $(q_1', w', \alpha', q_2')$ with $q_2' > 0$ is optimal for $\mathcal{P}$. Since $w \leq r$, by part (ii) of Proposition 1 (and since $b_2 = \infty$ and $q_2' > 0$), we must have that q_2 uniquely solves the FOC (4) for the given q_1', w', and α'.

Now, let $q_1'' = q_1' + q_2'$ and $q_2'' = 0$, and consider the alternative solution $(q_1'', w', \alpha', q_2'')$. Clearly $0 \leq q_1'' < \infty$, satisfying the second constraint of $\mathcal{P}$. Next, we know that q_2' uniquely solves the FOC (4) for a first-stage acquisition quantity of q_1', and inspection of the same equation thus reveals that $q_2'' = 0$ must uniquely solve the FOC when the first-stage acquisition quantity is q_1'' (for the same wage w' and joining fraction α'); in other words, $0 \in \arg\max_{0\leq q_2<\infty} \Pi(q_1'', w', \alpha', q_2)$, satisfying the third constraint of $\mathcal{P}$. Moreover, we have $w\gamma(\alpha, q_1'') = w\gamma(\alpha, q_1' + q_2') = v$, where the second equality holds by the optimality (hence feasibility) of (q_1, w, α, q_2) for $\mathcal{P}$; thus, the first constraint of $\mathcal{P}$ is also satisfied. All three constraints are satisfied, so $(q_1'', w', \alpha', q_2'')$ is feasible for $\mathcal{P}$.

If $c_1 < c_2$, then inspection of Equation (2) reveals that $\Pi(q_1', w', \alpha', q_2') < \Pi(q_1'', w', \alpha', q_2'') = \Pi(q_1' + q_2', w', \alpha', 0)$, contradicting the optimality of $(q_1', w', \alpha', q_2')$. Hence, a solution with $q_2 > 0$ cannot be optimal, so we must have $q_2 = 0$ at optimality.

If instead $c_1 = c_2$, then returning to Equation (2) shows that $\Pi(q_1', w', \alpha', q_2') = \Pi(q_1'', w', \alpha', q_2'') = \Pi(q_1' + q_2', w', \alpha', 0)$. Thus, any feasible solution with $q_2 > 0$ can be equaled in profit by another feasible solution with $q_2 = 0$, implying that an optimal solution must exist with $q_2 = 0$. ■

Proof of Proposition 4. We first prove part (ii). Consider a feasible solution (q_1, w, α, q_2) to $\mathcal{P}_{b_2}$. If $q_2 = 0$, then (ii) trivially holds. Now suppose $q_2 > 0$, and let $q_1' = q_1 + q_2$. By the feasibility of (q_1, w, α, q_2) for $\mathcal{P}_{b_2}$, we have $w\gamma(\alpha, q_1') = v$, and thus the candidate solution $(q_1', w, \alpha, 0)$ satisfies the first constraint of $\mathcal{P}_0$. For the second constraint, we have $0 \le q_1' \le b_1$ because q_1 and q_2 are nonnegative by their feasibility for $\mathcal{P}_{b_2}$ and $b_1 = \infty$. For the third constraint, we trivially have $0 \in \arg\max_{0 \le q_2 \le 0} \Pi(q_1', w, \alpha, q_2)$. Finally, since $c_1 \le c_2$ and the wage and joining fraction are the same, we have $\Pi(q_1', w, \alpha, 0) \ge \Pi(q_1, w, \alpha, q_2)$ by Equation (2). Part (ii) is thus established, and part (i) immediately follows.

We last prove part (iii). Take $x = 0$ and $\alpha = 0$. By Lemma EC.8, we can find $\hat{w}(0,0)$ such that $(0, \hat{w}(0,0), 0, 0)$ is feasible for $\mathcal{P}_0$. Now, for $q_1 = q_2 = 0$ and $\alpha = 0$, by Equation (EC.8) in the proof of Proposition 1, we have

$$\left.\frac{\partial \Pi}{\partial q_2}\right|_{q_2=0} = r - c_2 > 0.$$

This implies that to satisfy the last constraint in $\mathcal{P}_{b_2}$ requires $q_2 > 0$, implying that no feasible solution to $\mathcal{P}_{b_2}$ exists with $q_1 + q_2 = 0$ and $\alpha = 0$. ■

Proof of Proposition 5. Consider the equilibrium condition for $\alpha > 0$ (we drop q_1 from our notation for this proof since it is zero):

$$w(\alpha)\gamma(\alpha, q_2(\alpha)) = v, \tag{EC.10}$$

where to simplify notation we are using $q_2(\alpha)$ to denote $q_2(w(\alpha), \alpha)$. Taking derivative on both sides above—this is possible thanks to Lemma EC.1—gives

$$\dot{w}(\alpha)\gamma(\alpha, q_2(\alpha)) + w(\alpha)\frac{d}{d\alpha}\gamma(\alpha, q_2(\alpha)) = 0. \tag{EC.11}$$

We will now take the limit as $\alpha \downarrow 0$ for each term in the previous equation which will give us an expression for the derivative of $w(\cdot)$ at zero. We have:

$$\begin{aligned}\frac{d}{d\alpha}\gamma(\alpha, q_2(\alpha)) &= \frac{1}{\alpha M}\left[\bar{F}(q_2(\alpha) + \alpha M)(\dot{q}_2(\alpha) + M) - \bar{F}(q_2(\alpha))\dot{q}_2(\alpha)\right] - \frac{1}{\alpha^2 M}\int_{q_2(\alpha)}^{q_2(\alpha)+\alpha M} \bar{F}(x)dx \\ &= \dot{q}_2(\alpha) \cdot \frac{1}{\alpha M}\left[\bar{F}(q_2(\alpha) + \alpha M) - \bar{F}(q_2(\alpha))\right] + \frac{1}{\alpha^2 M}\left\{\bar{F}(q_2(\alpha) + \alpha M)\alpha M - \int_{q_2(\alpha)}^{q_2(\alpha)+\alpha M} \bar{F}(x)dx\right\}\end{aligned}$$

We use L'Hôpital's rule to obtain the limit of each term above. For the first term, we have

$$\lim_{\alpha \downarrow 0} \dot{q}_2(\alpha) \cdot \frac{1}{\alpha M}\left[\bar{F}(q_2(\alpha) + \alpha M) - \bar{F}(q_2(\alpha))\right] = -\dot{q}_2(0) \cdot f(q_2(0)),$$

for the second term we have

$$\begin{aligned}
\lim_{\alpha\downarrow 0}\frac{1}{\alpha^2 M}\left\{\bar F(q_2(\alpha)+\alpha M)\alpha M-\int_{q_2(\alpha)}^{q_2(\alpha)+\alpha M}\bar F(x)dx\right\} &= \lim_{\alpha\downarrow 0}\left\{\frac{\bar F(q_2(\alpha)+\alpha M)M-f(q_2(\alpha)+\alpha M)(\dot q_2(\alpha)+M)\alpha M}{2\alpha M}\right.\\
&\quad\left.-\frac{\bar F(q_2(\alpha)+\alpha M)(\dot q_2(\alpha)+M)-\bar F(q_2(\alpha))\dot q_2(\alpha)}{2\alpha M}\right\}\\
&=\lim_{\alpha\downarrow 0}\frac{-f(q_2(\alpha)+\alpha M)(\dot q_2(\alpha)+M)}{2}\\
&\quad-\lim_{\alpha\downarrow 0}\dot q_2(\alpha)\frac{\bar F(q_2(\alpha)+\alpha M)-\bar F(q_2(\alpha))}{2\alpha M}\\
&=-\frac{f(q_2(0))(\dot q_2(0)+M)}{2}+\frac{\dot q_2(0)\cdot f(q_2(0))}{2}\\
&=-f(q_2(0))\frac{M}{2}.
\end{aligned}$$

Hence,

$$\lim_{\alpha\downarrow 0}\frac{d}{d\alpha}\gamma(\alpha,q_2(\alpha))=f(q_2(0))\left[-\frac{M}{2}-\dot q_2(0)\right] \tag{EC.12}$$

Also for $q_2(\alpha)$ we have

$$(r-w(\alpha))F(q_2(\alpha)+\alpha M)+w(\alpha)F(q_2(\alpha))=r-c_2.$$

Differentiating on both sides and then letting $\alpha\downarrow 0$ we have

$$\dot q_2(0)=\frac{w(0)\cdot M}{r}-M.$$

Using this in (EC.12), that $q_2(0)=q_{c_2}^{\mathtt{NV}}$ and letting $\alpha\downarrow 0$ yields

$$\frac{d}{d\alpha}\gamma(\alpha,q_2(\alpha))\big|_{\alpha=0}=f(q_2(0))\left[-\frac{M}{2}-\dot q_2(0)\right]=Mf(q^{\mathtt{NV}})\left[\frac{1}{2}-\frac{w(0)}{r}\right].$$

Using L'Hôpital's rule for $\gamma(\alpha,q_2(\alpha))$ gives

$$\lim_{\alpha\downarrow 0}\gamma(\alpha,q_2(\alpha))=\lim_{\alpha\downarrow 0}\frac{\bar F(q_2(\alpha)+\alpha M)(\dot q_2(\alpha)+M)-\bar F(q_2(\alpha))\dot q_2(\alpha)}{M}=\bar F(q_{c_2}^{\mathtt{NV}}).$$

Hence, taking the limit as $\alpha\downarrow 0$ in (EC.10) yields:

$$w(0)=\frac{v}{\bar F(q_{c_2}^{\mathtt{NV}})}.$$

Combining these results with (EC.11), we have

$$\dot w(0)\bar F(q_{c_2}^{\mathtt{NV}})+\frac{v}{\bar F(q_{c_2}^{\mathtt{NV}})}\cdot Mf(q_{c_2}^{\mathtt{NV}})\left[\frac{1}{2}-\frac{v}{r\bar F(q_{c_2}^{\mathtt{NV}})}\right]=0.$$

Recalling that $\bar F(q_{c_2}^{\mathtt{NV}})=c_2/r$ and rearranging terms gives

$$\dot w(0)=\frac{vr^2}{c_2^2}\cdot Mf(q_{c_2}^{\mathtt{NV}})\left[\frac{v}{c_2}-\frac{1}{2}\right].$$

That is, $w(\alpha)$ is decreasing in a neighborhood of 0 if $c_2>2v$, and $w(\alpha)$ is increasing in a neighborhood of 0 if $c_2\in(v,2v)$. ■

Proof of Proposition 6. Let α be an equilibrium. If $\alpha = 0$, then the result directly holds. Suppose then that $\alpha > 0$. For tidier notation, let q_2 denote the optimal quantity $q_2(q_1, w, \alpha)$. Then, from Proposition 1 and (EQ) we have that

$$(r-w)\bar{F}(q_1+q_2+\alpha M) + w\bar{F}(q_1+q_2) \le c_2$$
$$w\int_{q_1+q_2}^{q_1+q_2+\alpha M} \frac{(u-(q_1+q_2))}{\alpha M} f(u)\mathrm{d}u + w\bar{F}(q_1+q_2+\alpha M) = v.$$

Note that the first equation above becomes an equality when $q_2 > 0$, but it is an inequality when $q_2 = 0$. For $w \le r$, the profit function is concave, and $q_2 = 0$ is optimal only when the solution to the first order condition is negative (see (EC.7)). Define

$$L(x) \triangleq \mathsf{E}[(D-x)^+] = \mathsf{E}[D-x|D\ge x]\bar{F}(x).$$

Then, by writing the equilibrium condition in terms of $L(\cdot)$, we have that

$$\begin{aligned}\frac{c_2}{v} &\ge \frac{(r-w)\bar{F}(q_1+q_2+\alpha M) + w\bar{F}(q_1+q_2)}{w\left(L(q_1+q_2) - L(q_1+q_2+\alpha M)\right)}\alpha M \\ &\ge \frac{w\bar{F}(q_1+q_2)}{wL(q_1+q_2)}\alpha M \\ &= \frac{\alpha M}{\mathsf{E}[D-(q_1+q_2)|D\ge q_1+q_2]}.\end{aligned}$$

In turn, we deduce that

$$\alpha \le \frac{c_2\mathsf{E}[D-(q_1+q_2)|D\ge (q_1+q_2)]}{vM} \le \frac{c_2\mathsf{E}[D]}{vM},$$

where in the last inequality we have used that the mean residual life $\mathsf{E}[D-x|D\ge x]$ is non-increasing. ■

Proof of Proposition 7. Let us fix q_1 and consider $w \in [c_2/\bar{F}(q_1), r]$. First, define

$$H(q_1,w,x) \triangleq \int_{q_1+q_2(q_1,w,x)}^{q_1+q_2(q_1,w,x)+x} \bar{F}(y)dy \quad \text{and} \quad R(q_1,w) \triangleq \max_x H(q_1,w,x).$$

We need to prove that $wR(q_1,w)$ is decreasing for $w \in [c_2, r]$. From Proposition 1 part *(ii)*, we have that $q_2(q_1,w,x)$ is non-increasing in x. Hence, $H(q_1,w,x) \le H(q_1,w,\infty)$. Note that

$$q_2(q_1,w,\infty) \in \arg\max_q \left\{ r\int_0^{q_1+q} \bar{F}(y)dy + (r-w)\int_{q_1+q}^{\infty}\bar{F}(y)dy - c_2 q\right\}.$$

Thus

$$q_2(q_1,w,\infty) = \left(F^{-1}\left(1-\frac{c_2}{w}\right) - q_1\right)^+ = F^{-1}\left(1-\frac{c_2}{w}\right) - q_1,$$

where the last equality comes from $w \in [c_2/\bar{F}(q_1), r]$. We have

$$wR(q_1,w) = w\int_{q_1+\left(F^{-1}\left(1-\frac{c_2}{w}\right)-q_1\right)}^{\infty} \bar{F}(y)dy = \frac{c_2}{\bar{F}(u(w))}\int_{u(w)}^{\infty}\bar{F}(y)dy = c_2\mathsf{E}[D-u(w)|D>u(w)],$$

where $u(w) = F^{-1}\left(1-\frac{c_2}{w}\right)$. Since $u(w)$ is increasing in w and the mean residual life $\mathsf{E}[D-u|D>u]$ is decreasing, we conclude that $wR(q_1,w)$ is decreasing, as desired. ■

Proof of Proposition 8. That $q_1 = 0$ and $F(M) \le (r - c_2)/r$ together imply that the solution to the FOC (4) is nonnegative. As $q_1 = 0$ and $b_2 = \infty$, for the remainder of the proof we drop the dependence on q_1 from our notation. From Corollary 1, we can then write

$$q_2(w,\alpha) = \mu\left(\log\left((r-w)e^{-\alpha M/\mu} + w\right) - \log c_2\right). \tag{EC.13}$$

Additionally, note that we can simplify Equation (6) to yield a closed form for the human driver matching rate γ. For exponential demand, Equation (6) becomes

$$\gamma(\alpha, q_2(w,\alpha)) = \mu e^{-q_2(w,\alpha)/\mu}\left(\frac{1-e^{-\alpha M/\mu}}{\alpha M}\right). \tag{EC.14}$$

After substituting the RHS of equation (EC.14), the driver equilibrium condition (EQ) becomes

$$w\mu e^{-q_2(w,\alpha)/\mu}\left(\frac{1-e^{-\alpha M/\mu}}{\alpha M}\right) = v. \tag{EC.15}$$

Equations (EC.13) and (EC.15) form a system of equations for the unknowns w and $q_2(w,\alpha)$. We can substitute Equation (EC.13) for $q_2(w,\alpha)$ in equation (EC.15) and simplify to yield

$$w(\alpha) = \frac{\alpha M r v}{(e^{\alpha M/\mu} - 1)(c_2\mu - \alpha M v)}.$$

Plugging this into Equation (EC.13) yields

$$q_2(w(\alpha),\alpha) = \mu \log\left(\frac{e^{-\alpha\lambda M} r}{c_2 - \alpha M v/\mu}\right).$$

Finally, the bound on human driver expected earnings and the resulting bound on the human joining fraction is proved in the paragraph preceding Proposition EC.1 in Appendix A. ■

Proof of Proposition 9. As an existence result, we can fix parameters as desired. Accordingly, consider $c_1 = c_2 = c$. Let $\mathcal{I}$ denote an instance of our model characterized by r, v, c, M, and F. Also, let Π^E denote the optimal profit when $b_1 = \infty$ and $b_2 = 0$ (Π^E is merely a convenience notation for $\Pi^\star(0)$), and let Π^L denote the optimal profit when $b_1 = 0$ and $b_2 = \infty$. Since $c_1 = c_2 = c$, Lemma EC.6 implies that the optimal profit is the same for any $b_1 \ge 0$, and thus $\Pi^L = \Pi^\star(\infty)$. We will show that

$$\inf_{\mathcal{I}} \frac{\Pi^L}{\Pi^E} \le \frac{\sqrt{5}-1}{2},$$

which implies

$$\sup_{\mathcal{I}} \frac{\Pi^E - \Pi^L}{\Pi^E} \ge \frac{3-\sqrt{5}}{2} \approx 0.381.$$

It is enough to exhibit an instance of our problem for which the infimum above is below the desired bound. We consider $F(x) = 1 - e^{-x/\mu}$. Consider (c, M) such that

$$F(M) \le \frac{r-c}{r} \quad \text{and} \quad \frac{\mu\cdot(c-v)}{Mv} \le 1.$$

From (EC.2) in Proposition EC.1 and the conditions above, we have

$$w^\star = \frac{r(c-v)}{v(e^{\frac{c-v}{v}} - 1)}, \quad \alpha^\star = \frac{\mu(c-v)}{Mv}, \quad \text{and} \quad q_2^\star = \mu\log\left[\frac{(r-w^\star)e^{-\frac{c-v}{v}} + w^\star}{c}\right]^+,$$

and from Proposition EC.3 we have

$$\tilde{\alpha} = 1 \quad \text{and} \quad F(\tilde{q}_1 + M) = \frac{r-c}{r}. \tag{EC.16}$$

We also impose that $q_2 \geq 0$. That is, we impose that $(r-w^\star)e^{-\frac{c-v}{v}} + w^\star \geq c$. The ratio we are analyzing then becomes

$$\frac{\Pi^L}{\Pi^E} = \frac{\Pi_c^{\mathtt{NV}}(q_2^\star + \alpha^\star M) + \alpha^\star M(c-v)}{\Pi_c^{\mathtt{NV}}(\tilde{q}_1 + M) + M(c-v)} = -\frac{\mu\left(c\log\left(\frac{re^{1-\frac{c}{v}}}{v}\right) + c - r\right)}{c(M-\mu) + c\mu\log\left(\frac{c}{r}\right) - Mv + \mu r}.$$

Hence, the ratio $\inf_{\mathcal{I}} \frac{\Pi^L}{\Pi^E}$ is bounded above by the value of the following optimization problem:

$$\begin{aligned} \inf_{M,c,\mu,r,v} \quad & -\frac{\mu\left(c\log\left(\frac{re^{1-\frac{c}{v}}}{v}\right) + c - r\right)}{c(M-\mu) + c\mu\log\left(\frac{c}{r}\right) - Mv + \mu r} \\ \text{s.t} \quad & 1 - e^{-M/\mu} \leq \frac{r-c}{r}, \quad c \in (v, r), \\ & \frac{\mu \cdot (c-v)}{Mv} \leq 1, \\ & \frac{re^{1-\frac{c}{v}}}{v} \geq 1, \end{aligned} \tag{EC.17}$$

where the last constraint ensures that $q_2 \geq 0$.

To conclude the proof it is enough to exhibit a feasible solution to the problem above that achieves the desired bound. Fix μ, r and $a \in (1,2)$ and consider the following solution for $\delta > 0$ small:

$$c = r - \delta, \quad v = r - a\delta, \quad M = \delta.$$

We note that the above leads to a feasible solution as long as $r \in [\mu(a-1), \mu]$. Indeed, we clearly have that $c \in (v, r)$ and we also have that

$$1 - e^{-M/\mu} \leq \frac{r-c}{r} \Leftrightarrow r - r\frac{\delta}{2\mu} + o(\delta^2) \leq \mu,$$

which is satisfied for $\delta > 0$ small and r, μ bounded above and below. Also, $\frac{\mu\cdot(c-v)}{Mv} \leq 1$ is equivalent to $\mu(a-1) - \delta a \leq r$ which holds because we are taking $r \in [\mu(a-1), \mu]$. Finally, $\frac{re^{1-\frac{c}{v}}}{v} \geq 1$ is equivalent to

$$\frac{(a^2-1)\delta}{r} + o(\delta^2) \leq 1,$$

which holds for δ small for r bounded above and below. Moreover, we take

$$a = \frac{\sqrt{\mu^2 + 4r^2} - \mu + 2r}{2r},$$

which is in $(1,2)$ as previously assumed. Let $R(\mu, \delta, r)$ be the objective in (EC.17) at the feasible solution we have constructed. By replacing the values of c, v, M and a, it is possible to verify that

$$\lim_{(r,\delta)\to(\mu,0)} R(\mu, \delta, r) = \frac{\sqrt{5}-1}{2},$$

as desired. ■

D Proofs of Results in Appendices A and B

Proof of Proposition EC.1. Recall that $b_1 = 0$, so $q_1 = 0$; accordingly, we drop the dependence on q_1 from our notation. After substituting $w(\alpha)$ and $q_2(w(\alpha), \alpha)$ (from Proposition 8) and the exponential CDF into the expected profit (2), we obtain $\Pi(w(\alpha), \alpha, q_2(w(\alpha), \alpha))$. Differentiating this single-variable function of α then gives

$$\frac{\mathrm{d}\Pi}{\mathrm{d}\alpha} = c_2 M\left(1 + \frac{v}{\alpha M v/\mu - c_2}\right),$$

along with the FOC

$$\alpha = \mu\left(\frac{c_2 - v}{Mc_2}\right) > 0. \tag{EC.18}$$

Evaluating the derivative at $\alpha = 0$ gives

$$\frac{\mathrm{d}\Pi}{\mathrm{d}\alpha}\Big|_{\alpha=0} = c_2 M\left(1 - \frac{v}{c_2}\right) > 0, \tag{EC.19}$$

i.e., the function is increasing at $\alpha = 0$. Moreover, the derivative is decreasing in α for α such that $\alpha M v < c_2\mu$, which is the full range of interest: values of α outside this range cannot be induced for any wage, as noted previously. So, the function is concave in α on the relevant interval, and the FOC (EC.18) is sufficient for a global maximum. Moreover, the FOC can be rearranged to give

$$\alpha M v/\mu = c_2 - v < c_2,$$

implying that the solution to the FOC is contained in the relevant interval with a nonnegative required wage. If the solution to the FOC gives $\alpha > 1$, then the function is increasing on $[0,1]$, and the optimal solution is to set $\alpha = 1$, hence the minimum in equation (EC.2). The optimal wage and AV quantity are obtained by substituting equation (EC.18) into equation (8). ■

Proof of Proposition EC.2. Recall that $\Pi^{\mathtt{NV}}_{c_1}$ denotes the expected profit at the optimal newsvendor quantity with no human drivers with AV cost c_1. Since $q_2 = b_2 = 0$, the profit function evaluated at $q_1 = q^{\mathtt{NV}}_{c_1}$ is given by

$$\begin{aligned}\Pi(q^{\mathtt{NV}}_{c_1}, w, \alpha, 0) = r\Big[\int_0^{q^{\mathtt{NV}}_{c_1}+\alpha M} u f(u)\mathrm{d}u + (q^{\mathtt{NV}}_{c_1}+\alpha M)\bar{F}(q^{\mathtt{NV}}_{c_1}+\alpha M)\Big] \\ - w\Big[\int_{q^{\mathtt{NV}}}^{q^{\mathtt{NV}}_{c_1}+\alpha M}(u - q^{\mathtt{NV}})f(u)\mathrm{d}u + \alpha M\bar{F}(q^{\mathtt{NV}}_{c_1}+\alpha M)\Big] - c_1 q^{\mathtt{NV}}_{c_1}.\end{aligned} \tag{EC.20}$$

By separating the integral and the second term in the first set of square brackets, we can equivalently express equation (EC.20) as

$$\begin{aligned}\Pi(q^{\mathtt{NV}}_{c_1}, w, \alpha, 0) =& r\Big[\int_0^{q^{\mathtt{NV}}_{c_1}} u f(u)\mathrm{d}u + q^{\mathtt{NV}}_{c_1}\bar{F}(q^{\mathtt{NV}}_{c_1})\Big] \\ &+ r\Big[\int_{q^{\mathtt{NV}}_{c_1}}^{q^{\mathtt{NV}}_{c_1}+\alpha M} u f(u)\mathrm{d}u + (q^{\mathtt{NV}}_{c_1}+\alpha M)\bar{F}(q^{\mathtt{NV}}_{c_1}+\alpha M) - q^{\mathtt{NV}}_{c_1}\bar{F}(q^{\mathtt{NV}}_{c_1})\Big] \\ &- w\Big[\int_{q^{\mathtt{NV}}_{c_1}}^{q^{\mathtt{NV}}_{c_1}+\alpha M}(u - q^{\mathtt{NV}}_{c_1})f(u)\mathrm{d}u + \alpha M\bar{F}(q^{\mathtt{NV}}_{c_1}+\alpha M)\Big] - c_1 q^{\mathtt{NV}}_{c_1} \\ =& \Pi^{\mathtt{NV}}_{c_1} + r\Big[\int_{q^{\mathtt{NV}}_{c_1}}^{q^{\mathtt{NV}}_{c_1}+\alpha M} u f(u)\mathrm{d}u + (q^{\mathtt{NV}}_{c_1}+\alpha M)\bar{F}(q^{\mathtt{NV}}_{c_1}+\alpha M) - q^{\mathtt{NV}}_{c_1}\bar{F}(q^{\mathtt{NV}}_{c_1})\Big] \\ &- w\alpha M\Big[\int_{q^{\mathtt{NV}}_{c_1}}^{q^{\mathtt{NV}}_{c_1}+\alpha M}\frac{(u - q^{\mathtt{NV}}_{c_1})}{\alpha M}f(u)\mathrm{d}u + \bar{F}(q^{\mathtt{NV}}_{c_1}+\alpha M)\Big] \\ =& \Pi^{\mathtt{NV}}_{c_1} + r\Big[\int_{q^{\mathtt{NV}}_{c_1}}^{q^{\mathtt{NV}}_{c_1}+\alpha M} u f(u)\mathrm{d}u + (q^{\mathtt{NV}}_{c_1}+\alpha M)\bar{F}(q^{\mathtt{NV}}_{c_1}+\alpha M) - q^{\mathtt{NV}}_{c_1}\bar{F}(q^{\mathtt{NV}}_{c_1})\Big] \\ &- w\alpha M\gamma(\alpha, q^{\mathtt{NV}}_{c_1}).\end{aligned} \tag{EC.21}$$

The second equality in equation (EC.21) follows from isolating $\Pi^{\mathtt{NV}}_{c_1}$ and factoring out αM from the second bracketed term. Also, the second bracketed term in the second equality in equation (EC.21) is equal to $\gamma(\alpha, q^{\mathtt{NV}}_{c_1})$ by equation (6); making this substitution yields the last equality in equation (EC.21).

We can use the driver equilibrium condition (EC.3) to let the platform choose α, requiring in turn that the wage is set so that the chosen α satisfies the condition. Setting $q_1 = q^{\mathtt{NV}}_{c_1}$ and substituting the RHS of equation (EC.3) into the last equality in equation (EC.21), we have that the platform's expected profit from setting $q_1 = q_{c_1}^{\mathtt{NV}}$ and choosing a wage that induces a human driver joining fraction α (such wage exists by Lemma EC.8) is

$$\Pi(q_{c_1}^{\mathtt{NV}}, \hat{w}(\alpha, q_{c_1}^{\mathtt{NV}}), \alpha, 0) = \Pi_{c_1}^{\mathtt{NV}} + r\Big[\int_{q_{c_1}^{\mathtt{NV}}}^{q_{c_1}^{\mathtt{NV}}+\alpha M} u f(u)\mathrm{d}u + (q_{c_1}^{\mathtt{NV}} + \alpha M)\bar{F}(q_{c_1}^{\mathtt{NV}} + \alpha M) - q_{c_1}^{\mathtt{NV}}\bar{F}(q_{c_1}^{\mathtt{NV}})\Big] - \alpha M v. \quad \text{(EC.22)}$$

Differentiating with respect to α gives

$$\frac{\partial \Pi(q_{c_1}^{\mathtt{NV}}, \hat{w}(\alpha, q_{c_1}^{\mathtt{NV}}), \alpha, 0)}{\partial \alpha} = M(r\bar{F}(q_{c_1}^{\mathtt{NV}} + \alpha M) - v), \quad \text{(EC.23)}$$

and evaluating at $\alpha = 0$ yields

$$\frac{\partial \Pi(q_{c_1}^{\mathtt{NV}}, \hat{w}(\alpha, q_{c_1}^{\mathtt{NV}}), \alpha, 0)}{\partial \alpha}\Big|_{\alpha=0} = M(r\bar{F}(q_{c_1}^{\mathtt{NV}}) - v) = M(r(1 - \frac{r-c_1}{r}) - v) = M(c_1 - v). \quad \text{(EC.24)}$$

Our assumption that $v < c_1$ implies that the partial derivative of the profit function with respect to the human joining fraction is strictly positive at $q_1 = q_{c_1}^{\mathtt{NV}}$ and $\alpha = 0$. Because we have $\Pi(q_{c_1}^{\mathtt{NV}}, \hat{w}(0, q_{c_1}^{\mathtt{NV}}), 0, 0) = \Pi_{c_1}^{\mathtt{NV}}$, the positivity of the partial derivative with respect to α implies that the platform can achieve a strictly higher profit by inducing a positive α than it would without human drivers (we also note that it could easily achieve $\Pi_{c_1}^{\mathtt{NV}}$ without human drivers by setting $q_1 = q_{c_1}^{\mathtt{NV}}$ and $w = 0$). ■

Proof of Proposition EC.3. By Lemma EC.3, if $F(q_1) < (r-v)/r$ and $F(q_1 + M) \geq (r-v)/r$, then $0 < \alpha(q_1) < 1$ and at the optimal $\alpha(q_1)$ we have

$$F(q_1 + \alpha(q_1)M) = \frac{r-v}{r} \iff \bar{F}(q_1 + \alpha(q_1)M) = \frac{v}{r} \iff q_1 + \alpha(q_1)M = F^{-1}(\frac{r-v}{r}). \quad \text{(EC.25)}$$

We can then apply the results of Lemma EC.3 for this and the other cases to get

$$\Pi(q_1, \hat{w}(\alpha(q_1), q_1), \alpha(q_1), 0) = \begin{cases} r\Big[\int_0^{q_1+M} u f(u)\mathrm{d}u + (q_1+M)\bar{F}(q_1+M)\Big] & \\ \quad -c_1 q_1 - vM & \text{if } F(q_1+M) \leq \frac{r-v}{r}, \\ r\Big[\int_0^{F^{-1}(\frac{r-v}{r})} u f(u)\mathrm{d}u + \frac{v}{r}F^{-1}(\frac{r-v}{r})\Big] & \\ \quad -c_1 q_1 - v\Big(F^{-1}(\frac{r-v}{r}) - q_1\Big) & \text{if } F(q_1) < \frac{r-v}{r} < F(q_1+M), \\ r\Big[\int_0^{q_1} u f(u)\mathrm{d}u + q_1\bar{F}(q_1)\Big] - c_1 q_1 & \text{if } F(q_1) \geq \frac{r-v}{r}. \end{cases} \quad \text{(EC.26)}$$

We continue by cases.

Case 1: $F(M) \leq (r - c_1)/r$. Differentiating the expression in the first piece of equation (EC.26), we get

$$\frac{\partial \Pi(q_1, \hat{w}(\alpha(q_1), q_1), \alpha(q_1), 0)}{\partial q_1} = r\bar{F}(q_1+M) - c_1, \quad \text{(EC.27)}$$

yielding the FOC (EC.5). The second derivative is

$$\frac{\partial^2 \Pi(q_1, \hat{w}(\alpha(q_1), q_1), \alpha(q_1), 0)}{\partial q_1^2} = -rf(q_1+M) < 0,$$

implying that the function is strictly concave in q_1 and that the FOC is sufficient for the global maximum of this piece of the function.

Moreover, the FOC (EC.5) has a solution in this case because the LHS is increasing in q_1 and $\lim_{q_1\to\infty} F(q_1+M) = 1 > (r-c_1)/r$. This solution falls in the interval in which the first piece of equation (EC.26) governs the profit because it satisfies

$$F(\tilde{q}_1+M) = \frac{r-c_1}{r} < \frac{r-v}{r}. \tag{EC.28}$$

Finally, by Lemma EC.4, the profit is strictly decreasing in q_1 for larger q_1 that fall into one of the other two intervals. Because the profit function is continuous (inspection reveals that the pieces coincide at the boundaries), the profit at the solution to the FOC is larger than any profit from the second two pieces, and the solution to the FOC (EC.5) thus achieves the global maximum over $q_1 \geq 0$. By equation (EC.28), we have $F(\tilde{q}_1) < F(\tilde{q}_1+M) < (r-v)/r$. Therefore, we will have $\hat{\alpha}(\tilde{q}_1) > 1$ in equation (EC.30), so by Lemma EC.3, it is optimal for the platform to induce $\alpha = 1$.

Case 2: $(r-c_1)/r < F(M) < (r-v)/r$. In this case, the FOC (EC.5) has no solution for $q_1 \geq 0$, and the function is decreasing over the whole range in which the first piece of equation (EC.26) governs the function because the derivative is always negative on this interval by equation (EC.27). Since the function is also decreasing over the second two intervals by Lemma EC.4, we conclude that the global maximum of the function occurs at $q_1 = 0$. Furthermore, because $F(q_1) = 0$ and $F(M) < (r-v)/r$, by Lemma EC.3, it is optimal to induce $\alpha = 1$.

Case 3: $F(M) \geq (r-v)/r$. In this case, the first piece of equation (EC.26) never governs the function for any $q_1 \geq 0$. So, by Lemma EC.4, the profit is strictly decreasing in q_1 for $q_1 \geq 0$, and it is optimal to set $q_1 = 0$. By Lemma EC.3, it is optimal in this case to induce a joining fraction $\hat{\alpha}(0) < 1$, the unique solution to equation (EC.30). ■

E Auxiliary Results and Proofs

In this section, for cases where $q_1 = 0$, we will drop it from our notation to give tidier expressions.

Lemma EC.1 **(Continuity and Differentiability of q_2).** *For given q_1, $q_2(q_1, w, \alpha)$ is continuous for all $w \leq r$ and $\alpha \in [0,1]$. Additionally, there exists $\hat{\alpha} > 0$ (independent from w) small enough such that $q_2(0, w, \alpha)$ has partial derivative with respect to w and α for all $(w,\alpha) \in (0,r) \times (0,\hat{\alpha})$, where the partial derivatives satisfy*

$$\frac{\partial q_2(w,\alpha)}{\partial w} = \frac{F(q_2(w,\alpha)+\alpha M) - F(q_2(w,\alpha))}{rf(q_2(w,\alpha)+\alpha M) + w(f(q_2(w,\alpha)) - f(q_2(w,\alpha)+\alpha M))}$$

and

$$\frac{\partial q_2(w,\alpha)}{\partial \alpha} = -M\frac{(r-w)f(q_2(w,\alpha)+\alpha M)}{rf(q_2(w,\alpha)+\alpha M) + w(f(q_2(w,\alpha)) - f(q_2(w,\alpha)+\alpha M))}.$$

Proof. The continuity of $q_2(q_1, w, \alpha)$ comes from Proposition 1. Indeed, because $w \leq r$, Proposition 1 establishes that $\hat{q}_2(q_1, w, \alpha)$ is the unique solution to the FOC (4). Moreover, since the right-hand-side in (4) is continuous in (q_2, w, α), q_1 is fixed, and the solution is unique, we must have that $\hat{q}_2(q_1, w, \alpha)$ is continuous which in turn implies that $q_2(q_1, w, \alpha)$ is continuous.

In order to show the differentiability (as noted, from here we drop q_1 from the notation as it is zero), we first establish that $\hat{q}_2(w,\alpha) > 0$ which will imply that $\hat{q}_2(w,\alpha) = q_2(w,\alpha)$. That is, $q_2(w,\alpha)$ coincides with the solution to (4). Then, the differentiability follows from the Implicit Function Theorem. To see that $\hat{q}_2(w,\alpha) > 0$ note that from (4) we have

$$rF(q_2+\alpha M) = (r-c_2) + w(F(q_2+\alpha M) - F(q_2)) \geq (r-c_2),$$

where the inequality holds because F is non-decreasing, and thus

$$q \geq F^{-1}\left(\frac{r-c_2}{r}\right) - \alpha M.$$

Since $c_2 < r$, the above is positive for $\alpha > 0$ small enough (independent from w), as desired. ■

Lemma EC.2 **(Possible to Recruit Some Humans and Wage Characterization)**. *For $q_1 = 0$, there exists $\hat{\alpha} > 0$ such that for all $\alpha \in [0, \hat{\alpha}]$ there exists a unique $w(\alpha) \in [0, r]$ continuous in α with*

$$w(\alpha)\gamma(\alpha, q_2(w(\alpha), \alpha)) = v.$$

Proof. We first note that we can simplify the expression for drivers' matching rate to

$$\gamma(\alpha, q_2(w,\alpha)) = \frac{1}{\alpha M}\int_{q_2(w,\alpha)}^{q_2(w,\alpha)+\alpha M} \bar{F}(x)dx. \tag{EC.29}$$

From Equation (EC.29), for a fixed wage $w < r$, from Lemma EC.1 we have

$$\lim_{\alpha\to 0^+} \gamma(\alpha, q_2(w,\alpha)) = \lim_{\alpha\to 0^+} \frac{1}{\alpha M}\int_{q_2(w,\alpha)}^{q_2(w,\alpha)+\alpha M} \bar{F}(x)dx = \frac{0}{0},$$

which is indeterminate, so we apply L'Hôpital's rule. The numerator is the integral in the above equation, and differentiating gives

$$\frac{d}{d\alpha}\Big(\int_{q_2(w,\alpha)}^{q_2(w,\alpha)+\alpha M} \bar{F}(x)dx\Big) = \bar{F}(q_2(w,\alpha)+\alpha M)(\frac{\partial q_2(w,\alpha)}{\partial \alpha} + M) - \bar{F}(q_2(w,\alpha))\frac{\partial q_2(w,\alpha)}{\partial \alpha},$$

noting that the derivatives are well defined by Lemma EC.1 for $w < r$ and $\alpha > 0$ small enough. Taking the limit gives

$$\lim_{\alpha\to 0^+}\frac{d}{d\alpha}\Big(\int_{q_2(w,\alpha)}^{q_2(w,\alpha)+\alpha M} \bar{F}(x)dx\Big) = M\bar{F}(q_2(w,0)) = M\bar{F}(q^{\texttt{NV}}) = M\Big(\frac{c_2}{r}\Big),$$

where the last two equalities follow from Equation (3) and the surrounding discussion. The denominator is αM, with derivative M, so we conclude that

$$\lim_{\alpha\to 0^+}\gamma(\alpha, q_2(w,\alpha)) = \bar{F}(q^{\texttt{NV}}) = \frac{c_2}{r}.$$

Accordingly, $\gamma(0, q_2(w,0)) = \bar{F}(q^{\texttt{NV}}_{c_2})$ for any $w < r$. For $\alpha = 0$, the driver equilibrium condition $w\gamma(0, q_2(w,0)) = v$ then becomes

$$w\bar{F}(q^{\texttt{NV}}_{c_2}) = v \iff w = v\Big(\frac{r}{c_2}\Big).$$

Letting $w(\alpha)$ be a wage that satisfies the equilibrium condition $w\gamma(\alpha, q_2(w,\alpha)) = v$, we have $w(0) = v(r/c_2)$. Let $\varepsilon > 0$ be such that $w(0) + \varepsilon < r$; note that this is well defined because $v < c_2$.

Now, let

$$f(w,\alpha) = w\gamma(\alpha, q_2(w,\alpha)) - v.$$

Because $\gamma(0, q_2(w(0)+\varepsilon, 0)) = c_2/r$ and $w(0)\gamma(0, q_2(w(0),0) = v$, we have $f(w(0)+\varepsilon, 0) = \varepsilon(c_2/r) > 0$. Because f is continuous, there then exists $\hat{\alpha} > 0$ with $f(w(0)+\varepsilon, \hat{\alpha}) > 0$. Also, we have $f(0,\hat{\alpha}) = -v$. And the latter is true for any $\alpha \le \hat{\alpha}$. Moreover, for any $w \in [0, w(0)+\varepsilon]$ we have that

$$\begin{aligned}
\frac{\partial f(w,\alpha)}{\partial w} &= \gamma(\alpha, q_2(w,\alpha)) + w\left(\bar{F}(q_2(w,\alpha)+\alpha M) - \bar{F}(q_2(w,\alpha))\right)\frac{\partial q_2(w,\alpha)}{\partial w} \\
&= \frac{1}{\alpha M}\int_{q_2(w,\alpha)}^{q_2(w,\alpha)+\alpha M}\bar{F}(x)dx - \frac{1}{\alpha M}w\frac{\left(\bar{F}(q_2(w,\alpha)+\alpha M) - \bar{F}(q_2(w,\alpha))\right)^2}{(r-w)f(q_2(w,\alpha)+\alpha M) + wf(q_2(w,\alpha))} \\
&\ge \frac{1}{\alpha M}\int_{q_2(w(0)+\varepsilon,\alpha)}^{q_2(w(0)+\varepsilon,\alpha)+\alpha M}\bar{F}(x)dx - \frac{1}{\alpha M}w\frac{\left(\bar{F}(q_2(w,\alpha)+\alpha M) - \bar{F}(q_2(w,\alpha))\right)^2}{(r-w)f(q_2(w,\alpha)+\alpha M) + wf(q_2(w,\alpha))} \\
&= \gamma(\alpha, q_2(w(0)+\varepsilon,\alpha)) - w\frac{f(\xi)^2\alpha M}{(r-w)f(q_2(w,\alpha)+\alpha M) + wf(q_2(w,\alpha))} \\
&\ge \gamma(\alpha, q_2(w(0)+\varepsilon,\alpha)) - (w(0)+\varepsilon)\frac{\bar{f}}{r\underline{f}}\alpha M,
\end{aligned}$$

where $\xi \in [q_2(w,\alpha), q_2(w,\alpha)+\alpha M]$ exists by virtue of the mean value theorem. In the first inequality we used that $q_2(w,\alpha)$ is non-decreasing in w. In the second inequality we used $\bar{f}$ and $\underline{f}$ as upper and lower bounds for $f(\cdot)$. We define the latter as follows. Note that since we are considering $\alpha \le \hat{\alpha}$ and $w \le w(0)+\varepsilon$, and because $q_2(w,\alpha)$ is continuous, then we can find $\bar{q}$ such that $q_2(w,\alpha) \le \bar{q}$ for all (w,α) under consideration. Then, we define $\bar{f} = \sup_{x\in[0,\bar{q}+\hat{\alpha}M]} f(x)$ and $\underline{f} = \inf_{x\in[0,\bar{q}+\hat{\alpha}M]} f(x)$, note that both bounds are finite and positive. Hence, we have

$$\inf_{w\in[0,w(0)+\varepsilon]} \frac{\partial f(w,\alpha)}{\partial w} \ge \gamma(\alpha, q_2(w(0)+\varepsilon,\alpha)) - (w(0)+\varepsilon)\frac{\bar{f}}{r\underline{f}}\alpha M,$$

and the term on the right-hand-side above is strictly positive for all $\alpha > 0$ small enough because $\gamma(\alpha, q_2(w(0)+\varepsilon,\alpha))$ converges to $c_2/r > 0$ as $\alpha \downarrow 0$. With some abuse of notation we suppose that this is true for all $\alpha \le \hat{\alpha}$. This implies that $f(w,\alpha)$ is an increasing function in $w \in [0, w(0)+\varepsilon]$ for all $\alpha \le \hat{\alpha}$. Therefore, there is a unique and continuous $w(\alpha) \in [0, w(0)+\varepsilon]$ such that $f(w(\alpha),\alpha) = 0$. Note that in the above we can take ε such that $w(0)+\varepsilon$ is as close as desired to r. Hence, the above proves that we can find a unique $w(\alpha) \le r$ for all α small enough. ∎

Lemma EC.3 (**Optimal Induced Human Joining Fraction in $\mathcal{P}_0$**). *For $b_2 = 0$, suppose that the platform chooses an AV quantity q_1, and let $\hat{\alpha}(q_1)$ be the unique solution in α to*

$$F(q_1+\alpha M) = \frac{r-v}{r}. \tag{EC.30}$$

If $F(q_1) < (r-v)/r$, then it is optimal for the platform to induce an equilibrium human joining fraction $\alpha(q_1)$, where

$$\alpha(q_1) = \min\{\hat{\alpha}(q_1), 1\}.$$

Otherwise, the platform should not employ human drivers. If $q_1 = q_{c_1}^{\mathtt{NV}}$, then the condition $F(q_1) < (r-v)/r$ reduces to $v < c_1$.

Proof. We can substitute a generic q_1 for $q_{c_1}^{\mathtt{NV}}$ in equation (EC.23) to get the derivative of the profit function for general q_1. Differentiating the function a second time gives

$$\frac{\partial^2 \Pi(q_1, \hat{w}(\alpha, q_1), \alpha, 0)}{\partial \alpha^2} = -rMf(q_1+\alpha M) < 0,$$

implying that the function is strictly concave in α and the FOC is sufficient for a global maximum.

If $F(q_1) \ge (r-v)/r$, then the derivative of the profit function is nonpositive at $\alpha = 0$ by the generic form of equation (EC.23). The concavity of the function then implies that the profit is strictly decreasing on $[0,1]$, so it is optimal to induce $\alpha = 0$.

On the other hand, if $F(q_1) < (r-v)/r$, then inspection of the same equation reveals that the derivative of the profit function is positive at $\alpha = 0$. In this case, equation (EC.23) gives the FOC (EC.30), which has a unique solution because (i) here $F(q_1) < (r-v)/r$, (ii) the LHS approaches $1 > (r-v)/r$ as α grows large, and (iii) the LHS is increasing in α while the RHS is constant. If $0 < \hat{\alpha}(q_1) < 1$, then it is optimal to induce $\alpha = \hat{\alpha}(q_1)$. If instead $\hat{\alpha}(q_1) \ge 1$, then the concavity of the function implies that it is increasing on $[0,1]$, so it is optimal to induce $\alpha = 1$.

Finally, equation (EC.24) shows that the condition for the derivative to be positive at $\alpha = 0$ reduces to $v < c_1$ if $q_1 = q_{c_1}^{\mathtt{NV}}$. ∎

Lemma EC.4 (**Shape of Profit in q_1**). *The profit $\Pi(q_1, \hat{w}(\alpha(q_1), q_1), \alpha(q_1), 0)$ is strictly decreasing in q_1 for q_1 such that*

$$F(q_1 + M) > \frac{r - v}{r}.$$

Proof. **Case 1: $F(q_1) < (r - v)/r < F(q_1 + M)$.** Differentiating the second piece of equation (EC.26) with respect to q_1 gives $v - c_1 < 0$, so the function is strictly decreasing in q_1.

Case 2: $F(q_1) \geq (r - v)/r$. In this case, the third piece of equation (EC.26) applies, which is equivalent to the newsvendor profit function for overage cost c_1 and underage cost $r - c_1$. The critical fractile for this problem is $(r - c_1)/r < (r - v)/r$. Thus, in this case we are always above the optimal quantity for the equivalent newsvendor problem, implying that the profit is strictly decreasing in q_1. ■

Lemma EC.5 (**Matching Rate Continuous at $\alpha = 0$**). *For given q_1 and q_2, let $\gamma(0, q_1 + q_2) = \bar{F}(q_1 + q_2)$. With this definition, $\gamma(\alpha, q_1 + q_2)$ is continuous in α in a neighborhood of zero.*

Proof. We first note that we can simplify the expression for drivers' matching rate to

$$\gamma(\alpha, q_1 + q_2) = \frac{1}{\alpha M} \int_{q_1+q_2}^{q_1+q_2+\alpha M} \bar{F}(u) \mathrm{d}u. \tag{EC.31}$$

We then have

$$\lim_{\alpha \to 0^+} \gamma(\alpha, q_1 + q_2) = \lim_{\alpha \to 0^+} \int_{q_1+q_2}^{q_1+q_2+\alpha M} \bar{F}(u) \mathrm{d}u = \frac{0}{0},$$

which is indeterminate, so we apply L'Hôpital's rule. The numerator is the integral in the above equation, and differentiating gives

$$\frac{d}{d\alpha}\Big(\int_{q_1+q_2}^{q_1+q_2+\alpha M} \bar{F}(u) \mathrm{d}u \Big) = M\bar{F}(q_1 + q_2 + \alpha M).$$

Taking the limit gives

$$\lim_{\alpha \to 0^+} \frac{d}{d\alpha}\Big(\int_{q_2(w,\alpha)}^{q_2(w,\alpha)+\alpha M} \bar{F}(x) dx \Big) = M\bar{F}(q_1 + q_2).$$

The denominator is αM, with derivative M, so we conclude that $\lim_{\alpha \to 0^+} \gamma(\alpha, q_1 + q_2) = \bar{F}(q_1 + q_2)$. Accordingly, we define $\gamma(0, q_1 + q_2) = \bar{F}(q_1 + q_2)$, and the result is proved. ■

Lemma EC.6 (**Early AV Acquisition Does Not Fix the Race to the Top**). *If $c_1 = c_2$, then the optimal profit is the same for all $b_1 \geq 0$.*

Proof. In this proof, we augment our notation to let $\mathcal{P}_{b_1,b_2}$ ($\Pi^\star(b_1, b_2)$) denote the platform's problem (optimal profit) for given b_1 and b_2. We will prove the result by showing that $\Pi^\star(b_1, \infty) = \Pi^\star(0, \infty)$ for all $b_1 > 0$.

Consider $b_1 > 0$. If $q_1 = 0$ in the optimal solution to $\mathcal{P}_{b_1,\infty}$, then the result immediately holds. For the rest of the proof, suppose otherwise, i.e., that the optimal q_1 is strictly positive.

First, observe that any feasible solution $(0, w, \alpha, q_2)$ to $\mathcal{P}_{0,\infty}$ is also feasible for $\mathcal{P}_{b_1,\infty}$. Since both problems have the same objective function, we thus have $\Pi^\star(0, \infty) \leq \Pi^\star(b_1, \infty)$.

To complete the proof, we leverage the following claim, the proof of which is relegated after the proof of the lemma.

Claim EC.1. *If $c_1 = c_2 = c$, then at optimality $(q_1^\star, w^\star, \alpha^\star, q_2^\star)$, we have $q_1^\star \leq q_2(0, w^\star, \alpha^\star)$.*

Now, consider an optimal solution $(q_1^\star, w^\star, \alpha^\star, q_2^\star)$ to $\mathcal{P}_{b_1,\infty}$. If $\alpha^\star = 0$, then we must have $q_1^\star + q_2^\star = q_c^{\texttt{NV}} = q_2(0, w, 0)$. Since this solution satisfies (EQ), the solution $(0, w^\star, 0, q_2(0, w, 0))$ also satisfies (EQ), and by definition it satisfies the optimality constraint for q_2, so it is feasible for $\mathcal{P}_{0,\infty}$ and achieves the same profit $\Pi^\star(b_1, \infty)$, implying that $\Pi^\star(b_1, \infty) \le \Pi^\star(0, \infty)$.

If instead $\alpha^\star > 0$, then we must have $w^\star \le r$, as otherwise the platform could strictly improve its profit by not recruiting any human drivers. Thus, by Proposition 1, the FOC (4) has a unique solution. We proceed by cases.

Case 1: $\boldsymbol{q_2(0, w^\star, \alpha^\star) = 0}$. Since $q_1^\star \le q_2(0, w^\star, \alpha^\star)$ by Claim EC.1, in this case, we have $q_1^\star = q_2^\star = 0$. By similar arguments to the $\alpha^\star = 0$ case, we have that $(0, w, \alpha, 0)$ is feasible for $\mathcal{P}_{0,\infty}$, and it achieves the same profit, so $\Pi^\star(b_1, \infty) \le \Pi^\star(0, \infty)$.

Case 2: $\boldsymbol{q_2(0, w^\star, \alpha^\star) > 0}$. In this case, by Proposition 1, $q_2^\star$ is the unique solution of the FOC (4) for $q_1^\star$, $w^\star$, and $\alpha^\star$. Inspection of this FOC reveals that for $q_1 = 0$ and the same $w^\star$ and $\alpha^\star$, setting $q_1^\star + q_2^\star$ uniquely solves this FOC and is thus optimal, satisfying the optimality constraint for $\mathcal{P}_{0,\infty}$. Moreover, $q_1 + q_2$ is the same as in $(q_1^\star, w^\star, \alpha^\star, q_2^\star)$, so (EQ) is also satisfied for $\mathcal{P}_{0,\infty}$. This implies that $(0, w^\star, \alpha^\star, q_1^\star + q_2^\star)$ is feasible for $\mathcal{P}_{0,\infty}$, and it achieves the same profit of $\Pi^\star(b_1, \infty)$, implying that $\Pi^\star(b_1, \infty) \le \Pi^\star(0, \infty)$ in this case also.

We have shown both that $\Pi^\star(b_1, \infty) \le \Pi^\star(0, \infty)$ and that $\Pi^\star(0, \infty) \le \Pi^\star(b_1, \infty)$, so we conclude $\Pi^\star(b_1, \infty) = \Pi^\star(0, \infty)$. Since $b_1 > 0$ was chosen arbitrarily, this completes the proof. ■

Proof of Claim EC.1. First, suppose that $\alpha^\star = 0$. In this case, we must have $q_1^\star + q_2^\star = q_c^{\texttt{NV}} = q_2(0, w, 0)$, and the result holds. For the rest of the proof, we treat the case with $\alpha^\star > 0$. Note that if $\alpha^\star > 0$ at optimality, then we must have $w^\star \le r$, as otherwise the platform could strictly improve its profit by not recruiting any human drivers. We can thus restrict our attention to solutions with $w \le r$.

Consider a solution (q_1', w, α, q_2') feasible to $\mathcal{P}_{b_1,\infty}$ such that $q_1' > q_2(0, w, \alpha)$. The matching rate γ is strictly decreasing in its second argument by equation (6), so we have $w\gamma(\alpha, q_2(0, w, \alpha)) > w\gamma(\alpha, q_1' + q_2') = v$. Thus, there exists $w' < w$ such that $w'\gamma(\alpha, q_2(0, w, \alpha)) = v$.

Moreover, $q_2(0, w, \alpha)$ is non-decreasing in w for fixed q_1 and α by Proposition 1, which implies that $w'\gamma(\alpha, q_2(0, w', \alpha)) \ge w'\gamma(\alpha, q_2(0, w, \alpha)) = v$. Now, let $g(x) := x\gamma(\alpha, q_2(0, x, \alpha))$ for the considered α. We have just shown that $g(w') \ge v$, and we also have $g(0) = 0 < v$. Thus, by the Intermediate Value Theorem, there exists $w'' \le w' < w$ such that $w''\gamma(\alpha, q_2(0, w'', \alpha)) = v$. Thus, the solution $(0, w'', \alpha, q_2(0, w'', \alpha))$ is also feasible for $\mathcal{P}_{b_1,\infty}$.

Finally, observe that we have

$$
\begin{aligned}
\Pi(q_1', w, \alpha, q_2') = \Pi(0, w, \alpha, q_1' + q_2') \le \Pi(0, w, \alpha, q_2(0, w, \alpha)) &< \Pi(0, w', \alpha, q_2(0, w, \alpha)) \\
\textit{by optimality of } q_2(\cdot): \qquad &\le \Pi(0, w', \alpha, q_2(0, w', \alpha)) \\
\textit{because } w'' \le w': \qquad &\le \Pi(0, w'', \alpha, q_2(0, w', \alpha)) \\
\textit{by optimality of } q_2(\cdot): \qquad &\le \Pi(0, w'', \alpha, q_2(0, w'', \alpha)).
\end{aligned}
$$

The equality holds by equation (2) because $c_1 = c_2 = c$. The first weak inequality holds by the optimality of $q_2(0, w, \alpha)$, and the strict inequality holds because $w' < w$ and Π is strictly decreasing in w for $\alpha > 0$.

Thus, for any feasible solution (q_1', w, α, q_2') to $\mathcal{P}_{b_1,\infty}$ such that $q_1' > q_2(0, w, \alpha)$, we can find another feasible solution with strictly higher profit. This implies that at optimality, we must have $q_1^\star \le q_2(0, w^\star, \alpha^\star)$, as desired. ■

F Human Driver Equilibrium

Note that the content of this section holds for any $b_1, b_2 \geq 0$. We first define the human driver equilibrium.

Definition EC.1 (Human Driver Equilibrium). For given q_1 and w, a human joining fraction α is a *driver equilibrium* if and only if one of the following is satisfied:

(i) $\alpha = 0$ and $w\gamma(0, q_1 + q_2(q_1, w, 0)) \leq v$;
(ii) $0 < \alpha < 1$ and $w\gamma(\alpha, q_1 + q_2(q_1, w, \alpha)) = v$;
(iii) $\alpha = 1$ and $w\gamma(1, q_1 + q_2(q_1, w, 1)) \geq v$.

This definition requires that all human drivers be acting optimally. In any of (i), (ii), or (iii) above, no infinitesimal human driver could improve her utility by switching from joining the platform to not joining, or vice versa. On the other hand, if (for instance), $0 < \alpha < 1$ and $w\gamma(0, q_1 + q_2(q_1, w, 0)) < v$, then (in contrast to point (ii) of the definition), the drivers who are joining the platform could strictly improve their utility by switching to not joining.

Using the equilibrium definition above, the following lemma shows that it is optimal for the platform to exactly equate a joining human driver's expected earnings with the outside option (rather than strictly exceed the outside option), implying the driver equilibrium condition (EQ) stated in Section 5.

Lemma EC.7 **(Driver Equilibrium Holds with Equality)**. *Consider $0 < \alpha \leq 1$ and $q_1 \geq 0$, and suppose that it is possible to satisfy* (EQ)*, i.e., that there exists w such that $w\gamma(\alpha, q_1 + q_2(q_1, w, \alpha)) \geq v$. For the given α and q_1, it is optimal for the platform to exactly satisfy the equilibrium condition, i.e., to choose a wage such that* (EQ) *holds with equality.*

Proof. For $\alpha = 0$ and given q_1, the wage w drops out of the FOC for the optimal q_2 (see Proposition 1), and thus changes in w do not affect the solution or the platform's expected profit. That is, $q_2(q_1, w, 0)$ is the same for any wage w; let $\tilde{q}_2(q_1, 0)$ denote this value. Letting $w' = v/\gamma(0, q_1 + \tilde{q}_2(q_1, 0))$ (using the definition of $\gamma(0, \cdot)$ from Lemma EC.5), we therefore have $w'\gamma(0, q_1 + q_2(q_1, w, 0)) = v$, and thus for a wage of w', $\alpha = 0$ is an equilibrium by part (i) of Definition EC.1. Any other wage such that $\alpha = 0$ is an equilibrium will have the same expected profit, so w', which satisfies (EQ) at equality, is indeed optimal for the given q_1 and $\alpha = 0$. This establishes the result for $\alpha = 0$.

For $\alpha > 0$, first assume that $w \leq r$, and note that $q_2(q_1, w, \alpha)$ is continuous in w by Proposition 1. Let $g(w) := w\gamma(\alpha, q_1 + q_2(q_1, w, \alpha))$. Since q_2 is continuous in w, q_1 and α are fixed, and γ is continuous in its second argument, we have that $g(w)$ is continuous in w. We have $g(0) = 0 < v$, and by assumption, there exists a wage w' such that $g(w') \geq v$. By the Intermediate Value Theorem, there also exists a wage w'' and $g(w'') = v$. If $0 < \alpha < 1$, then the condition must hold at equality because otherwise more drivers would join, contradicting the equilibrium, so the result is established for $0 < \alpha < 1$. For $\alpha = 1$, if it is impossible to strictly satisfy the equilibrium condition, then the result is similarly established.

Suppose otherwise, i.e., for $\alpha = 1$, assume that there exists a wage w' such that $g(w') > v$; $\alpha = 1$ is a driver equilibrium in this case by part (iii) of the definition above. The strict inequality implies that we can choose $\epsilon > 0$ such that $g(w' - \epsilon) > v$, and thus $\alpha = 1$ is a driver equilibrium also for the lower wage $w' - \epsilon$. It is clear from Equation (2) that the platform's expected profit is strictly decreasing in w for fixed $\alpha > 0$ and q_2, which combined with the optimality of $q_2(\cdot, \cdot, \cdot)$ gives us that

$$\Pi(q_1, w' - \epsilon, 1, q_2(q_1, w' - \epsilon, 1)) \geq \Pi(q_1, w' - \epsilon, 1, q_2(q_1, 1, w')) > \Pi(q_1, w', 1, q_2(q_1, 1, w')).$$

Therefore, any wage w' with $g(w') > v$ is strictly sub-optimal. As shown above, it is possible to satisfy the condition at equality, and it therefore must be optimal to do so.

Finally, suppose that $w > r$. Since $\alpha > 0$, setting $w > r$ can never be globally optimal for the platform (it would always be strictly better off not recruiting any human drivers at all), so without loss of optimality we can ignore solutions with $w > r$ and $\alpha > 0$. ■

Finally, we show that any human joining fraction $\alpha \in [0, 1]$ is achievable in $\mathcal{P}_0$.

LEMMA EC.8 **(Wage Exists for Any Human Joining Fraction in $\mathcal{P}_0$)**. *For $b_2 = 0$ and any human joining fraction $0 \leq \alpha \leq 1$ and AV quantity q_1 with $F(q_1) < 1$, a unique wage $\hat{w}(q_1, \alpha)$ exists that satisfies Equation* (EC.3)*, i.e., with $\hat{w}(q_1, \alpha)\gamma(\alpha, q_1) = v$.*

Proof. Because $F(q_1) < 1$, we have $\gamma(\alpha, q_1) > 0$ for any $0 \leq \alpha \leq 1$ by Equation (6) (and Lemma EC.5 for $\alpha = 0$). Therefore, we have that $\hat{w}(q_1, \alpha) = v/\gamma(\alpha, q_1)$, which is unique and well-defined because $\gamma(\alpha, q_1) > 0$. ■