Getting Out of Your Own Way: Introducing Autonomous Vehicles on a Ride-Hailing Platform

Francisco Castro
UCLA Anderson School of Management, francisco.castro@anderson.ucla.edu

Andrew E. Frazelle
Jindal School of Management, The University of Texas at Dallas, andrew.frazelle@utdallas.edu

Problem definition: After autonomous vehicles (AVs) are deployed for ride-hailing platforms but before their costs decrease enough to push human drivers off the road entirely, human drivers will compete for rides with AVs. We consider a ride-hailing platform's strategy to recruit human drivers while also operating a private fleet of AVs. **Methodology/results:** We formulate and solve a game-theoretic model of a ride-hailing platform with a private AV fleet that also recruits self-interested human drivers. The platform sets the human-driver wage and the AV deployment quantity, and human drivers make strategic joining decisions based on a rational anticipation of their expected earnings. We show that growing its AV fleet too quickly while the AV cost is still relatively high can be a costly mistake for the platform. Doing so triggers a feedback loop of increasing wages and increasing AV deployment, such that the platform prices itself out of the market for human drivers to the detriment of its own profits. **Managerial implications:** This "race to the top" effectively prevents the platform from attracting more than a limited number of human drivers, and it increases the cost of attracting a given number. Nonetheless, we prove that the platform can break the feedback loop by optimally tuning its AV fleet size, tempering the competition for rides and achieving a profitable balance of AVs and human drivers. Our findings suggest that even while their costs remain high, AVs can be a valuable tool for ride-hailing platforms, as long as the fleet size is carefully set.

Key words: innovation, gig economy, game theory, ride-hailing, autonomous vehicles, market design
This version: November 4, 2022.

---

1. Introduction

As the ride-hailing market has grown and self-driving technology has advanced rapidly in recent years, speculation has mounted about the future of autonomous vehicles (AVs) in this market. Indeed, such a future looms closer than ever. Waymo One, an experimental AV-operated ride-hailing service in Phoenix, Arizona (with planned expansion to San Francisco), has even dispensed with human safety drivers.[1] However, while multiple firms are testing AVs, the race to full autonomy for ride-hailing and automobility in general has not been without challenges. Several fatal accidents have been reported, and "Level 5" autonomy (when humans will not be needed for any driving tasks) has been called "one of the hardest problems we have."[2] Indeed, high-profile early proponents

[1] https://blog.waymo.com/2022/03/taking-our-next-step-in-city-by-bay.html?m=1

[2] https://www.thedrive.com/tech/31816/key-volkswagen-exec-admits-level-5-autonomous-cars-may-never-happen

of AV-operated ride-hailing services, including Uber[3] and Ford,[4] have recently abandoned their attempts to commercialize the technology due to out-of-control costs. These challenges raise the question of whether and how ride-hailing platforms can benefit from AV technology during the long journey—during part of which AV costs are likely to remain high—toward a future with no human-driven vehicles.

Although it is widely accepted that eventually all cars will be driven by software, human-driven vehicles still dominate the road at present. Moreover, the transition between these extremes will not happen overnight. A recent New York Times article described the small-scale fully-driverless ride-hailing experiment operated by GM's Cruise in San Francisco (Metz 2022). Cruise "[monitors driverless rides] from a remote operations center," and technicians are sometimes deployed to assist with rides, virtually or physically. The article also describes the currently high costs of deploying and operating AVs: "The development costs, back-end computing infrastructure and technicians needed to support these cars increase costs by hundreds of millions of dollars—at least for now." Note that these costs include not only fixed acquisition costs (extra hardware) but also additional variable operating costs to run the cloud servers and pay the human technicians who remotely monitor rides. Accordingly, although the long-run value proposition for AVs revolves around lower operating costs achieved by removing humans from the equation, this promise is unlikely to be realized in the near-term future, not even once the AV roll-out starts to scale up (see Nunes and Hernandez 2019, 2020).

In addition, platforms must plan for a new form of competition on the supply side of their marketplaces: man vs. machine. It is not clear how gig-economy workers will react to AVs joining the market, or how ride-hailing platforms should best balance the two sources of supply. We consider the problem of how a platform should manage its ride-hailing marketplace to simultaneously leverage a private AV fleet and successfully recruit human-driven vehicles. Since AV technology is still expensive to operate, the question may reasonably be asked why AVs are yet needed, if rides could be served more cheaply with human drivers. First, as evidenced by their heavy investments in AV technology and initial deployments of it discussed above, ride-hailing firms indeed see value in serving rides with AVs, even though costs are still quite high as the technology is in active development. Second, as usually happens with emerging technology (e.g., the original "horseless carriage," personal computers, etc.), AVs will need to be perfected over time in order for the technology to decrease in cost and achieve full market penetration. During this process, platforms can and should aim for the best mix of AV- and human-served rides. The goal of our work is to

[3] https://www.wsj.com/articles/uber-sells-self-driving-car-unit-to-autonomous-driving-startup-11607380167

[4] https://www.theverge.com/2022/10/26/23423998/argo-ai-shut-down-ford-vw-av-self-driving

inform a platform's strategy in the near-term future, i.e., after AVs are on the road for ride-hailing platforms (beyond the current experimental operations), but before AV costs decrease enough that human drivers are pushed out of the market and off the road. Indeed, we find that even with AVs that are relatively expensive to operate, an AV fleet can benefit a platform if the fleet size is appropriately sized and deployed. However, we also identify some critical problems that can arise if AV acquisition and deployment are not executed optimally.

A platform with a mixed fleet must manage several tradeoffs. One benefit of a company-owned AV is that the cost of deploying it is known and fixed. By contrast, a human driver must be recruited to join the platform, and hence, the wages and earning rate offered must satisfy an individual rationality constraint. The cost of a human-served ride is thus endogenous, and increasing human-driver participation may require higher wages, since the more drivers join the platform, the greater the competition among them for rides (they also must compete with AVs, as discussed below). On the other hand, a company-owned AV incurs an operating cost any time it is deployed on the road, while human drivers only earn money from the platform while with a passenger. So, operating with more human drivers and fewer AVs protects the platform against both lost sales (underage) and over-deployment (overage) costs. However, knowing that the platform may prefer to serve demand with AVs, human drivers may not join the platform because they cannot rely on being matched with fares. As noted by Jiang and Tian (2018) and Guda and Subramanian (2019), the two-sided marketplace of ride-hailing involves "customers" on both sides of the market in the sense that both passengers and drivers freely choose whether to use the platform's service based on its value. As such, the new element of AVs competing for rides necessarily affects the platform's strategy for recruiting human drivers. This novel supply-side competition motivates our two main research questions: (i) how should a ride-hailing platform set the human-driver wages and AV deployment quantity to exploit its AV fleet while simultaneously recruiting enough human drivers (at an affordable wage) to create an effective demand hedge, and (ii) in the presence of AVs, what effect do the self-interested joining decisions of human drivers have on the platform's bottom line?

To address these questions, we consider a ride-hailing platform with two supply sources: (i) a platform-operated fleet of AVs, and (ii) a population of human drivers that make strategic joining decisions. We model the decisions of the platform and drivers as an extensive-form game. Given its AV fleet of a certain size, the platform first sets the wage rate that it will pay human drivers for each ride. Then, human drivers decide whether to join the platform in equilibrium, determining the available pool of human labor. Finally, knowing the wage and the human labor pool size, the platform sets its AV deployment quantity (how many AVs will be on the road to be matched with passengers), and demand is realized. In equilibrium, the human drivers' expected earnings must match or exceed the outside option, and these earnings depend on the labor pool size, the wage rate, and a rational anticipation of the platform's AV deployment quantity.

Contributions. We find that the necessary and sufficient condition for the platform to employ human drivers in equilibrium is that the human drivers' outside option be less than the deployment cost of an AV (which can be thought of as the combined depreciation and operating cost for the AV for a given amount of driving time), aligning with our focus on the near-term future in which AVs are still relatively expensive to operate. In turn, our results suggest that AVs and human drivers can share the road for ride-hailing platforms during the transition period before AVs become cheap enough to make human drivers obsolete.

Importantly, the platform must carefully consider the consequences of human drivers' equilibrium behavior. The platform's optimal AV deployment quantity is increasing in the human-driver wage, so the positive impact of a wage increase on human drivers' expected earnings is dampened (possibly even negated) by the increased competition from AVs. This reflects the endogenous relationship between wages, human driver joining rate, and AV deployment quantity that is governed by the equilibrium participation condition for human drivers and the optimality condition for the platform's AV deployment. Perhaps counterintuitively, the equilibrium wage can actually *decrease* in the human joining fraction because at low joining fractions, increasing this fraction reduces the AV deployment quantity and thus increases human drivers' matching rate.[5] However, we also show that human participation may be intrinsically limited. As more humans join and push out AVs, eventually not many AVs are left, and the human-driver matching rate begins to decrease. The platform then must increase the wage to satisfy the equilibrium participation condition, but this makes human drivers less attractive, so it will deploy more AVs, which again increases competition and decreases the matching rate, so the wage must increase even more. Succinctly, we establish that the platform may price *itself* out of the market for human drivers by triggering a feedback loop of increasing wages and increasing AV deployment. This *race to the top* effectively prevents the platform from attracting more than a limited number of human drivers, and it increases the cost of attracting a given number.

A larger AV fleet can thus actually harm the platform's bottom line by hindering human driver recruitment. Because its AV deployment quantity is a best response to the wage and human joining fraction, the platform cannot credibly commit to a quantity that ensures human drivers a high matching rate. This leads to the feedback loop mentioned above and keeps the platform from achieving the ideal AV/human-driver mix. Indeed, the platform's profit can be as much as 38% less than if it could commit upfront to an AV deployment quantity. So, if the platform obtains too large

[5] This is the measure of the expected number of ride requests that each human driver will serve.

an AV fleet too early, then the threat that the platform may deploy them can scare away human drivers while they remain an important labor source. This finding appears discouraging for the platform, but we prove that a simple remedy recovers the lost profit: tuning the AV fleet size. We show that by optimizing the AV fleet size to exploit the new technology while still guaranteeing human drivers a high enough matching rate, the platform can always match the profit it would earn with commitment power.

Our findings highlight the importance to ride-hailing platforms of appropriately pacing the introduction of AV technology. A properly sized AV fleet can be beneficial even while costs are relatively high, and a profitable balance of AVs and human drivers can outperform both a human-only and an AV-only fleet. However, during the transition between only humans on the road and only AVs, platforms must carefully manage their AV fleets and the expectations of human drivers: too many AVs will trigger the race to the top and push humans out of the labor market, which in the near-term future will hurt profitability. Too few, on the other hand, will also leave money on the table, as well as put the platform behind in AV adoption, which is of strategic importance in the long term. In their strategy to introduce AVs to the marketplace, it is crucial for platforms to properly manage the endogenous interaction between wages, human participation, and AV deployment; significant profit losses await otherwise.

2. Related Literature

Only a few very recent papers in the operations management literature incorporate AVs in the context of ride-hailing. However, all have entirely different focus areas from ours, and notably, none of them consider a profit-maximizing platform's management of the endogenous relationship between wages, human driver participation, and AV deployment quantity, as we do. In the setting of Siddiq and Taylor (2022) with competition between platforms, they assume linear demand and supply functions. By contrast, we model human drivers as strategic agents who make participation decisions based on their expected earnings given the posted wage and a rational anticipation of their matching rate ahead of the platform's AV deployment decision. This creates a non-linear, endogenous relationship in our model between wage and human driver supply through the impact of both on the optimal AV deployment that is not seen in Siddiq and Taylor (2022). We also distinguish between fleet sizing (long-term) decisions and AV deployment (mid-term) decisions.

Lian and van Ryzin (2022) treat platforms as market-clearers and study the decisions of market participants (human drivers and AV owners). They consider two platform scenarios—common platform for AVs and human drivers or independent platforms for each—and two AV ownership scenarios—AVs owned by individuals or AVs owned by a monopolist. The combinations yield a total of four different market designs, and they analyze the resulting prices, wages, and utilization in each. The setting in Lian and van Ryzin (2022) differs from ours along several dimensions. First, in contrast to their model of ride-hailing platforms as market-clearers, we consider a profit-maximizing platform that sources rides from both humans and its own AV fleet, and we provide important insights about the platform's human-driver recruiting strategy. Second, they assume an unlimited human labor pool; we treat this pool as finite at the outset, which is important in our setting because the competition both among human drivers and between AVs and humans depends on the labor pool size. In contrast to both Siddiq and Taylor (2022) and Lian and van Ryzin (2022), in the present work, human drivers' rational anticipation of the platform's best-response deployment entails a unique endogenous relationship between wages, available human drivers, and AV deployment. Our findings also highlight the impact of commitment power on the platform's ability to recruit human drivers. For other recent studies involving AVs, we refer the reader to Liu (2018), Mirzaeian et al. (2021), and Baron et al. (2022), and for a study that looks into the granular spatial incentives that AVs can produce on human vehicle repositioning we refer the reader to Benjaafar et al. (2021).

Another related stream of work concerns blended workforces composed of full-time employees (similar to our AVs) and flexible agents (similar to our human drivers). Early works on this front consider how to optimally source contingent workers from an external labor supply agency, see e.g., Milner and Pinker (2001) and Pinker and Larson (2003). More recent studies, however, consider a related problem in which flexible agents are independent. For example, Hu et al. (2022) analyze the welfare implications of uniform and hybrid worker classification in on-demand platforms. This work establishes that having full-time employees and contractors can be more beneficial for some workers, consumers and the platform than uniform classification. Lobel et al. (2022) use a similar modeling framework to ours to study the impact of wages on the labor composition; in contrast, we focus on the endogenous competition in a blended workforce and its implications for the platform's bottom line. Dong and Ibrahim (2020), on the other hand, consider the multi-period staffing problem faced by an on-demand platform that employs a mix of full-time employees and randomly determined flexible agents. A key difference between our setting and that of Dong and Ibrahim (2020) is that the endogeneity in their flexible agents' joining decision is stochastic and not strategic; additionally, their endogeneity is related to the number of other flexible workers while ours is related to both the human drivers that join and the autonomous vehicles that the firm deploys. Chakravarty (2021) studies the viability of a blended workforce and the pricing implications of preferential rationing (employees are matched before independent drivers) in a two-stage model. While our study and that of Chakravarty (2021) are related, our modeling approach is different and our results focus on the analysis of potential equilibrium outcomes and not solely on the viability of a blended workforce.

Finally, this paper is also related to works that study how to incentivize independent, strategic supply units in on-demand platforms, see, e.g., Cachon et al. (2017), Daniels (2017), Bimpikis et al.

(2019), Guda and Subramanian (2019), Hu and Zhou (2020), and Besbes et al. (2021). In our model, supply units make their joining decision strategically given wages; however, they also consider the impact of AVs on their earnings. Also related to our paper are studies that compare contractors and full-time employees in on-demand platforms. Taylor (2018) studies how prices and wages are impacted by customers' delay sensitivity and agents' independence, and by uncertainty in customers' valuations and agents' opportunity cost. A key observation in this work is the *agent participation externality* which implies that equilibria with low wages and many agents can be sustained because an improved service (due to more agents) leads to larger demand, and thus better agent utilization. Interestingly, the latter effect dominates the competition effect which would lead to low utilization and higher wages. In our setting, as the platform increases the induced fraction of agents in the system, wages may decrease due not to an improved service but rather to a substitution effect. More agents in the system implies that the platform can rely less on autonomous vehicles, which can improve agent utilization despite generating more competition among humans, leading to lower wages. Gurvich et al. (2019) consider a related newsvendor setting without a mixed fleet, in which supply units are self-scheduling, and they establish that higher wages are needed to induce a higher number of self-interested supply units. The latter effect occurs because demand is not affected by service but also because, in contrast to our paper, the platform does not have access to its own supply. In Nikzad (2020), the author considers on-demand service platforms and establishes that in thin markets (small labor pool), the platform can sustain high wages because the marginal improvement in service level can outweigh the higher cost associated with those higher wages. In parallel work to Nikzad (2020), Benjaafar et al. (2022) study how different policies affect labor welfare in on-demand service platforms. They establish that nominal wages (without utilization) always decrease, albeit at a diminishing rate, in the size of the labor pool because more supply stimulates demand by decreasing delay. The latter leads to a non-monotone effective wage (nominal wage times utilization) which first increases and then decreases. In the present paper, we observe the reverse due to the substitution effect between human drivers and autonomous vehicles. The nominal wage can decrease in the induced fraction of supply at small values of this fraction. Then, for larger fractions of induced supply, the nominal wages can be arbitrarily large due to a feedback loop of increasing wages and increasing AV deployment.

To summarize, our work takes a fresh perspective on the already fresh problem of operating a ride-hailing service supported by human drivers in the presence of AVs, developing an optimal strategy to manage human-driver incentives through wages and matching rates In what follows, we reveal pitfalls that ride-hailing platforms may succumb to as they grow their AV fleets. We uncover a surprising "race to the top" of increasing wages and increasing AV deployment that fundamentally limits human driver recruitment, but we also show that the platform can escape this race to the top by optimally managing AV fleet growth and human driver expectations.

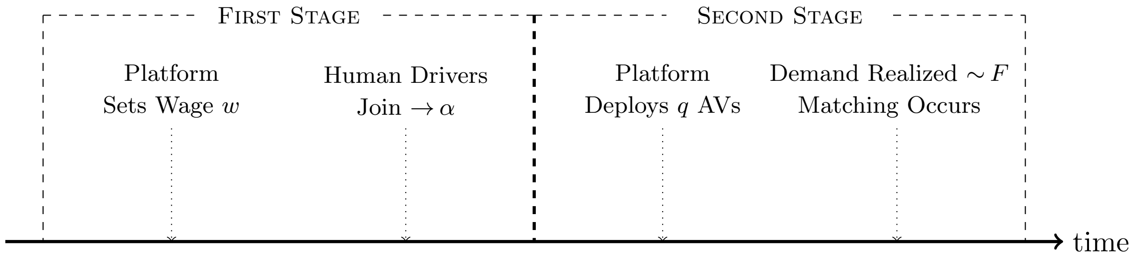


Figure 1 Sequence of Events

3. Model

We study a ride-hailing platform that connects passengers requesting rides with AVs or human drivers. We suppose that rides can be served with AVs or human-operated vehicles and that the platform's average revenue per ride is r for either type of vehicle.[6] Our model is a sequential game with two stages: in the first, the platform sets wages and human drivers make joining decisions, and in the second, the platform makes a deployment decision and demand is realized. The first stage captures the long-term interaction of drivers with the platform, while the second stage models the shorter-term, daily decisions of the platform. The detailed sequence of events and the relevant notation follow and are also summarized in Figure 1.

First stage. At the beginning of the first stage, the ride-hailing platform sets the wage w that human drivers will receive per ride. Next, the human drivers decide whether to join the platform. Because we consider drivers' longer-term joining decisions—i.e., whether to participate in the marketplace—rather than their real-time decisions of whether to drive at a given moment, the wage w should be viewed as an average over a suitable time horizon. For simplicity, we assume that all drivers who join the platform are available to serve passengers in the second stage, but our model could easily incorporate a fraction of drivers who join the platform but in the end are not available in the second stage. The drivers are homogeneous and have an outside option valued at $v > 0$. Drivers join the platform if their expected payoff from doing so (weakly) exceeds their outside option, given the wage and the anticipated AV deployment quantity, the latter being inferred by backward induction. The population of nonatomic human drivers has mass M. Denoting by α the fraction of human drivers who choose to join the platform (we will call this the joining fraction), the supply of human drivers in the second stage is αM. We defer the formal definition of driver equilibrium to Section 5.

Second stage. At the beginning of the second stage, the platform observes the fraction α of human drivers who have joined the platform. The platform has access to a fleet of $N \geq 0$ AVs. After

[6] To keep the model parsimonious, we assume the same revenue per ride for either vehicle type, but we have observed that our main insights extend to the case with different but relatively similar revenues.

observing the human joining fraction α, it chooses an AV deployment quantity $q \le N$ (deployment signifies that an AV is on the road and available to be matched with passengers). To deploy AVs, the platform incurs an operating cost of $c < r$ per unit, so the total AV operating cost is cq. Importantly, this operating cost is incurred for each deployed AV, whether matched with a passenger or not. Components of this operating cost include fuel, cloud computing expenses, technological and human costs for remote monitoring of rides (Metz 2022, Nunes and Hernandez 2019), wear and tear on the vehicles, etc., which result from the vehicle driving, as well as any liability insurance and depreciation cost of the asset.

Finally, demand is realized. The number of requested rides D is a random variable with cumulative distribution function (CDF) $F(\cdot)$ with $F(0) = 0$, corresponding probability density function (PDF) $f(\cdot)$ and mean μ. We use $\bar{F}(\cdot)$ to denote $1 - F(\cdot)$. Each unit of AV can serve one unit of demand, and the same holds for human drivers. The platform allocates demand first to the q deployed AVs, and human drivers serve excess demand. Note that, because the AV deployment cost is sunk, allocating demand to AVs first is optimal for the platform. The total available supply is $q + \alpha M$, and any demand exceeding this quantity is lost.

Platform's problem. Suppose the platform has set the wage at w and a fraction α of human drivers have joined the platform. If the platform deploys q AVs and the realized demand is d, then the platform's profit is

$$\pi(w,\alpha,q;d) \triangleq r\min\{d, q+\alpha M\} - cq - w\min\{[d-q]^+, \alpha M\}. \tag{1}$$

The number of passengers served is the minimum of the available supply $q + \alpha M$ and the realized demand d, and multiplying by the revenue per ride r gives the total revenue. There are two components of the cost, the AV operating cost and the human driver wages. The AV operating cost is cq. The wage cost is based on the *realized* number of human-served rides, which is the minimum of the excess demand not handled by AVs and the available human supply. Human drivers receive no compensation when not with a fare. In what follows we use $\Pi(w,\alpha,q)$ to denote the expected value of $\pi(w,\alpha,q;d)$, where the expectation is taken over the demand D.

Letting (EQ) be the human drivers' equilibrium condition (see Section 5), the platform's optimization problem is then

$$\begin{aligned} \Pi^\star \triangleq \max_{(w,\alpha,q)} \; & \Pi(w,\alpha,q) \\ s.t. \quad & \alpha \textit{ satisfies } \text{(EQ)} \\ & q \in \arg\max_{q \le N} \Pi(w,\alpha,q) \end{aligned} \tag{$\mathcal{P}$}$$

We will use $(w^\star, \alpha^\star, q^\star)$ to denote the optimal solution of $(\mathcal{P})$. Observe that the platform must solve a bi-level optimization problem because the deployment quantity is set after the wages and the joining fraction are determined. Note also that the human joining fraction α is treated as a decision variable for the firm, subject to the equilibrium condition (EQ).

4. Optimal AV Deployment

We first study the optimal AV deployment decision, denoted by $q(w,\alpha)$, when the wages and the fraction of human drivers that join the platform are fixed. This corresponds to solving the second stage of the platform's bi-level problem.

With no human drivers (i.e., for $\alpha = 0$), the platform's deployment problem reduces to a standard newsvendor problem with overage (over-deployment) cost c and underage (lost-sales) cost $r-c$. Let $\Pi^{\mathtt{NV}}(q)$ and $q^{\mathtt{NV}}$ denote the expected profit and the optimal quantity in the standard newsvendor problem with the above parameters—we also use the shorthand $\Pi^{\mathtt{NV}}$ for $\Pi^{\mathtt{NV}}(q^{\mathtt{NV}})$. The optimal deployment $q^{\mathtt{NV}}$ uniquely solves

$$F(q^{\mathtt{NV}}) = \frac{r-c}{r}. \tag{2}$$

The exception is if $q^{\mathtt{NV}} > N$, in which case the optimal deployment is N.

For $\alpha > 0$, the problem departs from the newsvendor model in that we have two tiers of underage cost. Demand between q and $q + \alpha M$ is served by human drivers, with a cost difference of $w - c$ compared to an AV-served ride. But demand above $q + \alpha M$ is lost entirely, incurring a lost-sales cost of $r - c$. Due to this tiered cost structure, the platform's first-order condition (FOC) does not reduce to a simple critical ratio as in (2).

The following result characterizes the optimal deployment quantity $q(w,\alpha)$ in quasi-closed form. Let $x^+ = \max\{x, 0\}$.

Proposition 1 **(Optimal AV Deployment)**. *Let $\hat{q}(w,\alpha)$ be a solution in q to*

$$(r-w)F(q+\alpha M) + wF(q) = r - c. \tag{3}$$

Then:

(i) If $w > r$, then either $q(w,\alpha)$ coincides with a solution to (3) *such that $0 < \hat{q}(w,\alpha) \le N$, or we have $q(w,\alpha) = N$, and $q(w,\alpha)$ is non-decreasing in α.*

(ii) If $w \le r$, then $\hat{q}(w,\alpha)$ is unique and the platform's optimal AV deployment quantity is

$$q(w,\alpha) = \min\{\hat{q}(w,\alpha)^+, N\},$$

and $q(w,\alpha)$ is non-increasing in α.

Moreover, $q(w,\alpha)$ is non-decreasing in w.

Condition (3) is the FOC obtained from the platform's AV deployment problem. We note that in *(i)* there could be multiple solutions because when $w > r$, the left-hand side in (3) is not necessarily monotone. For $w \leq r$, the left-hand side in (3) is monotone in q and, therefore, there is a unique solution. Additionally, the proposition establishes the monotonicity of $q(w,\alpha)$ with respect to α and w. For a given wage w, $q(w,\alpha)$ optimally balances the profit coming from AVs and human-driven vehicles. As the joining fraction α increases, the platform can satisfy a larger fraction of the demand with humans which leads to a (weakly) larger human-driver profit if and only if $r \geq w$. In turn, to compensate, it is optimal for the platform to (weakly) decrease the optimal deployment quantity if and only if $r \geq w$. Now, for a given joining fraction α, the platform is always better off increasing the deployment quantity as the wage increases because human-served rides become more expensive and they can substituted by AVs. Interestingly, in the next section, we will see that when we consider the equilibrium wage associated with each joining fraction α, the optimal deployment quantity can increase in α even when $w \leq r$.

In what follows, we will rely on the implicit characterization of $q(w,\alpha)$ given by Proposition 1 to analyze the optimal wage and joining fraction. Nevertheless, in the special case of exponential demand, we obtain a closed-form expression for the optimal AV deployment quantity. We provide this expression in the following corollary, and we will later return to the exponential distribution as a running example to make our results more concrete and to build intuition. For exponentially distributed demand, the cumulative distribution function F is given by $F(x) = 1 - e^{-x/\mu}$ (recall that μ is the mean demand).

Corollary 1 **(AV Deployment with Exponential Demand)**. *If demand is exponentially distributed, then the optimal AV deployment quantity is*

$$q(w,\alpha) = \min\left\{\mu \log\left[\frac{(r-w)e^{-\alpha M/\mu} + w}{c}\right]^+, N\right\}. \tag{4}$$

The corollary follows immediately from substituting the exponential CDF into the FOC (3) and isolating its unique solution $\hat{q}(w,\alpha)$. Note that for exponential demand, the FOC has a unique solution even when $w > r$.

5. Driver Equilibrium

Human drivers join the platform if and only if their expected payoff exceeds their outside option. Let $\gamma(\alpha,q)$ be the *matching rate* of a driver to passengers when a fraction α of humans joins the platform and the platform deploys q AVs, that is,

$$\gamma(\alpha,q) \triangleq \frac{\mathsf{E}\left[\min\left\{(D-q)^+, \alpha M\right\}\right]}{\alpha M}. \tag{5}$$

Note that $\gamma(\alpha,q)$ represents the anticipated number of requests per unit of time (up to a constant that we normalize to one) that each driver receives in the horizon considered in their joining decision.

The expression for $\gamma(\alpha, q)$ captures the matching decision of the platform. If $D < q$, the platform only matches AVs; if $D \in [q, q+\alpha M]$, the platform randomizes the excess demand among human drivers; and finally, if $D > q+\alpha M$, then all drivers are matched.

Recalling that $q(w, \alpha)$ is the optimal AV deployment quantity for wage w and human joining fraction α, we can write our driver equilibrium condition as

$$w\gamma(\alpha, q(w, \alpha)) = v.^{7} \tag{EQ}$$

Later, we will discuss in detail the endogenous relationship between w, α, and q in equilibrium. For now, it suffices to point out this endogeneity and note that the equilibrium condition (EQ) plays a critical role in our analysis, as well as precipitating some counterintuitive findings.

In what follows, it will be useful to define the wage w that can induce a certain joining fraction α. That is, we fix α in (EQ) and use $w(\alpha)$ to denote a value of w that induces a fraction α of drivers to join the platform. In general, $w(\alpha)$ may not be unique; however, it is possible to find conditions under which for any joining fraction α there is a unique w that induces such α (see, e.g., Lemma 2 in the appendix) Additionally, until otherwise stated, we assume that the platform operates with ample AVs, that is, $N = \infty$, to allow the platform maximum flexibility in the second stage in deploying AVs and bring the supply-side competition to center stage. Later, in Section 6.3, we consider a setting in which the platform can exert control on N and compare it to the ample-AV case. Also, unless otherwise specified, $\Pi^\star$ refers to the optimal ample-AV profit.

5.1. Need for Human Drivers

A future in which AVs completely replace human drivers aligns in our model with AV costs decreasing below the humans' outside option v. At that point, our model suggests that human drivers will be obsolete—see Proposition 2 below. Before then, a transition period is expected during which AVs and humans share the road. This period reflects a decrease in AV operating costs to the point where AVs are commercially viable, but not so cheap that humans cannot earn a decent wage from the platform. In our model, this regime corresponds to $v < c$, with possibly $w > c$, depending on the particular equilibrium. Indeed, as discussed in the introduction, ride-hailing platforms are currently investing in trial deployments of AVs, even though the technology is still inactive development and thus expensive. Because AVs are strategically important in the long term, platforms are willing to operate them at a higher cost for some period of time to achieve economies of scale and eventually bring down costs.

[7] In some cases, it is possible to satisfy the equilibrium condition strictly (driver expected earnings strictly larger than the outside option); Lemma 7 in Appendix E shows that it is always optimal to satisfy it at equality.

Proposition 2 **(Need for Human Drivers)**. *If $c \le v$, then the optimal solution to $(\mathcal{P})$ is such that $\alpha^\star = 0$, $q^\star = q^{NV}$ and $\Pi^\star = \Pi^{NV}$. However, if $v < c$, then it is optimal for the platform to induce some human drivers to participate in equilibrium i.e., $\alpha^\star > 0$.*

The condition in Proposition 2 is remarkably simple given the complex forces driving the equilibrium. We emphasize that this condition does *not* imply that the platform pays humans less per ride than the AV operating cost; the ordering of c and the equilibrium wage $w(\alpha^\star)$ depends on the relationship between r, c, and v. Also, different levels of human participation imply different wages, so for the platform to recruit more drivers, say a joining fraction $\alpha' > \alpha^\star$, the required wage would be different and possibly much higher. Aligning with our stated focus on the near-term future with relatively expensive AVs, henceforth we assume that $v < c$.

5.2. Effect of Substitution and Competition on Wages

We now uncover the interplay between the substitution of AVs with human drivers and the competition among human drivers. When humans join, the platform reduces its AV deployment (when $w \le r$—see Proposition 1) because humans cover some of the demand, entailing two opposing effects on the matching rate. As human participation increases, competition from AVs initially attenuates, while competition among human drivers intensifies; the net effect is ambiguous, and the next proposition characterizes it for small human joining fractions.

Proposition 3 **(Substitution vs. Competition)**. *The wage $w(\alpha) \le r$ is decreasing in a neighborhood of zero if $c > 2v$ and increasing in a neighborhood of zero if $c \in (v, 2v)$.*

The proposition establishes that when the operating cost is sufficiently high, the required wage to induce a small fraction of human drivers to join the platform is decreasing in such fraction because the substitution of AVs with humans dominates the added competition among humans. In fact, because c is relatively large, the platform chooses to reduce its deployment quantity enough so that the matching rate $\gamma(\alpha, q)$ increases despite more humans joining the platform. The equilibrium constraint (EQ) then forces the wages to decrease.

Wages that locally decrease in the labor supply have been observed in the literature (e.g., in Taylor 2018 and Benjaafar et al. 2022) but with a fundamentally different origin. Typically, the reason for decreasing wages in thicker markets is an increased service level. In our case, increased supply from one source causes the platform to decrease supply from the other source, and the net effect on wages can be negative. In contrast with small human joining fractions, we will see next that the behavior at higher fractions is of a different nature, with adverse consequences for the platform.

5.3. Platform's Race to the Top with Itself

In choosing the wage w, the platform must consider its impact on the human participation level and, by extension, on the optimal AV deployment. The next proposition characterizes a uniform bound for the impact of wages on the human participation.

PROPOSITION 4 **(Limited Human Driver Recruitment)**. *If F has decreasing mean residual life*[8] *and $w \le r$, then the human driver equilibrium joining fraction α is bounded above by*

$$\alpha \le \frac{c\mu}{Mv}. \tag{6}$$

Proposition 4 establishes a cap on human driver recruitment, which depends on the problem parameters. For a low outside option v, it is easier for the platform to attract more human drivers. When the average demand μ is high, the platform may also be able to attract more humans simply because there are more potential passengers. However, as the operating cost c of AVs decreases, fewer humans can be convinced to enter the market because they will anticipate more AVs being deployed.

When the upper bound in Eq. (6) is less than 1, the platform cannot possibly induce all human drivers to participate and break even on human-served rides. Intuitively, to induce a very high level of human participation, the platform will have to offer higher wages to compensate for the increased competition among human drivers. However, for very high wages, the platform strongly prefers to meet demand with AVs, and the optimal deployment quantity in the last stage will increase, which reduces the human matching rate γ. So, increasing the wage increases the earnings conditional on matching, but it may reduce the equilibrium matching rate. The platform can attempt to counter this effect by increasing the wage even more, but that would increase the optimal AV deployment quantity still further and decrease the human matching rate in a feedback loop. The next proposition makes this intuition precise.

Note that $\alpha M\gamma(\alpha, q(w,\alpha))$ corresponds to the expected aggregate demand served by human drivers. We define the maximum of this quantity for a given wage, w, by

$$R(w) \triangleq \max_{\alpha M} \ \alpha M \cdot \gamma(\alpha, q(w,\alpha)),$$

and thus $wR(w)$ is the maximum possible aggregate human driver earnings at wage w.

PROPOSITION 5 **(Race to the Top)**. *If $c \le w \le r$ and F has decreasing mean residual life, then the maximum possible aggregate human driver earnings are non-increasing in the wage w.*

[8] The mean residual life (MRL) of a random variable D is $\mathsf{E}[D - x | D > x]$. Decreasing MRL represents a broad class of distributions, including all distributions with increasing hazard rate.

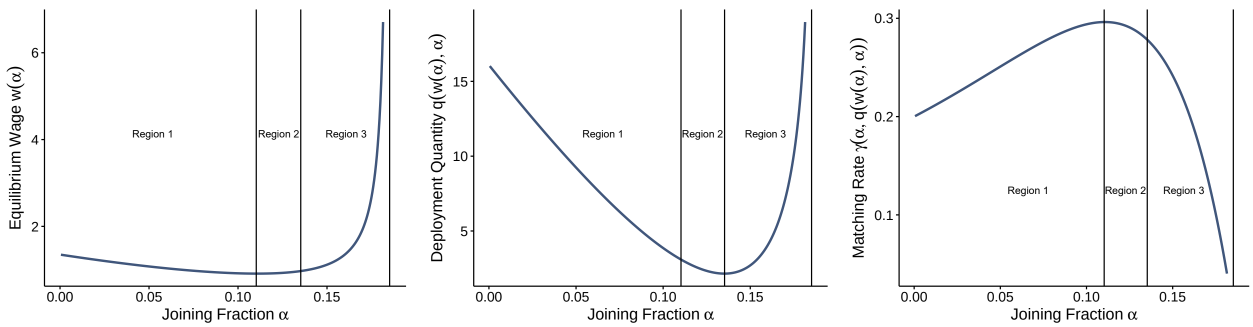
Proposition 5 reveals the surprising fact that (an upper bound on) aggregate human driver earnings is actually *decreasing* in the wage offered to human drivers (weakly or strictly so depending on the distribution, as we will see). This result has implications for the ability of the platform to recruithuman drivers. By (EQ), in equilibrium we must have $\alpha M v = w\alpha M\gamma(\alpha, q(w,\alpha)) \leq wR(w)$. By showing that the rightmost quantity in this relation is decreasing in w (and recalling that M and v are constants), Proposition 5 thus establishes that the largest human joining fraction α that could possibly be achieved in equilibrium is *decreasing* in the wage. In other words, and counterintuitively, increasing the human-driver wage deters human drivers! We proceed to investigate this phenomenon further.

For larger human joining fractions, the platform must raise the wage to increase the joining fraction. However, more human drivers at a higher wage increases the optimal AV deployment quantity because AVs become relatively more preferable to the platform. In effect, the platform prices itself out of the market for human drivers. At high wages, human drivers anticipate intense competition from AVs and a low matching rate. So, to increase human participation, the platform must raise wages even higher, creating a *race to the top with itself* that thwarts recruitment as shown by Propositions 4 and 5: the feedback loop of increasing wages and increased AV deployment means that increasing wages can actually *reduce* human participation, effectively capping the amount of human drivers that can be incentivized into the system. Put simply, because its AV deployment quantity must be a best response to the wage and human-driver joining fraction, the platform cannot credibly commit to a low quantity while also setting wages high enough to induce very high human participation.

As can be seen from their proofs, Propositions 4 and 5 require careful analysis to establish key properties of the interaction between the platform and its human drivers for a broad class of demand distributions. That these findings hold for a broad class of demand distributions places their structural implications on firm ground. With the structural results as a guide, for the remainder of this section we temporarily specialize to the exponential distribution, for which we can characterize all the main quantities in closed form. The precise expressions illuminate the driving forces behind the race to the top.

Proposition 6 **(Wage, Deployment Quantity and Matching Rate)**. *If demand is exponential and $F(M) \leq (r-c)/r$, then the equilibrium joining fraction α is bounded above by $c\mu/(Mv)$. Moreover, inducing an equilibrium human joining fraction $0 < \alpha < c\mu/(Mv)$ is consistent with a wage and deployment quantity of*

$$w(\alpha) = \frac{\alpha M r v}{(e^{\alpha M/\mu} - 1)(c\mu - \alpha M v)}, \quad q(w(\alpha), \alpha) = \mu \log\left(\frac{\mu e^{-\alpha\lambda M} r}{c\mu - \alpha M v}\right), \tag{7}$$



(a) Wage (b) AV Deployment Quantity (c) Human Matching Rate

Figure 2 **Equilibrium choices vs. α ($M = 200$, $r = 5$, $c = 1$, $v = .27$, exponential demand with $\mu = 10$).**

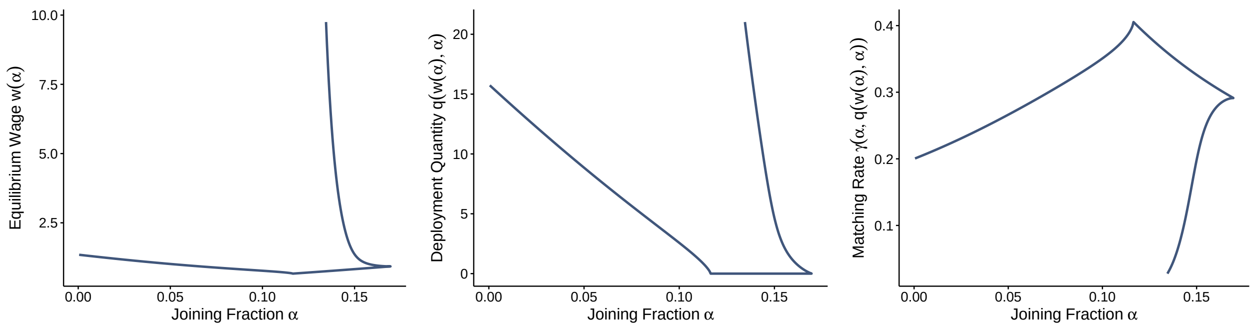
and a matching rate given by

$$\gamma(\alpha, q(w(\alpha), \alpha)) = \frac{(c\mu - \alpha M v)}{r}\Big(\frac{1 - e^{-\alpha M/\mu}}{\alpha M}\Big). \tag{8}$$

Proposition 6 reveals the inner workings of the race to the top, and we plot each of the relevant quantities in Fig. 2. As α increases from zero, at first, the platform decreases its AV deployment. Reduced competition from AVs dominates the increased competition among human drivers, and thus the required wage actually decreases as α increases (see Proposition 3). However, this trend eventually reverses as competition among humans intensifies. Moreover, eventually (near $c\mu/(Mv) \approx 0.18$, indicated by the rightmost vertical line in each plot), the platform must set the wage *higher* than its revenue per ride ($w(\alpha)$ increases without bound as α approaches $c\mu/(Mv)$, which can be seen by inspecting the denominator of $w(\alpha)$ in Eq. 7), so that it would rather lose a unit of demand entirely than serve it with a human driver. The optimal deployment quantity thus increases ($q(w(\alpha), \alpha)$ can also be arbitrarily large), so that as α increases, eventually the matching rate decreases fast enough that the platform cannot further increase human participation—hence the first part of Proposition 6. Note that this extends Proposition 4 for the exponential case by showing that the upper bound on human driver recruitment holds for arbitrarily high wages; that is, the platform could not escape the race to the top even by paying wages higher than the revenue per ride (which, naturally, would never be optimal anyway). Additional details about the case of exponential demand can be found in Appendix A; this includes the platform's optimal solution, which is found by substituting the expressions in Proposition 6 into the platform's expected profit function, which then becomes a function only of α, and optimizing.

Returning our attention to Fig. 2, we observe three distinct regions of α. In the first region, as α increases from zero, the optimal deployment quantity (Fig. 2b) decreases because human drivers can cover some demand, outweighing the increased competition among human drivers and yielding a net increase in γ, the human matching rate (see Fig. 2c). With a higher matching rate, the platform can pay a lower wage (compare Fig. 2a and 2c). So, the firm gets more human drivers at a lower cost, unambiguously positive. Eventually, the optimal deployment quantity slows its descent (Fig. 2b), so the additional competition among humans is no longer offset by the reduction in AVs. At this point, we enter the second region. The matching rate γ now decreases in α, so the wage must increase. However, the wage is still low enough that more human drivers benefits the platform, so the optimal AV deployment quantity still decreases in α. Finally, there is a critical point at which the increased competition causes the matching rate γ to decrease more rapidly, so the wage must increase more rapidly to counter, bringing us into the third region. But when the required wage becomes too high, the firm begins to increase its AV deployment quantity, triggering the runaway race to the top of increasing wages and increasing AV deployment. Note that, at this critical point, the only potential way for the platform to increase human participation would be to increase wages; however, the wages are already so high that the platform must counter with a further elevated deployment of AVs. The net recruitment effect is negative in that it fundamentally limits the amount of human drivers that join the platform and, as a consequence, the platform's ability to garner additional revenue.

As mentioned, the exponential distribution yields closed-form results that paint a clear picture of the race to the top and the resulting limits on human-driver recruitment. Interestingly, for other distributions, recruitment can be even more severely hindered, and next we provide an example to illustrate this additional richness. We have just seen that for exponential demand, the human joining fraction has an upper bound but does continue to increase (albeit at a declining rate) with the wage asymptotically toward this bound. This is a consequence of the constant failure rate. By contrast, for distributions with strictly increasing failure rate, the human joining fraction eventually begins to "curl back" on itself as the wage increases, reflecting an even more severe version of the race to the top. This behavior is reflected in Fig. 3, for a Weibull distribution with an increasing failure rate. As with exponential demand, the wage is decreasing for small α, then increasing for larger α. The two main differences are that (i) the optimal deployment quantity is zero for a range of α and (ii) near the bound of Proposition 4, instead of the wage increasing without bound as α increases to a limit, here the joining fraction "curls back" and starts to *decrease* as the wage increases further. In this case, at the critical point where higher wages are the only option for the platform to recruit more human drivers, the joining fraction is not merely fundamentally limited but actually strictly decreases with the wages. Below, we study the effect of the race to the top on the platform's profit and devise a means of mitigating the profit loss due to lack of commitment power.



(a) Wage (b) AV Deployment Quantity (c) Human Matching Rate

Figure 3 **Equilibrium choices vs. α ($M = 200$, $r = 5$, $c = 1$, $v = .27$, Weibull demand: $\mu = 10$, $k = 1.2$).**

6. Commitment Power, Profit Loss, and Recovery

In this section, we reveal how costly the race to the top can be by showing that the profit can be substantially less than if the platform could credibly commit to a deployment quantity. We then propose a remedy to recover the higher profit, even without commitment power.

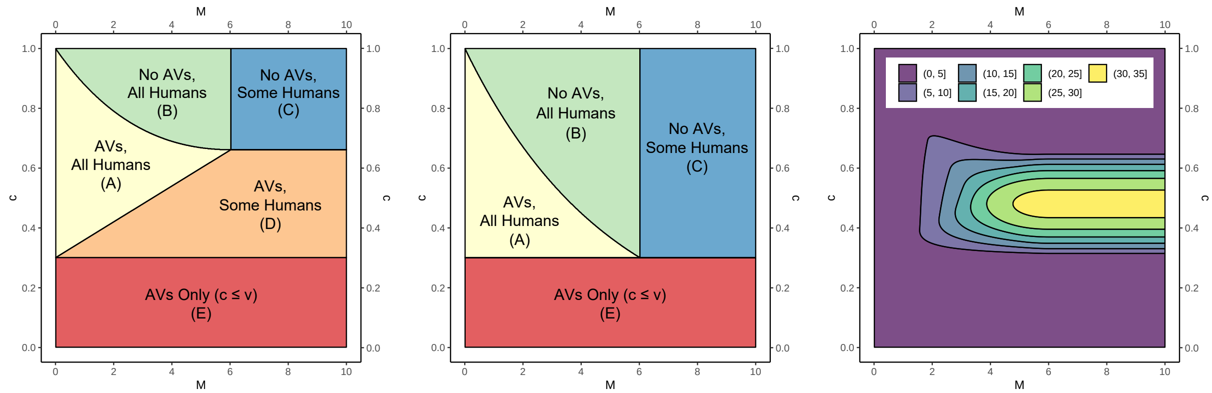
6.1. A Modified Game with Commitment Power

We now briefly discuss a modified version of our game that adjusts the sequence of events to endow the platform with commitment power: the platform chooses the deployment quantity before drivers make their joining decision. The optimal profit $\widetilde{\Pi}$ for this modified game serves as a benchmark—an upper bound—against which to compare the outcomes of the original game. Technical details can be found in Appendix B, where we provide the complete analytical solution to the platform's modified problem.

Proposition 7 **(Modified Game Upper Bounds Original Game)**. *The platform's equilibrium profit in the modified game is an upper bound on its equilibrium profit in the original game, i.e., we have* $\Pi^\star \leq \widetilde{\Pi}$.

The modified game achieves higher profits and avoids the race to the top because the AV deployment quantity does not react to the wage and human joining fraction. We reveal the nature of the difference between the games by illustrating the structure of the optimal solutions in Figure 4.

Figure 4a characterizes the original game's equilibrium over ranges of AV operating cost c and human driver population size M, and Figure 4b depicts the equilibrium in the modified game (see Appendices A and B for closed-form expressions). For small c, it is optimal in both the original and modified games for the platform not to use human drivers—see region E (AVs Only ($c \leq v$)). For larger c, the story is more nuanced. The area in which all human drivers are active is larger in the modified game, due primarily to the race to the top in the original game. There is a sizable overlap between region D (AVs, Some Humans) in the original game with region A (AVs, All Humans) in the modified game. In region D (AVs, Some Humans) of Figure 4a, the AV operating cost is



(a) Original Game (b) Modified Game (c) Pct. Profit Difference

Figure 4 Equilibrium comparison and profit loss for $r = 1$, $v = .3$, exponential demand with $\mu = 5$.

intermediate, so human drivers are valuable, but if the wage increases then the platform will quickly switch to AVs, hence the race to the top. In this region, the platform employs some but not all humans in the original game. By contrast, in the modified game, the quantity does not respond to changes in the wage, so the platform can profitably convince all humans to join the platform over a wider range of parameters, allowing it to achieve the ideal AV/human-driver mix and the corresponding higher profit implied by Proposition 7.

6.2. Profit Loss with Ample AVs

We now demonstrate that the value of commitment power can lead to a substantial difference in optimal profits. In Figure 4c, the contour bands reflect the percentage of the modified game's optimal profit that is sacrificed in the original game. In regions with similar solutions in both games, the loss is minimal. However, for other parameters, the difference in profit is more than 30%. Particularly large profit losses occur with large M and intermediate c. This regime implies plenty of potential human drivers and an AV cost high enough that overage cost is significant, so the hedge offered by humans is indeed valuable, but not so high that AVs are not viable. In the modified game, the platform can commit to deploying few AVs; then, human drivers expect higher earnings and are willing to join in larger numbers. However, in the original game, the race to the top hinders recruitment; if the wage rises, then because the AV operating cost is not too high, the best-response deployment quantity will increase substantially. This increase reduces the human-driver matching rate and offsets any increase in their expected earnings, and either the bound on human drivers of Proposition 4 is tight, or the required wages to induce full human participation are too high to be optimal. The platform must then rely more on AVs than it would like. The next proposition complements these observations by providing a lower bound on the maximum possible profit loss.

Proposition 8 **(Worst Case Profit Loss)**. *Let $\mathcal{I}$ denote an instance of our problem characterized by r, v, c, M, and F. We have*

$$\sup_{\mathcal{I}} \frac{\widetilde{\Pi} - \Pi^{\star}}{\widetilde{\Pi}} \geq \frac{3-\sqrt{5}}{2} \approx 0.381.$$

In light of Proposition 8, we next tackle the problem of recovering the lost profit.

6.3. Limited AV Fleet Size Recovers Lost Profit

We have previously noted that ride-hailing platforms have long-term strategic reasons to deploy AVs. These incentives might motivate a platform to operate an AV fleet even in the face of the race to the top. However, notwithstanding the race to the top, we now proceed to show that a platform need not sacrifice short-term profitability to avoid falling behind technological developments, and that AVs can form a profitable part of a ride-hailing platform's fleet even while the deployment costs are relatively high. To wit, a properly sized AV fleet can escape the race to the top and achieve the full profit of the modified game benchmark.

In the modified game benchmark, since q is determined before human drivers make joining decisions, to incorporate an AV fleet-sizing decision we can suppose that both q and N are jointly determined at the beginning of the game. On the other hand, in the original game augmented with an initial AV fleet-sizing decision, the fleet size and deployment quantity cannot be determined jointly because the deployment is a best response to the earlier stages of the game including the human joining fraction. In the next proposition, we establish that with the optimal fleet size N' for the original game with fleet sizing, the platform can match the optimal profit from the modified game.

Proposition 9 **(Limited Fleet Recovers Lost Profit)**. *In the original game with fleet sizing, it is optimal for the platform to set $N' = \widetilde{q}$, the optimal quantity from the modified game. Moreover, by doing so the platform achieves the same optimal profit as with full commitment power.*

The benefit of a limited fleet size is that it partially decouples the deployment quantity q from the FOC (3), breaking the chain of endogeneity that created the race to the top in wages. The platform's choice to limit its AV fleet credibly signals to human drivers the maximum number of AVs that it will deploy; this increases the anticipated matching rate, reducing the required wage for a given α and eliminating the race to the top.

By Proposition 9, if the platform has access to $N' = \widetilde{q}$ AVs and has the opportunity to obtain additional AVs, then it would hurt the platform's profit to accept them. Thus, a larger AV fleet may be harmful because it can hinder human driver recruitment, imposing substantial profit losses. Hence, platforms should size their AV fleets carefully: too few AVs does not exploit the value of the technology, while too many will deter human drivers from even considering joining the platform.

Finally, we remark that we have incorporated the fixed acquisition cost of an AV into the deployment cost c as depreciation, and in fact, initial investments in AVs can be viewed as research and development costs as much as assets. However, if an additional fixed cost were included, then the optimal fleet size would be even smaller, aligning with our core message that platforms should grow their AV fleets carefully.

7. Conclusion

We have studied a ride-hailing platform supported by both a fleet of autonomous vehicles (AVs) and self-interested human drivers with their own vehicles. The platform faces a tradeoff between these sources for serving rides. AVs are fully in the platform's control with known and fixed deployment cost (they are also strategically valuable for the long term), but if deployed, each one incurs a cost even if not matched with a passenger. On the other hand, human drivers only represent a cost when matched, but the platform must manage incentives successfully to recruit them, which requires promising high enough expected earnings; indeed, the required wage is endogenous and is tightly bound to the platform's AV deployment decision.

Our key findings relate to the value of a shared road for ride-hailing platforms, the confounding race to the top, and the remedy of a smaller fleet. Until AV costs decrease below a threshold, human drivers can be a valuable hedge against demand risk because they incur no cost when not serving a passenger. However, when a ride-hailing platform serves rides with both human drivers and AVs, complicated dynamics govern the endogenous relationship between human driver wages, the human driver participation level, and the optimal AV deployment quantity.

For relatively low levels of human participation, the equilibrium wage can actually be *decreasing* in the participation level: as human participation increases, the optimal AV deployment quantity initially decreases, reducing the competition from AVs so that a lower wage is required to satisfy human drivers. At higher levels of human participation, eventually not many AVs remain, so the net effect on the matching rate of an increase in human participation switches from positive to negative, and hence the required wage switches from decreasing to increasing. Above this point, as human participation—and hence the required wage—increases, human-served rides become less profitable for the platform, which increases its AV deployment quantity to compensate. Increased competition from AVs negates the benefit to human drivers of increased wages, necessitating still higher wages and creating a feedback loop that leads the platform into a race to the top with itself. The feedback loop arises because the platform cannot credibly commit to an AV deployment quantity that will be sub-optimal given the equilibrium wage and human participation level. Indeed, the platform's profit is always higher in a modified game with commitment power, and we showed that the difference in profit can be more than 38%. So, in planning for the shared road, the platform must beware not to price itself out of the market for human drivers by obtaining too large an AV fleet and making the market unattractive for humans. Fortunately, we proved that by limiting its AV fleet size, the platform can recover the same profit it would earn if it did have commitment power. Put simply, a larger AV fleet can actually *hurt* the platform. Thus, in the near-term future while human drivers are still on the road and AVs are relatively expensive, it is crucial for a ride-hailing platform to precisely tune the size of its AV fleet, lest it trigger the race to the top and harm its own profits by pushing human drivers out of the market.

References

Baron O, Berman O, Nourinejad M (2022) Introducing autonomous vehicles: Adoption patterns and impacts on social welfare. *Manufacturing & Service Operations Management* 24(1):352–369.

Benjaafar S, Ding JY, Kong G, Taylor T (2022) Labor welfare in on-demand service platforms. *Manufacturing & Service Operations Management* 24(1):110–124.

Benjaafar S, Wang Z, Yang X (2021) Autonomous vehicles for ride-hailing. *Available at SSRN 3919411* .

Besbes O, Castro F, Lobel I (2021) Surge pricing and its spatial supply response. *Management Science* 67(3):1350–1367.

Bimpikis K, Candogan O, Saban D (2019) Spatial pricing in ride-sharing networks. *Operations Research* 67(3):744–769.

Cachon GP, Daniels KM, Lobel R (2017) The role of surge pricing on a service platform with self-scheduling capacity. *Manufacturing & Service Operations Management* 19(3):368–384.

Chakravarty AK (2021) Blending capacity on a rideshare platform: Independent and dedicated drivers. *Production and Operations Management* 30(8):2522–2546.

Daniels KM (2017) Capacity constrained two-sided markets. *Working Paper, WUSTL* .

Dong J, Ibrahim R (2020) Managing supply in the on-demand economy: Flexible workers, full-time employees, or both? *Operations Research* 68(4):1238–1264.

Guda H, Subramanian U (2019) Your Uber is arriving: Managing on-demand workers through surge pricing, forecast communication, and worker incentives. *Management Science* 65(5):1995–2014.

Gurvich I, Lariviere M, Moreno A (2019) Operations in the on-demand economy: Staffing services with self-scheduling capacity. Hu M, ed., *Sharing Economy*, 249–278 (Springer).

Hu M, Wang J, Zhang ZJ (2022) Implications of worker classification in on-demand economy. *Available at SSRN* .

Hu M, Zhou Y (2020) Price, wage, and fixed commission in on-demand matching. *Available at SSRN 2949513*.

Jiang B, Tian L (2018) Collaborative consumption: Strategic and economic implications of product sharing. *Management Science* 64(3):1171–1188.

Lian Z, van Ryzin G (2022) Capturing the benefits of autonomous vehicles in ride-hailing: The role of dispatch platforms and market structure. *Available at SSRN 3716491*.

Liu W (2018) An equilibrium analysis of commuter parking in the era of autonomous vehicles. *Transportation Research Part C: Emerging Technologies* 92:191–207.

Lobel I, Martin S, Song H (2022) Employees, contractors, or hybrid: An operational perspective. *Available at SSRN 3878215* .

Metz C (2022) Stuck on the streets of San Francisco in a driverless car. *New York Times* URL `https://www.nytimes.com/2022/09/28/technology/driverless-cars-san-francisco.html`, accessed on October 31, 2022.

Milner JM, Pinker EJ (2001) Contingent labor contracting under demand and supply uncertainty. *Management Science* 47(8):1046–1062.

Mirzaeian N, Cho SH, Scheller-Wolf A (2021) A queueing model and analysis for autonomous vehicles on highways. *Management Science* 67(5):2904–2923.

Nikzad A (2020) Thickness and competition in on-demand service platforms. *Working Paper, USC.*

Nunes A, Hernandez K (2019) The cost of self-driving cars will be the biggest barrier to their adoption. *Harvard Business Review* .

Nunes A, Hernandez KD (2020) Autonomous taxis & public health: High cost or high opportunity cost? *Transportation Research Part A: Policy and Practice* 138:28–36.

Pinker EJ, Larson RC (2003) Optimizing the use of contingent labor when demand is uncertain. *European Journal of Operational Research* 144(1):39–55.

Siddiq A, Taylor T (2022) Ride-hailing platforms: Competition and autonomous vehicles. *Manufacturing & Service Operations Management* 24(3):1511–1528.

Taylor TA (2018) On-demand service platforms. *Manufacturing & Service Operations Management* 20(4):704–720.

Appendix. Technical Details and Proofs

In this appendix, we first provide additional details for the case of exponentially distributed demand, including the complete solution for the equilibrium of the original game. Then, we give technical details for the modified game with commitment power. After that, we provide the proofs of the main results in the paper, and finally we give auxiliary results and proofs, including a separate section with some technical details for the human driver equilibrium.

A. Full Solution for Exponential Demand

First, we note that the driver equilibrium condition (EQ) may have multiple solutions or no solution at all in α. If the equation has no solution, then in equilibrium either $\alpha = 0$ or $\alpha = 1$, while if there are multiple solutions to the equation, then all are equilibria. However, although there may be multiple equilibria for a given wage, with exponential demand, for any given equilibrium joining fraction α, there is a unique wage that can induce this fraction. In case of multiple equilibrium fractions for the same wage, we assume that the platform can induce the equilibrium fraction of its choosing. Because the platform prefers more human drivers all else being equal, it should always choose the highest fraction corresponding to a given wage. In summary, instead of describing how the joining fraction, the deployment quantity and the matching rate vary with the wage, we can express the latter two quantities and the wage in terms of the joining fraction.

The wage may exceed the unit revenue r, entailing a negative overage "cost" such that matching passengers with human drivers actually represents a net loss for the platform. This possibility may seem far-fetched because the platform would be foolish to facilitate rides on which it loses money (apart from special discounts to gain market share, which are beyond the scope of this work), but it is relevant because of the equilibrium condition (EQ). In some cases, to induce a particular α, the firm must pay a wage higher than the revenue per ride.

Proposition 6 in the main body provides closed-form solutions for all the quantities of interest. This result parallels Propositions 4 and 5 for the case of exponential demand. It first establishes that regardless of the wage that the platform sets (cf. Proposition 4, which requires $w \le r$), the joining fraction α cannot exceed $c\mu/(Mv)$. The platform can never provide enough incentive for drivers to induce them all to join (when this bound is smaller than 1). To see why this is true, first note that the matching rate defined in Equation (5) specialized to the exponential case is

$$\gamma(\alpha, q) = \mu e^{-q/\mu}\Big(\frac{1 - e^{-\alpha M/\mu}}{\alpha M}\Big).$$

For a given wage w, combining the above with the FOC for the optimal AV quantity in Proposition 1 gives the following expression for a driver's expected earnings (the wage multiplied by the matching rate)

$$w\gamma(\alpha, q(w,\alpha)) = \frac{c\mu}{\alpha M}\Big(1 - \frac{r}{r + w(e^{\alpha M/\mu} - 1)}\Big). \tag{9}$$

Note that this quantity is increasing in the wage w. However, because the optimal deployment quantity adjusts to changes in w, the platform is restricted in its ability to increase the expected earnings, even for arbitrarily high wages. Indeed,

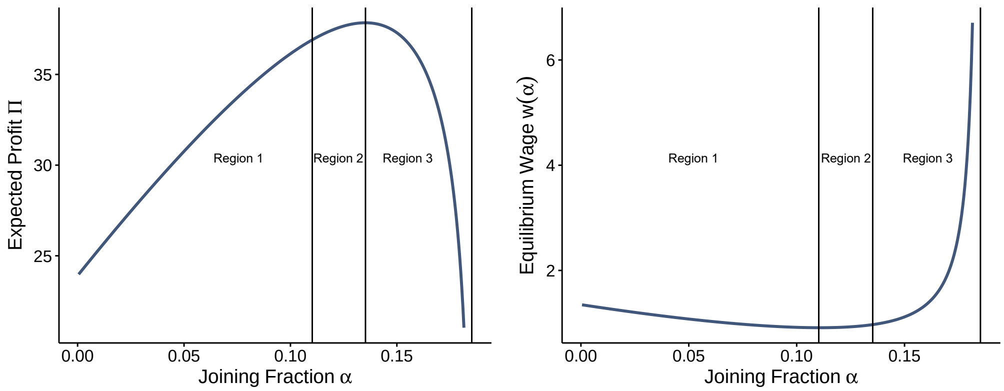
$$\lim_{w\to\infty} \frac{c\mu}{\alpha M}\Big(1 - \frac{r}{r + w(e^{\alpha M/\mu} - 1)}\Big) = \frac{c\mu}{\alpha M}.$$

Because the expected human driver earnings are increasing in the wage, the limiting value $c/(\lambda\alpha M)$ is an upper bound on the human driver expected earnings for any wage. This bound on human driver expected earnings constrains the level of human participation that the platform can induce. In fact, when we account for the equilibrium outcome, i.e., we set Eq. (9) equal to the outside option v, we see that the joining fraction α can be no larger than $c/(\lambda M v)$.

We next provide in closed form the optimal wage, human joining fraction, and AV deployment quantity.

PROPOSITION 10 **(Platform's Optimal Solution)**. *If $v < c$, $F(M) \le (r - c)/r$, and the demand is exponentially distributed with mean μ, then the optimal solution to $(\mathcal{P})$ is*

$$w^\star = \frac{r(c-v)}{v(e^{\frac{c-v}{v}} - 1)}, \quad \alpha^\star = \min\Big\{\frac{\mu(c-v)}{Mv}, 1\Big\}, \quad \text{and} \quad q^\star = \mu \log\Big[\frac{(r - w^\star)e^{-\alpha^\star M/\mu} + w^\star}{c}\Big]^+. \tag{10}$$



(a) Platform Profit $\Pi(w(\alpha),\alpha,q(w(\alpha),\alpha))$ (b) Equilibrium Wage

Figure 5 **Relationship between wage, joining fraction, and profit ($M=200$, $r=5$, $c=1$, $v=.27$, $\mu=10$).**

The solution in Proposition 10 achieves a balance between the required wages and deployment quantity. Consider Figure 5 and Figure 2 as the joining fraction increases from 0. First, both the required wage and the deployment quantity decrease while the matching rate increases. This is profitable for the platform because it can get more human drivers at a smaller wage while saving on operating costs for the AVs that are not deployed. But as more human drivers enter the market, the drops in wage diminish until they become zero. In fact, to further induce more human drivers to join, the platform must start increasing wages while reducing the deployment quantity as the matching rate decreases due to competition. This is still profitable for the platform because it prefers to serve customers with human drivers and it can cover a larger portion of the demand. There is a tipping point α at which the required wage becomes large enough that it is no longer profitable for the platform to continue incentivizing human drivers into the market, at which point the platform would prefer to use AVs to serve demand. At this point, it is better for the firm to increase its AV deployment quantity so that more of the demand is served by higher-margin AVs. In fact, inducing a finite fraction of human drivers could require an arbitrarily large wage. Interestingly, $\alpha^\star$ is different from the joining fraction at which the wage starts increasing and the matching rate being decreasing. That is, there is a region in which it is yet profitable for the platform to induce more humans into the market despite having to pay them more because their wage is still sufficiently small.

Note that the induced equilibrium joining fraction $\alpha^\star$ is increasing in the operating cost c. That is, the more expensive it is to operate AVs, the more the platform wishes to offload some of the demand (or at least, demand risk) to human drivers. Additionally, $\alpha^\star$ decreases with the outside option reflecting the fact that a higher outside option makes human drivers more costly for the platform. The optimal wage is increasing in $c/v>1$. For larger operating cost or for a lower outside option, the optimal wage increases because the platform aims to induce into the market a higher fraction of humans which, in turn, requires a higher wage.

B. A Modified Game with Commitment Power

In the modified game, the platform first (irrevocably) selects and announces its deployment quantity q, *then* it sets wages, after which human drivers decide whether to join the platform. The functional form of the platform's expected profit is the same in this game as in the original game; what changes are the equilibrium strategies. Human drivers do not need to anticipate the platform's deployment decision; instead, they make their decision based on the pre-announced deployment quantity and wage. We will see that this setting is amenable to closed-form characterizations because there is no longer a non-linear interaction between the equilibrium joining fraction and the deployment quantity. We assume ample AVs in the modified game, so the platform can select any $q \geq 0$.

We now compare the optimal profits of the modified and original games. In the modified game, the platform's optimization problem is

$$\widetilde{\Pi} \triangleq \max_{(w,\alpha,q)} \quad \Pi(w,\alpha,q) \qquad \text{s.t.} \quad w\gamma(\alpha,q)=v. \tag{$\widetilde{\mathcal{P}}$}$$

We will use $(\widetilde{w},\widetilde{\alpha},\widetilde{q})$ to refer to the optimal solution of $(\widetilde{\mathcal{P}})$. For the original problem $(\mathcal{P})$, note that the equilibrium condition (EQ) can be equivalently expressed via the two constraints $w\gamma(\alpha,q)=v$ and $q=q(w,\alpha)$. Thus, it is clear that any feasible solution to $(\mathcal{P})$ is also feasible for $(\widetilde{\mathcal{P}})$, and the problems also have the same objective function.

Proposition 7 in the main body implies that the profit in the modified game is at least as large as that in the original game. As a first step toward demonstrating the difference between the two games, we next solve the platform's problem in the modified game. The proposition also highlights the role of commitment power. In the original game, the platform cannot commit to a deployment quantity; however, in the modified game, the platform does commit and, consequently, can obtain a higher profit.

In the modified game, to induce an equilibrium human joining fraction α for a given deployment quantity q, the platform must choose a wage w satisfying the driver equilibrium condition

$$w\gamma(\alpha,q)=v.^{9} \tag{11}$$

This driver equilibrium condition differs from (EQ): here, the deployment quantity q replaces $q(w(\alpha),\alpha)$ because the quantity is chosen before wages are set and human drivers make their joining decisions. Note that this reversed order reflects the change from the original game. In the original game, the platform first announced wages to induce a human joining fraction, then optimized its deployment quantity knowing the human participation level. The response of the deployment quantity to the wage and human participation level is what constrained the platform in its human driver recruitment efforts. By contrast, here in the modified game, the platform first commits to an AV deployment quantity, then chooses its wage to induce a desired human joining fraction. Nonetheless, in the modified game, the conditions under which it is optimal to induce some human drivers to participate are the same as those for the original game.

Proposition 11 **(Need for Human Drivers in the Modified Game)**. *If $c\le v$, then the optimal solution to $(\widetilde{\mathcal{P}})$ is such that $\widetilde{\alpha}=0$, $\widetilde{q}=q^{NV}$ and $\widetilde{\Pi}=\Pi^{NV}$. However, if $v<c$, then it is optimal for the platform to induce some human drivers to participate in equilibrium i.e., $\widetilde{\alpha}>0$.*

In the original game, a closed-form characterization of the optimal solution is not achievable for general distributions due to the equilibrium condition and the dependence of the deployment quantity on the joining fraction; by contrast, in the modified game, we can completely characterize the optimal solution as follows. First, let $\hat{w}(\alpha,q)$ be a wage that satisfies equation (11) for a joining fraction α and deployment quantity q (note that a unique $\hat{w}(\alpha,q)$ exists for any $0\le\alpha\le 1$ and deployment quantity q with $F(q)<1$: see Lemma 9 in Appendix E). We can then write the firm's expected profit in the modified game as

$$\Pi(\hat{w}(\alpha,q),\alpha,q)=\Pi^{\mathtt{NV}}(q+\alpha M)+\alpha M(c-v). \tag{12}$$

Then, for given q, we can optimize this function in α to reveal the optimal human joining fraction for the platform to induce. The optimality equation for α for a given q takes the form of a newsvendor-like critical fractile (see Lemma 4 in Appendix D). After substituting the optimality equation for α as a function of q to give $\alpha(q)$ (along with the corresponding wage), we can optimize the platform's profit in q.

Proposition 12 **(Optimal Solution in the Modified Game)**. *Suppose $v<c$. In the modified game, $\widetilde{w}=v/\gamma(\tilde{\alpha},\tilde{q})$. For the optimal deployment and induced human joining fractions:*

(i) if $F(M)\le(r-c)/r$, then the platform optimally chooses a quantity $\widetilde{q}$ and induces full human participation (i.e., it induces $\tilde{\alpha}=1$), where $\widetilde{q}$ is the unique solution to

$$F(\widetilde{q}+M)=\frac{r-c}{r} \quad \textit{and} \quad \widetilde{q}=q^{NV}-M; \tag{13}$$

[9] By Lemma 8 in Appendix E, it is optimal to satisfy this constraint at equality.

(ii) if $(r-c)/r < F(M) < (r-v)/r$, then the platform optimally deploys no AVs ($\tilde{q}=0$) and induces full participation from human drivers ($\tilde{\alpha}=1$);

(iii) if $F(M) \geq (r-v)/r$, then the platform optimally deploys no AVs ($\tilde{q}=0$), and it induces a human joining fraction $\tilde{\alpha}$ such that

$$F(\tilde{\alpha}M) = \frac{r-v}{r}. \tag{14}$$

We can understand Proposition 12 by probing the origins of the critical fractiles in Eqs. (13) and (14). If $F(M) \leq (r-c)/r$, then AVs are relatively inexpensive in the sense that the original newsvendor critical fractile is large relative to the number of human drivers (human drivers alone would not even be enough to cover the original newsvendor critical fractile). The solution in part *(i)* of the proposition implies that the platform employs all of the human drivers, but it covers the same fractile of the demand distribution as it would without them—compare to Eq. (2). In other words, the platform achieves the same probability of shortage as it would with only AVs, but it replaces M AVs with human drivers by setting $\tilde{q} = q^{\mathtt{NV}} - M$ and inducing $\tilde{\alpha}=1$, paying the required wage to accomplish this. These human drivers provide the platform with a hedge against demand risk because it need pay wages only to the human drivers who are matched with a fare. Relative to the case with no human drivers, the platform reduces the probability that it will be on the hook for unused supply, without increasing the probability of shortage. This level of human participation ($\alpha=1$) might not be feasible in the original game (as we saw for exponential demand in Appendix A), and, even if feasible, it might not be profitable: if $M \geq (c/v)\mu$, then by Proposition 4, it cannot be optimal to induce all human drivers to participate in the original game because the equilibrium wage will be higher than the unit revenue.

Conversely, if $(r-c)/r < F(M)$, then AVs are expensive enough (and there are enough human drivers) that the platform actually prefers not to use AVs at all. This is a "conservative" outcome for the platform: because AVs are relatively expensive and it has access to human drivers who present no overage risk, it is better to fulfill demand only with human drivers. Whether to induce all or only some of the human drivers to participate depends on the number of human drivers and the value of their outside option v. Fixing the number of human drivers, with a low outside option human drivers are cheap and, therefore, inducing all of them to join is optimal. With a higher outside option, it is not optimal to induce all of them to join, but it is optimal to cover the demand fractile $(r-v)/r$ using only human drivers. Summarizing, in the modified game, it is always optimal for the platform to cover a fractile of the demand between $(r-c)/r$ and $(r-v)/r$. The optimal point within this interval to choose, and what mix of human drivers and AVs to use to reach it, depends on the cost parameters, the demand distribution, and the number of human drivers.

C. Proofs of Main Results

Proof of Proposition 1. Taking the expectation over demand realizations in equation (1), we can compute the firm's expected profit for a given deployment quantity q, given w and α. Specifically, we have

$$\begin{aligned}\Pi(w,\alpha,q) = r\Big[\int_0^{q+\alpha M} uf(u)\mathrm{d}u + (q+\alpha M)\bar{F}(q+\alpha M)\Big] \\ - w\Big[\int_q^{q+\alpha M} (u-q)f(u)\mathrm{d}u + \alpha M\bar{F}(q+\alpha M)\Big] - cq,\end{aligned} \tag{15}$$

Differentiating the expected profit with respect to q, we get

$$\frac{\partial \Pi}{\partial q} = r\bar{F}(q+\alpha M) + w\Big[F(q+\alpha M) - F(q)\Big] - c. \tag{16}$$

The first-order condition (FOC) (3) follows by rearranging equation (16). We first argue that the first order condition always characterizes the optimal quantity (note that here we are allowing for negative quantities as well even though those are not feasible). Suppose that $w > r$. At $q=0$, from equation (16) we have

$$\frac{\partial \Pi}{\partial q}\Big|_{q=0} = r - c + (w-c)F(\alpha M) > 0,$$

and we always have that

$$\lim_{q\to\infty} \frac{\partial \Pi}{\partial q} = -c.$$

Hence, by the Intermediate Value Theorem there exists $q \in (0,\infty)$ such that $\frac{\partial \Pi}{\partial q} = 0$ and such q is optimal.

Next, suppose that $w \le r$. At $q = -\alpha M$ we have $\frac{\partial \Pi}{\partial q} > 0$. Hence, again, by the Intermediate Value Theorem there exists $q \in [-\alpha M, \infty)$ such that $\frac{\partial \Pi}{\partial q} = 0$. Differentiating the profit Π a second time, from equation (16) we get

$$\begin{aligned}\frac{\partial^2 \Pi}{\partial q^2} &= -rf(q+\alpha M) + w\Big[f(q+\alpha M) - f(q)\Big]\\ &= (w-r)f(q+\alpha M) - wf(q)\\ &< 0,\end{aligned}$$

where the last inequality follows because $w \le r$. Thus, the function is concave, and the FOC (3) is sufficient for a global maximum. If the global maximum is negative then because of the concavity, the optimal feasible deployment quantity is zero. If the global maximum is positive and below N then the optimal deployment quantity coincides with $\hat{q}(w,\alpha)$; but if $\hat{q}(w,\alpha) > N$ then the optimal deployment quantity is N. In summary, if $w \le r$ then the optimal deployment quantity is given by $\min\{\hat{q}(w,\alpha)^+, N\}$.

Finally, to see the monotonicity of $q(w,\alpha)$ with respect to α and w in the two cases, it is enough to study the modularity of Π. We have

$$\frac{\partial^2 \Pi}{\partial q \partial \alpha} = (w-r)Mf(q+\alpha M), \quad \text{and} \quad \frac{\partial^2 \Pi}{\partial q \partial w} = F(q+\alpha M) - F(q),$$

that is, given w, Π is super-modular when $w > r$ and sub-modular when $w \le r$. Hence, by Topkis Theorem, in the former case $q(w,\alpha)$ is non-decreasing in α and in the latter case $q(w,\alpha)$ is non-increasing in α. Also, given α, Π is always super-modular. Thus, by Topkis Theorem $q(w,\alpha)$ is non-decreasing in w. This concludes the proof. □

Proof of Corollary 1. If $c < r \le w$, then at $q = 0$, the LHS of equation (3) is equal to

$$(r-w)F(\alpha M) \le 0 < r - c.$$

Thus, the LHS is less than the RHS at $q = 0$. Also, we have

$$\frac{\partial \Pi}{\partial q}\Big|_{q=0} = r\bar{F}(\alpha M) - c + wF(\alpha M) = r - c + (w-r)F(\alpha M) > 0. \tag{17}$$

Because also $\lim_{q\to\infty} \partial \Pi / \partial q = -c$, a local maximum must exist by the intermediate value theorem. Differentiating the LHS of equation (3) gives

$$(r-w)f(q+\alpha M) + wf(q) \ge 0,$$

where the inequality holds because the exponential density function F is decreasing. Thus, the LHS is increasing in q and the RHS is constant, so the solution to the FOC is unique and is the global maximizer.

If $\hat{q} \le N$, then the optimal deployment is $\hat{q}$. If $\hat{q} > N$, then the function is still increasing at N and therefore the optimal deployment is N.

For $w < r$ the result follows from using the exponential distribution in Proposition 1 part (ii). □

Proof of Proposition 2. We begin with the case $c \le v$. We first show that in this case $\Pi^\star \le \Pi^{\texttt{NV}}$, and then we show that the solution in the statement achieves a profit of $\Pi^{\texttt{NV}}$.

Note that a total of αM human drivers join the platform. For the drivers who join, the matching rate is $\gamma(\alpha, q(w,\alpha))$. As a result, in expectation the platform pays w to a total of $\alpha M \gamma(\alpha, q(w,\alpha))$ human drivers. We can then write the platform's objective as

$$\Pi(w,\alpha,q) = r\Big[\int_0^{q+\alpha M} uf(u)\mathrm{d}u + (q+\alpha M)\bar{F}(q+\alpha M)\Big] - w\alpha M\gamma(\alpha, q(w,\alpha)) - cq,$$

Using (EQ) in the equation above, we have

$$\begin{aligned}\Pi(w,\alpha,q) &= r\Big[\int_0^{q+\alpha M} uf(u)\mathrm{d}u + (q+\alpha M)\bar{F}(q+\alpha M)\Big] - \alpha M v - cq\\ &= r\Big[\int_0^{q+\alpha M} uf(u)\mathrm{d}u + (q+\alpha M)\bar{F}(q+\alpha M)\Big] - \alpha M v - c\,(q+\alpha M) + c\alpha M\\ &\le \max_q \Big\{ r\Big[\int_0^{q} uf(u)\mathrm{d}u + q\bar{F}(q)\Big] - cq\Big\} + \alpha M(c-v)\\ &= \Pi^{\texttt{NV}} + \alpha M(c-v)\\ &\le \Pi^{\texttt{NV}},\end{aligned}$$

where in the last inequality we used that $c \le v$. Moreover, in the above we replaced $w\alpha M\gamma(\alpha, q(w,\alpha))$ by $\alpha M v$, which is possible thanks to (EQ). Next we show that this upper bound is attained. Note that any solution such that $\alpha > 0$ will yield a profit strictly below $\Pi^{\texttt{NV}}$. Now, consider a solution as in the statement. Because $w^\star < v$, from (EQ) we can deduce that $\alpha = 0$ is the only possible equilibrium. By Proposition 1 and (2) the platform's best response quantity is $q^{\texttt{NV}}$, and the objective value is $\Pi^{\texttt{NV}}$.

Now assume that $v < c$. Define the function

$$\Pi^{\texttt{NV}}(q) \triangleq r\Big[\int_0^q uf(u)\mathrm{d}u + q\bar{F}(q)\Big] - cq.$$

Note that $\Pi^{\texttt{NV}}(q)$ is maximized at $q = q^{\texttt{NV}}$ and $\frac{d}{dq}\Pi^{\texttt{NV}}(q^{\texttt{NV}}) = 0$. From the previous part of the proof, we have

$$\Pi(w,\alpha,q) = \Pi^{\texttt{NV}}(q + \alpha M) + \alpha M(c - v)$$

Suppose that q is such that

$$q^{\texttt{NV}} - \alpha C_2 \le q \le q^{\texttt{NV}} - \alpha C_1, \tag{18}$$

for some positive constants C_1 and C_2. Then,

$$\begin{aligned}\Pi(w,\alpha,q) &= \Pi^{\texttt{NV}}(q^{\texttt{NV}}) + \frac{1}{2}\frac{d^2}{dq^2}\Pi^{\texttt{NV}}(\xi)O(\alpha^2) + \alpha M(c - v)\\ &= \Pi^{\texttt{NV}}(q^{\texttt{NV}}) - \frac{1}{2}rf(\xi)\cdot O(\alpha^2) + \alpha M(c - v).\end{aligned}$$

where we have performed a Taylor expansion with ξ in a neighborhood of $q^{\texttt{NV}}$ (that can be taken arbitrarily close to $q^{\texttt{NV}}$). So if we can establish that $q(w(\alpha),\alpha)$ satisfies Eq. (18), that $f(\xi)$ is bounded and that α can be made arbitrarily small then we would have proven that there exists $\alpha > 0$ such that $\Pi > \Pi^{\texttt{NV}}$. The latter is true because the last two terms above would be positive for α small enough since $c > v$. This is enough to show that the optimal solution should be such that $\alpha^\star > 0$ because from the first part of the proof we can deduce that $\alpha^\star = 0$ leads to a platform profit bounded above by $\Pi^{\texttt{NV}}$.

Now, we show how to set the wages to induce an equilibrium such that the above holds. From Lemma 2 we know that there exists $\hat{\alpha} > 0$ such that for all $\alpha \in [0,\hat{\alpha}]$ there exists a unique $w(\alpha) \le r$ with

$$w(\alpha)\gamma(\alpha, q(w(\alpha),\alpha)) = v.$$

That is, any $\alpha \le \hat{\alpha}$ can be achieved by setting a wage of $w(\alpha) \le r$ which is continuous in $[0,\hat{\alpha}]$. Let $\hat{q}(w(\alpha),\alpha)$ be the unique solution to the first order condition from Proposition 1 (later we will verify that $q(w(\alpha),\alpha) = \hat{q}(w(\alpha),\alpha)$ because we will have that $\hat{q}(w(\alpha),\alpha) > 0$):

$$(r - w(\alpha))F(q + \alpha M) + w(\alpha)F(q) = (r - c).$$

Observe because $w(\alpha)$ is continuous in $[0,\hat{\alpha}]$ and $\hat{q}(w,\alpha)$ is continuous (see Lemma 1) for $w \le r$, the first order condition above implies that $\hat{q}(w(\alpha),\alpha)$ must be bounded by some $\bar{q}$. This, in turn, implies Eq. (18). Indeed, from the first order condition and the mean value theorem, for $\chi_1 \in [q, q + \alpha M]$ we have

$$(r - w(\alpha))\left[F(q) + \alpha M f(\chi_1)\right] + w(\alpha)F(q) = (r - c),$$

which gives

$$F(q) = F(q^{\texttt{NV}}) - \alpha M\frac{(r - w(\alpha))}{r}f(\chi_1), \quad \chi_1 \in [q, q + \alpha M].$$

Note that the derivative of $F^{-1}(x)$ is $1/f(F^{-1}(x))$, hence by the mean value theorem we have that

$$q = q^{\texttt{NV}} - \frac{1}{f(F^{-1}(\chi_2))}\alpha M\frac{(r - w(\alpha))}{r}f(\chi_1), \quad \chi_2 \in [F(q^{\texttt{NV}}) - \alpha M\frac{(r - w(\alpha))}{r}f(\chi_1), F(q^{\texttt{NV}})].$$

The above corresponds to the deployment quantity that solves the first order condition $\hat{q}(w(\alpha),\alpha)$. We define $f_U = \sup_{x\in[0,\bar{q}+M]} f(x)$ and $f_L = \inf_{x\in[0,\bar{q}+M]} f(x)$. Note that $f_U, f_L > 0$ and are well defined. We also define $w_U = \sup_{\alpha\in[0,\hat{\alpha}]} w(\alpha)$ and $w_L = \inf_{\alpha\in[0,\hat{\alpha}]} w(\alpha)$ which are both well defined due to the continuity of $w(\cdot)$. We let $\bar{w}$ denote $\max\{|r - w_L|, |r - w_U|\}$. Hence we have the following bounds

$$q^{\texttt{NV}} - \alpha M\frac{\bar{w}}{r}\frac{f_U}{f_L} \le \hat{q}(w(\alpha),\alpha) \le q^{\texttt{NV}} + \alpha M\frac{\bar{w}}{r}\frac{f_U}{f_L},$$

which proves that $q(w(\alpha),\alpha)$ satisfies Equation (18). Additionally, note that $f(\xi)$ is bounded because ξ is in a neighborhood of $q^{\texttt{NV}}$ and f is continuous. □

Proof of Proposition 3. Consider the equilibrium condition for $\alpha > 0$:

$$w(\alpha)\gamma(\alpha, q(\alpha)) = v, \tag{19}$$

where to simplify notation we are using $q(\alpha)$ to denote $q(w(\alpha), \alpha)$. Taking derivative on both sides above gives

$$\dot{w}(\alpha)\gamma(\alpha, q(\alpha)) + w(\alpha)\frac{d}{d\alpha}\gamma(\alpha, q(\alpha)) = 0. \tag{20}$$

We will now take the limit as $\alpha \downarrow 0$ for each term in the previous equation which will give us an expression for the derivative of $w(\cdot)$ at zero. We have:

$$\begin{aligned}\frac{d}{d\alpha}\gamma(\alpha, q(\alpha)) &= \frac{1}{\alpha M}\left[\bar{F}(q(\alpha)+\alpha M)(\dot{q}(\alpha)+M) - \bar{F}(q(\alpha))\dot{q}(\alpha)\right] - \frac{1}{\alpha^2 M}\int_{q(\alpha)}^{q(\alpha)+\alpha M}\bar{F}(x)dx \\ &= \dot{q}(\alpha)\cdot\frac{1}{\alpha M}\left[\bar{F}(q(\alpha)+\alpha M) - \bar{F}(q(\alpha))\right] + \frac{1}{\alpha^2 M}\left\{\bar{F}(q(\alpha)+\alpha M)\alpha M - \int_{q(\alpha)}^{q(\alpha)+\alpha M}\bar{F}(x)dx\right\}\end{aligned}$$

We use L'Hôpital's rule to obtain the limit of each term above. For the first term, we have

$$\lim_{\alpha\downarrow 0}\dot{q}(\alpha)\cdot\frac{1}{\alpha M}\left[\bar{F}(q(\alpha)+\alpha M) - \bar{F}(q(\alpha))\right] = -\dot{q}(0)\cdot f(q(0)),$$

for the second term we have

$$\begin{aligned}\lim_{\alpha\downarrow 0}\frac{1}{\alpha^2 M}\left\{\bar{F}(q(\alpha)+\alpha M)\alpha M - \int_{q(\alpha)}^{q(\alpha)+\alpha M}\bar{F}(x)dx\right\} &= \lim_{\alpha\downarrow 0}\left\{\frac{\bar{F}(q(\alpha)+\alpha M)M - f(q(\alpha)+\alpha M)(\dot{q}(\alpha)+M)\alpha M}{2\alpha M}\right. \\ &\quad \left. - \frac{\bar{F}(q(\alpha)+\alpha M)(\dot{q}(\alpha)+M) - \bar{F}(q(\alpha))\dot{q}(\alpha)}{2\alpha M}\right\} \\ &= \lim_{\alpha\downarrow 0}\frac{-f(q(\alpha)+\alpha M)(\dot{q}(\alpha)+M)}{2} \\ &\quad - \lim_{\alpha\downarrow 0}\dot{q}(\alpha)\frac{\bar{F}(q(\alpha)+\alpha M) - \bar{F}(q(\alpha))}{2\alpha M} \\ &= -\frac{f(q(0))(\dot{q}(0)+M)}{2} + \frac{\dot{q}(0)\cdot f(q(0))}{2} \\ &= -f(q(0))\frac{M}{2}.\end{aligned}$$

Hence,

$$\lim_{\alpha\downarrow 0}\frac{d}{d\alpha}\gamma(\alpha, q(\alpha)) = f(q(0))\left[-\frac{M}{2} - \dot{q}(0)\right] \tag{21}$$

Also for $q(\alpha)$ we have

$$(r - w(\alpha))F(q(\alpha)+\alpha M) + w(\alpha)F(q(\alpha)) = r - c.$$

Differentiating on both sides and then letting $\alpha \downarrow 0$ we have:

$$\dot{q}(0) = \frac{w(0)\cdot M}{r} - M.$$

Using this in (21), that $q(0) = q^{\texttt{NV}}$ and letting $\alpha \downarrow 0$ yields:

$$\frac{d}{d\alpha}\gamma(\alpha, q(\alpha))\big|_{\alpha=0} = f(q(0))\left[-\frac{M}{2} - \dot{q}(0)\right] = Mf(q^{\texttt{NV}})\left[\frac{1}{2} - \frac{w(0)}{r}\right].$$

Using L'Hopital's rule for $\gamma(\alpha, q(\alpha))$ gives

$$\lim_{\alpha\downarrow 0}\gamma(\alpha, q(\alpha)) = \lim_{\alpha\downarrow 0}\frac{\bar{F}(q(\alpha)+\alpha M)(\dot{q}(\alpha)+M) - \bar{F}(q(\alpha))\dot{q}(\alpha)}{M} = \bar{F}(q^{\texttt{NV}}).$$

Hence, taking the limit as $\alpha \downarrow 0$ in (19) yields:

$$w(0) = \frac{v}{\bar{F}(q^{\texttt{NV}})}.$$

Combining these results with (20), we have

$$\dot{w}(0)\bar{F}(q^{\texttt{NV}}) + \frac{v}{\bar{F}(q^{\texttt{NV}})}\cdot Mf(q^{\texttt{NV}})\left[\frac{1}{2} - \frac{v}{r\bar{F}(q^{\texttt{NV}})}\right] = 0.$$

Recalling that $\bar{F}(q^{\texttt{NV}}) = c/r$ and rearranging terms gives

$$\dot{w}(0) = \frac{vr^2}{c^2}\cdot Mf(q^{\texttt{NV}})\left[\frac{v}{c} - \frac{1}{2}\right].$$

That is, $w(\alpha)$ is decreasing in a neighborhood of 0 if $c > 2v$, and $w(\alpha)$ is increasing in a neighborhood of 0 if $c \in (v, 2v)$.
□

Proof of Proposition 4. We have the same assumptions as in Proposition 1, with the additional assumption of ample AVs. Let α be an equilibrium. If $\alpha = 0$, then the result directly holds. Suppose then that $\alpha > 0$. For simplicity of notation let q denote the optimal deployment quantity $q(w, \alpha)$. Then, from Proposition 1 and (EQ) we have that

$$(r-w)\bar{F}(q+\alpha M) + w\bar{F}(q) \leq c$$
$$w\int_q^{q+\alpha M} \frac{(u-q)}{\alpha M} f(u)\mathrm{d}u + w\bar{F}(q+\alpha M) = v.$$

Note that the first equation above becomes an equality when the $q > 0$, but it is an inequality when $q = 0$ because the latter case occurs only when the profit function is concave and the solution to the first order condition is negative (see (16)). Define

$$L(q) \triangleq \mathsf{E}[(D-q)^+] = \mathsf{E}[D-q|D>q]\bar{F}(q).$$

Then, by writing the equilibrium condition in terms of $L(q)$, we have that

$$\begin{aligned}\frac{c}{v} &\geq \frac{(r-w)\bar{F}(q+\alpha M) + w\bar{F}(q)}{w\left(L(q) - L(q+\alpha M)\right)}\alpha M \\ &\geq \frac{w\bar{F}(q)}{wL(q)}\alpha M \\ &= \frac{\alpha M}{\mathsf{E}[D-q|D>q]}.\end{aligned}$$

In turn, we deduce that

$$\alpha \leq \frac{c\mathsf{E}[D-q|D\geq q]}{vM} \leq \frac{c\mathsf{E}[D]}{vM},$$

where in the last inequality we have used that the mean residual life $\mathsf{E}[D-q|D>q]$ is non-increasing. □

Proof of Proposition 5. Consider the following equation for $x \geq 0$

$$x\cdot v = w\int_{q(w,x)}^{q(w,x)+x} \bar{F}(y)dy. \tag{22}$$

If $x = \alpha M$, then the right-hand side of this expression is equal to $w\alpha M\gamma(w, q(w,\alpha))$, i.e., the expected aggregate human driver earnings. Note that any equilibrium α times M is a solution to the equation above by (EQ). Hence, in order to prove the result, it suffices to show that the right-hand-side above is bounded by a function (to be determined) which is decreasing for $w \in [c, r]$.

First, define

$$H(w,x) \triangleq \int_{q(w,x)}^{q(w,x)+x} \bar{F}(y)dy \quad \text{and} \quad R(w) \triangleq \max_x H(w,x).$$

Note that the right-hand-side in Eq. (22) is bounded above by $wR(w)$. Hence, we need to prove that $wR(w)$ is decreasing for $w \in [c, r]$. From Proposition 1 part *(ii)*, we have that $q(w,x)$ is non-increasing in x. Hence, $H(w,x) \leq H(w,\infty)$. Note that

$$q(w,\infty) \in \arg\max_q \left\{ r\int_0^q \bar{F}(y)dy + (r-w)\int_q^\infty \bar{F}(y)dy - cq \right\}.$$

Thus

$$q(w,\infty) = F^{-1}\left(1 - \frac{c}{w}\right).$$

We have

$$wR(w) = w\int_{F^{-1}\left(1-\frac{c}{w}\right)}^\infty \bar{F}(y)dy = \frac{c}{\bar{F}(u(w))}\int_{u(w)}^\infty \bar{F}(y)dy = c\mathsf{E}[D-u(w)|D>u(w)],$$

where $u(w) = F^{-1}\left(1 - \frac{c}{w}\right)$. Since $u(w)$ is increasing in w and the mean residual life $\mathsf{E}[D-u|D>u]$ is decreasing, we conclude that $wR(w)$ is decreasing, as desired. □

Proof of Proposition 6. From Corollary 1 we can write

$$q(w,\alpha)=\mu\Big[\log\Big((r-w)e^{-\alpha M/\mu}+w\Big)-\log c\Big]. \tag{23}$$

Additionally, note that we can simplify the integral in Equation (5) to yield a closed form for the human driver matching rate γ. Equation (5) becomes

$$\gamma(\alpha,q(w,\alpha))=\mu e^{-q(w,\alpha)/\mu}\Big(\frac{1-e^{-\alpha M/\mu}}{\alpha M}\Big). \tag{24}$$

After substituting the RHS of equation (24), the driver equilibrium condition (EQ) becomes

$$w\mu e^{-q(w,\alpha)/\mu}\Big(\frac{1-e^{-\alpha M/\mu}}{\alpha M}\Big)=v. \tag{25}$$

Equations (23) and (25) form a system of equations for the unknowns w and $q(w,\alpha)$. We can substitute Eq. (23) for $q(w,\alpha)$ in equation (25) and simplify to yield

$$w(\alpha)=\frac{\alpha M r v}{(e^{\alpha M/\mu}-1)(c\mu-\alpha M v)}.$$

Plugging this into Eq. (23) yields

$$q(w(\alpha),\alpha)=\mu\log\left(\frac{e^{-\alpha\lambda M}r}{c-\alpha M v/\mu}\right).$$

Finally, the bound on human driver expected earnings and the resulting bound on the human joining fraction is proved in the paragraph preceding Proposition 10 in Appendix A. □

Proof of Proposition 7. Any feasible values (w,α,q) for $(\mathcal{P})$ are feasible for $(\widetilde{\mathcal{P}})$ as $(\widetilde{\mathcal{P}})$ is a relaxation of $(\mathcal{P})$ (see the discussion immediately following the statement of $(\widetilde{\mathcal{P}})$). Thus, the optimal solution to $(\mathcal{P})$ is feasible for $(\widetilde{\mathcal{P}})$, implying that the optimal profit $\Pi^\star$ from the original game is achievable in the modified game. It is therefore a lower bound on the optimal profit in the modified game, i.e., we have $\Pi^\star\le\widetilde{\Pi}$. □

Proof of Proposition 8. It is enough to exhibit an instance of our problem for which the infimum is below the desired bound. We consider $F(x)=1-e^{-x/\mu}$. Furthermore, consider (c,M) such that

$$F(M)\le\frac{r-c}{r},\qquad \frac{\mu\cdot(c-v)}{Mv}\le 1.$$

From (10) in Proposition 10 and the conditions above, we have

$$w^\star=\frac{r(c-v)}{v(e^{\frac{c-v}{v}}-1)},\quad \alpha^\star=\frac{\mu(c-v)}{Mv},\quad\text{and}\quad q^\star=\mu\log\Big[\frac{(r-w^\star)e^{-\frac{c-v}{v}}+w^\star}{c}\Big]^+.$$

and from Proposition 12 we have

$$\widetilde{\alpha}=1\quad\text{and}\quad F(\widetilde{q}+M)=\frac{r-c}{r}. \tag{26}$$

We also impose that $q\ge 0$. That is, we impose that $(r-w^\star)e^{-\frac{c-v}{v}}+w^\star\ge c$. The ratio we are analyzing then becomes

$$\frac{\Pi^\star}{\widetilde{\Pi}}=\frac{\Pi^{\texttt{NV}}(q^\star+\alpha^\star M)+\alpha^\star M(c-v)}{\Pi^{\texttt{NV}}(\widetilde{q}+M)+M(c-v)}=-\frac{\mu\left(c\log\left(\frac{re^{1-\frac{c}{v}}}{v}\right)+c-r\right)}{c(M-\mu)+c\mu\log\left(\frac{c}{r}\right)-Mv+\mu r}.$$

Hence, the ratio $\inf_{\mathcal{I}}\frac{\Pi^\star}{\widetilde{\Pi}}$ is bounded above by the value of the following optimization problem:

$$\begin{aligned}\inf_{M,c,\mu,r,v}\ &-\frac{\mu\left(c\log\left(\frac{re^{1-\frac{c}{v}}}{v}\right)+c-r\right)}{c(M-\mu)+c\mu\log\left(\frac{c}{r}\right)-Mv+\mu r}\\ \text{s.t}\ \ &1-e^{-M/\mu}\le\frac{r-c}{r},\quad c\in(v,r),\\ &\frac{\mu\cdot(c-v)}{Mv}\le 1,\\ &\frac{re^{1-\frac{c}{v}}}{v}\ge 1,\end{aligned} \tag{27}$$

where the last constraint ensures that $q\ge 0$.

To conclude the proof it is enough to exhibit a feasible solution to the problem above that achieves the desired bound. Fix μ, r and $a \in (1,2)$ and consider the following solution for $\delta > 0$ small:

$$c = r - \delta, \quad v = r - a\delta, \quad M = \delta.$$

We note that the above leads to a feasible solution as long as $r \in [\mu(a-1), \mu]$. Indeed, we clearly have that $c \in (v, r)$ and we also have that

$$1 - e^{-M/\mu} \leq \frac{r-c}{r} \Leftrightarrow r - r\frac{\delta}{2\mu} + o(\delta^2) \leq \mu,$$

which is satisfied for $\delta > 0$ small and r, μ bounded above and below. Also, $\frac{\mu \cdot (c-v)}{Mv} \leq 1$ is equivalent to $\mu(a-1) - \delta a \leq r$ which holds because we are taking $r \in [\mu(a-1), \mu]$. Finally, $\frac{re^{1-\frac{c}{v}}}{v} \geq 1$ is equivalent to

$$\frac{(a^2-1)\delta}{r} + o(\delta^2) \leq 1,$$

which holds for δ small for r bounded above and below. Moreover, we take

$$a = \frac{\sqrt{\mu^2 + 4r^2} - \mu + 2r}{2r},$$

which is in $(1,2)$ as previously assumed. Let $R(\mu, \delta, r)$ be the objective in (27) at the feasible solution we have constructed. By replacing the values of c, v, M and a, it is possible to verify that the following limit holds

$$\lim_{(r,\delta)\to(\mu,0)} R(\mu, \delta, r) = \frac{\sqrt{5}-1}{2},$$

as desired. □

Proof of Proposition 9. By Propositions 2 and 11, human drivers are active in both games if $v < c$ and in neither game otherwise. If human drivers are active in neither game, then the platform's solution is the same in both games, namely to choose the newsvendor quantity $q^{\texttt{NV}}$, i.e., we have $\widetilde{q} = q^{\texttt{NV}}$, and it is optimal in the original game to set $N' = N = \widetilde{q} = q^{\texttt{NV}}$, so the result holds. For the rest of the proof, we assume that $v < c$, so that humans are active in both games. Note that $\widetilde{\alpha} > 0$ implies $\widetilde{w} \leq r$, as otherwise the platform could improve its profit in the modified game by inducing $\alpha = 0$.

In the original game without ample AVs, we can write the platform's optimization problem as

$$\begin{aligned} \Pi' := \max_{N,w,\alpha,q} \quad & \Pi(w,\alpha,q) \\ \text{s.t.} \quad & w\gamma(\alpha,q) = v \\ & q = q(w,\alpha) \end{aligned} \tag{28}$$

First, note that the definition of $q(w,\alpha)$ in Proposition 1 includes the upper bound of N on the deployment quantity, so the constraint $q = q(w,\alpha)$ enforces both the sequential rationality of the deployment and its upper bound implied by the fleet size. The triplet $(\widetilde{w}, \widetilde{\alpha}, \widetilde{q})$ is optimal and therefore feasible for the problem $(\widetilde{\mathcal{P}})$, implying that $\widetilde{w}\gamma(\widetilde{\alpha}, \widetilde{q}) = v$.

If $F(M) \leq (r-c)/r$, then Lemma 6 and its proof imply that the FOC (3) has a unique solution $\hat{q}(\widetilde{w}, \widetilde{\alpha}) \geq 0$, which corresponds to the ample-AV solution to the platform's deployment problem with this wage and human joining fraction. So, letting $N' = \widetilde{q}$, combined with the above, part (ii) of Proposition 1 and the fact that $\widetilde{w} \leq r$ imply that the solution $(N', \widetilde{w}, \widetilde{\alpha}, \widetilde{q})$ satisfies all three constraints for the problem (28), and it achieves the same optimal profit $\widetilde{\Pi} = \Pi(\widetilde{w}, \widetilde{\alpha}, \widetilde{q})$ as does the modified game. If $F(M) > (r-c)/r$, then $\widetilde{q} = 0$ and whatever the ample-AV deployment is in the original game, the platform can recover the modified game's profit by setting $N = q = 0$, $w = \widetilde{w}$, and $\alpha = \widetilde{\alpha}$. Setting $N = 0$ ensures that $q(\widetilde{w}, \widetilde{\alpha}) = 0$, satisfying the second constraint, and the first constraint is satisfied because this constraint also applies to the modified game, for which this solution is optimal and thus feasible.

For the converse, note that the problem $(\widetilde{\mathcal{P}})$ is a relaxation of (28) (where (28) has an additional decision variable N that affects its additional constraint but not the objective function), so we must have $\Pi' \leq \widetilde{\Pi}$. Having demonstrated above that the profit $\widetilde{\Pi}$ can be achieved in the original game without ample AVs, we conclude that this bound is tight, i.e., we have $\Pi' = \widetilde{\Pi} \geq \Pi^\star$. □

Proof of Proposition 10. After substituting $w(\alpha)$, $q(w(\alpha),\alpha)$, and the exponential CDF into the expected profit (15), we obtain $\Pi(w(\alpha),\alpha,q(w(\alpha),\alpha))$. Differentiating this single-variable function of α then gives

$$\frac{\mathrm{d}\Pi}{\mathrm{d}\alpha} = cM\Big(1+\frac{v}{\alpha Mv/\mu - c}\Big),$$

along with the FOC

$$\alpha = \mu\left(\frac{c-v}{Mv}\right) > 0. \tag{29}$$

Evaluating the derivative at $\alpha = 0$ gives

$$\frac{\mathrm{d}\Pi}{\mathrm{d}\alpha}\Big|_{\alpha=0} = cM\Big(1-\frac{v}{c}\Big) > 0, \tag{30}$$

i.e., the function is increasing at $\alpha = 0$. Moreover, the derivative is decreasing in α for α such that $\alpha Mv < c\mu$, which is the full range of interest: values of α outside this range cannot be induced for any wage, as noted previously. So, the function is concave in α on the relevant interval, and the FOC (29) is sufficient for a global maximum. Moreover, the FOC can be rearranged to give

$$\alpha Mv/\mu = c - v < c,$$

implying that the solution to the FOC is contained in the relevant interval with a nonnegative required wage. If the solution to the FOC gives $\alpha > 1$, then the function is increasing on $[0,1]$, and the optimal solution is to set $\alpha = 1$, hence the minimum in equation (10). The equilibrium wage and deployment quantity are obtained by substituting equation (29) into equation (7). □

Proof of Proposition 11. Analogous arguments to those in the first part of the proof of Proposition 2 establish the first part of this result for $c \le v$. For the case with $v < c$, recall that $\Pi^{\mathtt{NV}}$ denotes the expected profit at the optimal newsvendor quantity with no human drivers. Recall also that $\gamma(\alpha, q^{\mathtt{NV}})$ is the human matching rate if the firm is known to have chosen a deployment quantity $q^{\mathtt{NV}}$. Note that in the original game, the wage w influences the matching rate γ through the optimized deployment quantity $q(w,\alpha)$. However, in the modified game, the matching rate does not depend on the wage because the deployment quantity does not respond to α and w. The profit function for the modified game, for $q = q^{\mathtt{NV}}$, is given by

$$\begin{aligned}\Pi(w,\alpha,q^{\mathtt{NV}}) = r\Big[\int_0^{q^{\mathtt{NV}}+\alpha M} uf(u)\mathrm{d}u + (q^{\mathtt{NV}}+\alpha M)\bar{F}(q^{\mathtt{NV}}+\alpha M)\Big]\\ - w\Big[\int_{q^{\mathtt{NV}}}^{q^{\mathtt{NV}}+\alpha M}(u-q^{\mathtt{NV}})f(u)\mathrm{d}u + \alpha M\bar{F}(q^{\mathtt{NV}}+\alpha M)\Big] - cq^{\mathtt{NV}}.\end{aligned} \tag{31}$$

By separating the integral and the second term in the first set of square brackets, we can equivalently express equation (31) as

$$\begin{aligned}\Pi(w,\alpha,q^{\mathtt{NV}}) =& r\Big[\int_0^{q^{\mathtt{NV}}} uf(u)\mathrm{d}u + q^{\mathtt{NV}}\bar{F}(q^{\mathtt{NV}})\Big]\\ &+ r\Big[\int_{q^{\mathtt{NV}}}^{q^{\mathtt{NV}}+\alpha M} uf(u)\mathrm{d}u + (q^{\mathtt{NV}}+\alpha M)\bar{F}(q^{\mathtt{NV}}+\alpha M) - q^{\mathtt{NV}}\bar{F}(q^{\mathtt{NV}})\Big]\\ &- w\Big[\int_{q^{\mathtt{NV}}}^{q^{\mathtt{NV}}+\alpha M}(u-q^{\mathtt{NV}})f(u)\mathrm{d}u + \alpha M\bar{F}(q^{\mathtt{NV}}+\alpha M)\Big] - cq^{\mathtt{NV}}\\ =& \Pi^{\mathtt{NV}} + r\Big[\int_{q^{\mathtt{NV}}}^{q^{\mathtt{NV}}+\alpha M} uf(u)\mathrm{d}u + (q^{\mathtt{NV}}+\alpha M)\bar{F}(q^{\mathtt{NV}}+\alpha M) - q^{\mathtt{NV}}\bar{F}(q^{\mathtt{NV}})\Big]\\ &- w\alpha M\Big[\int_{q^{\mathtt{NV}}}^{q^{\mathtt{NV}}+\alpha M}\frac{(u-q^{\mathtt{NV}})}{\alpha M}f(u)\mathrm{d}u + \bar{F}(q^{\mathtt{NV}}+\alpha M)\Big]\\ =& \Pi^{\mathtt{NV}} + r\Big[\int_{q^{\mathtt{NV}}}^{q^{\mathtt{NV}}+\alpha M} uf(u)\mathrm{d}u + (q^{\mathtt{NV}}+\alpha M)\bar{F}(q^{\mathtt{NV}}+\alpha M) - q^{\mathtt{NV}}\bar{F}(q^{\mathtt{NV}})\Big]\\ &- w\alpha M\gamma(\alpha, q^{\mathtt{NV}}).\end{aligned} \tag{32}$$

The second equality in equation (32) follows from isolating $\Pi^{\mathtt{NV}}$ and factoring out αM from the second bracketed term. Also, the second bracketed term in the second equality in equation (32) is equal to $\gamma(\alpha, q^{\mathtt{NV}})$ by equation (5); making this substitution yields the last equality in equation (32).

We can use the driver equilibrium condition (11) to let the platform choose α, requiring in turn that the wage is set so that the chosen α satisfies the condition. Setting $q = q^{\mathtt{NV}}$ and substituting the RHS of equation (11) into the last equality in equation (32), we have that the platform's expected profit from deploying $q^{\mathtt{NV}}$ and choosing a wage that induces a human driver joining fraction α is

$$\Pi(\hat{w}(\alpha, q^{\mathtt{NV}}), \alpha, q^{\mathtt{NV}}) = \Pi^{\mathtt{NV}} + r\Big[\int_{q^{\mathtt{NV}}}^{q^{\mathtt{NV}}+\alpha M} uf(u)\mathrm{d}u + (q^{\mathtt{NV}} + \alpha M)\bar{F}(q^{\mathtt{NV}} + \alpha M) - q^{\mathtt{NV}}\bar{F}(q^{\mathtt{NV}})\Big] - \alpha M v. \tag{33}$$

Differentiating with respect to α gives

$$\frac{\partial \Pi(\hat{w}(\alpha, q^{\mathtt{NV}}), \alpha, q^{\mathtt{NV}})}{\partial \alpha} = M(r\bar{F}(q^{\mathtt{NV}} + \alpha M) - v), \tag{34}$$

and evaluating at $\alpha = 0$ yields

$$\frac{\partial \Pi(\hat{w}(\alpha, q^{\mathtt{NV}}), \alpha, q^{\mathtt{NV}})}{\partial \alpha}\Big|_{\alpha=0} = M(r\bar{F}(q^{\mathtt{NV}}) - v) = M(r(1 - \frac{r-c}{r}) - v) = M(c - v). \tag{35}$$

Our assumption that $v < c$ implies that the partial derivative of the profit function with respect to the human joining fraction is strictly positive at $q = q^{\mathtt{NV}}$ and $\alpha = 0$. Because we have $\Pi(\hat{w}(0, q^{\mathtt{NV}}), 0, q^{\mathtt{NV}}) = \Pi^{\mathtt{NV}}$, the positivity of the partial derivative with respect to α implies that the platform can achieve a strictly higher profit by inducing a positive α than it would without human drivers (and it could easily achieve $\Pi^{\mathtt{NV}}$ without human drivers by setting $q = q^{\mathtt{NV}}$ and $w = 0$). □

Proof of Proposition 12. By Lemma 4, if $F(q) < (r - v)/r$ and $F(q + M) \geq (r - v)/r$, then $0 < \alpha(q) < 1$ and at the optimal $\alpha(q)$ we have

$$F(q + \alpha(q)M) = \frac{r-v}{r} \iff \bar{F}(q + \alpha(q)M) = \frac{v}{r} \iff q + \alpha(q)M = F^{-1}(\frac{r-v}{r}). \tag{36}$$

We can then apply the results of Lemma 4 for this and the other cases to get

$$\Pi(\hat{w}(\alpha(q), q), \alpha(q), q) = \begin{cases} r\Big[\int_0^{q+M} uf(u)\mathrm{d}u + (q+M)\bar{F}(q+M)\Big] \\ \quad -cq - vM & \text{if } F(q+M) \leq \frac{r-v}{r}, \\ r\Big[\int_0^{F^{-1}(\frac{r-v}{r})} uf(u)\mathrm{d}u + \frac{v}{r}F^{-1}(\frac{r-v}{r})\Big] \\ \quad -cq - v\Big(F^{-1}(\frac{r-v}{r}) - q\Big) & \text{if } F(q) < \frac{r-v}{r} < F(q+M), \\ r\Big[\int_0^q uf(u)\mathrm{d}u + q\bar{F}(q)\Big] - cq & \text{if } F(q) \geq \frac{r-v}{r}. \end{cases} \tag{37}$$

We continue by cases. **Case 1: $F(M) \leq (r - c)/r$.** Differentiating the expression in the first piece of equation (37), we get

$$\frac{\partial \Pi(\hat{w}(\alpha(q), q), \alpha(q), q)}{\partial q} = r\bar{F}(q + M) - c, \tag{38}$$

yielding the FOC (13). The second derivative is

$$\frac{\partial^2 \Pi(\hat{w}(\alpha(q), q), \alpha(q), q)}{\partial q^2} = -rf(q + M) < 0,$$

implying that the function is concave and that the FOC is sufficient for the global maximum of this piece of the function.

Moreover, the FOC has a solution in this case because the LHS is increasing in q and $\lim_{q\to\infty} F(q+M) = 1 > (r-c)/r$. This solution falls in the interval in which the first piece of equation (37) governs the profit because it satisfies

$$F(\widetilde{q} + M) = \frac{r-c}{r} < \frac{r-v}{r}. \tag{39}$$

Finally, by Lemma 5, the profit is strictly decreasing in q for larger q that fall into one of the other two intervals. Because the profit function is continuous (inspection reveals that the pieces coincide at the boundaries), the profit at the solution to the FOC is larger than any profit from the second two pieces, and the solution to the FOC (13) thus achieves the global maximum over $q \geq 0$. By equation (39), we have $F(\tilde{q}) < F(\tilde{q}+M) < (r-v)/r$. Therefore, we will have $\hat{\alpha}(\tilde{q}) > 1$ in equation (41), so by Lemma 4, it is optimal for the platform to induce $\alpha = 1$.

Case 2: $(r-c)/r < F(M) < (r-v)/r$. In this case, the FOC has no solution for $q \geq 0$, and the function is decreasing over the whole range in which the first piece of equation (37) governs the function because the derivative is always negative on this interval by equation (38). Since the function is also decreasing over the second two intervals by Lemma 5, we conclude that the global maximum of the function occurs at $q = 0$. Furthermore, because $F(q) = 0$ and $F(M) < (r-v)/r$, by Lemma 4, it is optimal to induce $\alpha = 1$.

Case 3: $F(M) \geq (r-v)/r$. In this case, the first piece of equation (37) never governs the function for any q. So, by Lemma 5, the profit is strictly decreasing in q for $q \geq 0$, and it is optimal to set $q = 0$. By Lemma 4, it is optimal in this case to induce a joining fraction $\hat{\alpha}(0) < 1$, the unique solution to equation (41). □

D. Auxiliary Results and Proofs

LEMMA 1 **(Continuity and Differentiability of the Deployment Quantity)**. *Suppose that $c < r$, then $q(w,\alpha)$ is continuous for all $w \leq r$ and $\alpha \in [0,1]$. Additionally, there exists $\hat{\alpha} > 0$ (independent from w) small enough such that $q(w,\alpha)$ has partial derivative with respect to w and α for all $(w,\alpha) \in (0,r) \times (0,\hat{\alpha})$, where the partial derivatives satisfy*

$$\frac{\partial q(w,\alpha)}{\partial w} = \frac{F(q(w,\alpha)+\alpha M) - F(q(w,\alpha))}{rf(q(w,\alpha)+\alpha M) + w(f(q(w,\alpha)) - f(q(w,\alpha)+\alpha M))}$$

and

$$\frac{\partial q(w,\alpha)}{\partial \alpha} = -M \frac{(r-w)f(q(w,\alpha)+\alpha M)}{rf(q(w,\alpha)+\alpha M) + w(f(q(w,\alpha)) - f(q(w,\alpha)+\alpha M))}.$$

Proof. The continuity of $q(w,\alpha)$ comes from Proposition 1. Indeed, because $w \leq r$ then Proposition 1 establishes that $\hat{q}(w,\alpha)$ is the unique solution to the first order condition in (3). Moreover, since the right-hand-side in (3) is continuous in (q,w,α) and the solution is unique, we must have that $\hat{q}(w,\alpha)$ is continuous which in turn implies that $q(w,\alpha)$ is continuous.

In order to show the differentiability, we first establish that $\hat{q}(w,\alpha) > 0$ which will imply that $\hat{q}(w,\alpha) = q(w,\alpha)$. That is, $q(w,\alpha)$ coincides with the solution to (3). Then, the differentiability follows from the Implicit Function Theorem. To see that $\hat{q}(w,\alpha) > 0$ note that from (3) we have

$$rF(q+\alpha M) = (r-c) + w(F(q+\alpha M) - F(q)) \geq (r-c),$$

where the inequality holds because F is non-decreasing, thus

$$q \geq F^{-1}\left(\frac{r-c}{r}\right) - \alpha M.$$

Since $c < r$ the above is positive for $\alpha > 0$ small enough (independent from w), as desired.

□

LEMMA 2 **(Possible to Recruit Some Humans)**. *Suppose that $c < r$ and $v < c$. Then, there exists $\hat{\alpha} > 0$ such that for all $\alpha \in [0,\hat{\alpha}]$ there exists a unique $w(\alpha) \in [0,r]$ continuous in α with*

$$w(\alpha)\gamma(\alpha, q(w(\alpha),\alpha)) = v.$$

Proof. We first note that we can simplify the expression for drivers' matching rate to

$$\gamma(\alpha, q(w,\alpha)) = \frac{1}{\alpha M}\int_{q(w,\alpha)}^{q(w,\alpha)+\alpha M} \bar{F}(x)dx. \tag{40}$$

From Equation (40), for a fixed wage $w < r$, from Lemma 1 we have

$$\lim_{\alpha\to 0^+} \gamma(\alpha, q(w,\alpha)) = \lim_{\alpha\to 0^+} \frac{1}{\alpha M}\int_{q(w,\alpha)}^{q(w,\alpha)+\alpha M} \bar{F}(x)dx = \frac{0}{0},$$

which is indeterminate, so we apply L'Hôpital's rule. The numerator is the integral in the above equation, and differentiating gives

$$\frac{d}{d\alpha}\Big(\int_{q(w,\alpha)}^{q(w,\alpha)+\alpha M}\bar{F}(x)dx\Big)=\bar{F}(q(w,\alpha)+\alpha M)(\frac{\partial q(w,\alpha)}{\partial\alpha}+M)-\bar{F}(q(w,\alpha))\frac{\partial q(w,\alpha)}{\partial\alpha},$$

noting that the derivatives are well defined by Lemma 1 for $w<r$ and $\alpha>0$ small enough. Taking the limit gives

$$\lim_{\alpha\to0^+}\frac{d}{d\alpha}\Big(\int_{q(w,\alpha)}^{q(w,\alpha)+\alpha M}\bar{F}(x)dx\Big)=M\bar{F}(q(w,0))=M\bar{F}(q^{\texttt{NV}})=M\Big(\frac{c}{r}\Big),$$

where the last two equalities follow from Equation (2) and the surrounding discussion. The denominator is αM, with derivative M, so we conclude that

$$\lim_{\alpha\to0^+}\gamma(\alpha,q(w,\alpha))=\bar{F}(q^{\texttt{NV}})=\frac{c}{r}.$$

Accordingly, we define $\gamma(0,q(w,0))=\bar{F}(q^{\texttt{NV}})$ for any $w<r$. For $\alpha=0$, the driver equilibrium condition $w\gamma(0,q(w,0))=v$ then becomes

$$w\bar{F}(q^{\texttt{NV}})=v\iff w=v\Big(\frac{r}{c}\Big).$$

Letting $w(\alpha)$ be a wage that satisfies the equilibrium condition $w\gamma(\alpha,q(w,\alpha))=v$, we have $w(0)=v(r/c)$. Let $\varepsilon>0$ be such that $w(0)+\varepsilon<r$; note that this is well defined because $v<c$.

Now, let

$$f(w,\alpha)=w\gamma(\alpha,q(w,\alpha))-v.$$

Because $\gamma(0,q(w(0)+\epsilon,0))=c/r$ and $w(0)\gamma(0,q(w(0),0)=v$, we have $f(w(0)+\epsilon,0)=\epsilon(c/r)>0$. Because f is continuous, there then exists $\hat{\alpha}>0$ with $f(w(0)+\epsilon,\hat{\alpha})>0$. Also, we have $f(0,\hat{\alpha})=-v$. And the latter is true for any $\alpha\le\hat{\alpha}$. Moreover, for any $w\in[0,w(0)+\varepsilon]$ we have that

$$\begin{aligned}\frac{\partial f(w,\alpha)}{\partial w}&=\gamma(\alpha,q(w,\alpha))+w\left(\bar{F}(q(w,\alpha)+\alpha M)-\bar{F}(q(w,\alpha))\right)\frac{\partial q(w,\alpha)}{\partial w}\\&=\frac{1}{\alpha M}\int_{q(w,\alpha)}^{q(w,\alpha)+\alpha M}\bar{F}(x)dx-\frac{1}{\alpha M}w\frac{\left(\bar{F}(q(w,\alpha)+\alpha M)-\bar{F}(q(w,\alpha))\right)^2}{(r-w)f(q(w,\alpha)+\alpha M)+wf(q(w,\alpha))}\\&\ge\frac{1}{\alpha M}\int_{q(w(0)+\varepsilon,\alpha)}^{q(w(0)+\varepsilon,\alpha)+\alpha M}\bar{F}(x)dx-\frac{1}{\alpha M}w\frac{\left(\bar{F}(q(w,\alpha)+\alpha M)-\bar{F}(q(w,\alpha))\right)^2}{(r-w)f(q(w,\alpha)+\alpha M)+wf(q(w,\alpha))}\\&=\gamma(\alpha,q(w(0)+\varepsilon,\alpha))-w\frac{f(\xi)^2\alpha M}{(r-w)f(q(w,\alpha)+\alpha M)+wf(q(w,\alpha))}\\&\ge\gamma(\alpha,q(w(0)+\varepsilon,\alpha))-(w(0)+\varepsilon)\frac{\bar{f}}{r\underline{f}}\alpha M,\end{aligned}$$

where $\xi\in[q(w,\alpha),q(w,\alpha)+\alpha M]$ exists by virtue of the mean value theorem. In the first inequality we used that $q(w,\alpha)$ is non-decreasing in w. In the second inequality we used $\bar{f}$ and $\underline{f}$ as upper and lower bounds for $f(\cdot)$. We define the latter as follows. Note that since we are considering $\alpha\le\hat{\alpha}$ and $w\le w(0)+\varepsilon$, and because $q(w,\alpha)$ is continuous, then we can find $\bar{q}$ such that $q(w,\alpha)\le\bar{q}$ for all (w,α) under consideration. Then, we define $\bar{f}=\sup_{x\in[0,\bar{q}+\hat{\alpha}M]}f(x)$ and $\underline{f}=\inf_{x\in[0,\bar{q}+\hat{\alpha}M]}f(x)$, note that both bounds are finite and positive. Hence, we have

$$\inf_{w\in[0,w(0)+\varepsilon]}\frac{\partial f(w,\alpha)}{\partial w}\ge\gamma(\alpha,q(w(0)+\varepsilon,\alpha))-(w(0)+\varepsilon)\frac{\bar{f}}{r\underline{f}}\alpha M,$$

and the term on the right-hand-side above is strictly positive for all $\alpha>0$ small enough because $\gamma(\alpha,q(w(0)+\varepsilon,\alpha))$ converges to $c/r>0$ as $\alpha\downarrow0$. With some abuse of notation we suppose that this is true for all $\alpha\le\hat{\alpha}$. This implies that $f(w,\alpha)$ is an increasing function in $w\in[0,w(0)+\varepsilon]$ for all $\alpha\le\hat{\alpha}$. Therefore, there is a unique and continuous $w(\alpha)\in[0,w(0)+\varepsilon]$ such that $f(w(\alpha),\alpha)=0$. Note that in the above we can take ε such that $w(0)+\varepsilon$ is as close as desired to r. Hence, the above proves that we can find a unique $w(\alpha)\le r$ for all α small enough. □

LEMMA 3 **(AV Deployment Quantity Decreasing in α Around Zero)**. *If $v<c$, then the function $q(w(\alpha),\alpha)$ is decreasing in α at $\alpha=0$.*

Proof. By Lemma 2, there exists $\hat{\alpha} > 0$ such that the driver equilibrium condition can be satisfied at a wage $w(\alpha)$ for $0 \le \alpha \le \hat{\alpha}$. For simplicity of notation, in this proof we use $q(\alpha)$ to denote $q(w(\alpha), \alpha)$, and $\dot{q}(\alpha)$ to denote its derivative in α. Substituting $w(\alpha)$ into the FOC in Proposition 1 and differentiating, we have

$$\dot{q}(\alpha)\Big[(r-w)f(q+\alpha M)+wf(q)\Big]+(r-w)f(q+\alpha M)M-\dot{w}(\alpha)\Big[F(q+\alpha M)-F(q)\Big]=0.$$

Substituting $\alpha = 0$ and canceling terms gives

$$\dot{q}(0)r = (w(0)-r)M.$$

From the proof of Lemma 2, we have $w(0) = r(v/c)$, which then implies

$$\dot{q}(0) = M(\frac{v}{c}-1) < 0,$$

where the inequality holds by our assumption that $v < c$. The result follows. □

Lemma 4 **(Optimal Induced Human Joining Fraction in Modified Game)**. *Suppose that the platform chooses a deployment quantity q, and let $\hat{\alpha}(q)$ be the unique solution in α to*

$$F(q+\alpha M) = \frac{r-v}{r}. \tag{41}$$

In the modified game, if $F(q) < (r-v)/r$, then it is optimal for the platform to induce an equilibrium human joining fraction $\alpha(q)$, where

$$\alpha(q) = \min\{\hat{\alpha}(q), 1\}.$$

Otherwise, the platform should not employ human drivers. If $q = q^{\texttt{NV}}$, then the condition $F(q) < (r-v)/r$ reduces to $v < c$.

Proof. We can substitute a generic q for $q^{\texttt{NV}}$ in equation (34) to get the derivative of the profit function for general q. Differentiating the function a second time gives

$$\frac{\partial^2 \Pi(\hat{w}(\alpha,q),\alpha,q)}{\partial \alpha^2} = -rMf(q+\alpha M) < 0,$$

implying that the function is strictly concave and the FOC is sufficient for a global maximum.

If $F(q) \ge (r-v)/r$, then the derivative of the profit function is nonpositive at $\alpha = 0$ by the generic form of equation (34). The concavity of the function then implies that the profit is strictly decreasing on $[0,1]$, so it is optimal to induce $\alpha = 0$.

On the other hand, if $F(q) < (r-v)/r$, then inspection of the same equation reveals that the derivative of the profit function is positive at $\alpha = 0$. In this case, equation (34) gives the FOC (41), which has a unique solution because the LHS is increasing in α while the RHS is constant. If $0 < \hat{\alpha}(q) < 1$, then it is optimal to induce $\alpha = \hat{\alpha}(q)$. If instead $\hat{\alpha}(q) \ge 1$, then it is optimal to induce $\alpha = 1$.

Finally, equation (35) shows that the condition for the derivative to be positive at $\alpha = 0$ reduces to $v < c$ if $q = q^{\texttt{NV}}$. □

Lemma 5 **(Shape of Profit in q)**. *If $v < c$, then the profit $\Pi(\hat{w}(\alpha(q),q),\alpha(q),q)$ is strictly decreasing in q for q such that*

$$F(q+M) > \frac{r-v}{r}.$$

Proof. **Case 1: $F(q) < (r-v)/r < F(q+M)$.** Differentiating the second piece of equation (37) with respect to q gives $v - c < 0$, so the function is strictly decreasing in q.

Case 2: $F(q) \ge (r-v)/r$. In this case, the third piece of equation (37) applies, which is equivalent to the newsvendor profit function for overage cost c and underage cost $r - c$. The critical fractile for this problem is $(r-c)/r < (r-v)/r$. Thus, in this case we are always above the optimal quantity for the equivalent newsvendor problem, implying that the profit is strictly decreasing in q. □

Lemma 6 **(Bound on Optimal Quantity in Modified Game)**. *The optimal quantity for the modified game is weakly less than the ample-AV solution for the original game at the same human joining fraction and wage. Specifically, either* $\hat{q}(\widetilde{w}, \widetilde{\alpha}) \geq 0$ *is unique and* $\widetilde{q} \leq \hat{q}(\widetilde{w}, \widetilde{\alpha})$*, or we have* $\widetilde{q} = 0$.

Proof. **Case 1: $F(M) \leq (r-c)/r$.** In this case, by Proposition 12, we have that $\widetilde{\alpha} = 1$ and the optimal quantity $\widetilde{q}$ for the modified game satisfies

$$F(\widetilde{q} + M) = \frac{r-c}{r}. \tag{42}$$

Because $\widetilde{\alpha} = 1$, we must have $\widetilde{w} \leq r$, as otherwise the platform could improve its revenue by setting $w = \hat{w}(0)$ and recruiting no human drivers. Thus, by Proposition 1, the ample-AV solution to the original game is the unique solution to (3), namely $\hat{q}(\widetilde{w}, \widetilde{\alpha}) \geq 0$, where the nonnegativity holds because the derivative of the platform's last-stage profit is nonnegative at $q = 0$ for $F(M) \leq (r-c)/r$. By equation (3), and because $F(\hat{q}(\widetilde{w}, 1)) \leq F(\hat{q}(\widetilde{w}, 1) + M)$, we have

$$r - c = (r - \widetilde{w}) F(\hat{q}(\widetilde{w}, 1) + M) + \widetilde{w} F(\hat{q}(\widetilde{w}, 1)) \leq r F(\hat{q}(\widetilde{w}, 1) + M), \tag{43}$$

Equations (42) and (43) then imply that

$$F(\widetilde{q} + M) = \frac{r-c}{r} \leq F(\hat{q}(\widetilde{w}, 1) + M). \tag{44}$$

Because the CDF $F(\cdot)$ is an increasing function, equation (44) allows us to conclude, recalling that in this case $\widetilde{\alpha} = 1$, that

$$\widetilde{q} \leq \hat{q}(\widetilde{w}, \widetilde{\alpha}).$$

Case 2: $F(M) > (r-c)/r$. In this case, by Proposition 12, we have $\widetilde{q} = 0$. □

E. Driver Equilibrium Condition

Because of the endogeneity among wage, joining fraction, and AV deployment quantity, it is useful to verify that it cannot be optimal to strictly satisfy the driver equilibrium condition (EQ). Lemma 7 establishes this result for the original game, and Lemma 8 does the same for the modified game. These results also allow us to make a change of variable, using the driver equilibrium condition to substitute the wage out of the equilibrium profit function.

Lemma 7 **(Original Game: Driver Equilibrium Holds with Equality)**. *In the original game, consider* $0 < \alpha \leq 1$*, and suppose that it is possible to satisfy* (EQ) *for such* α*, i.e., that there exists* w *such that* $w\gamma(\alpha, q(w, \alpha)) \geq v$*. For the given* α*, it is optimal for the platform to exactly satisfy the equilibrium condition, i.e., to choose a wage such that* (EQ) *holds with equality.*

Proof. First, assume that $w \leq r$. Let $g_\alpha(w) := w\gamma(\alpha, q(w, \alpha))$. The function g_α is continuous in w because q is continuous in w and γ is continuous in its second argument q (γ is also continuous in α, but this fact is not needed here because we consider fixed α). For any α, we have $g_\alpha(0) = 0 < v$. Moreover, by assumption, there exists a wage w' such that $g_\alpha(w') \geq v$. By the intermediate value theorem, there also exists a wage w'' with $g_\alpha(w'') = v$. For any $0 < \alpha < 1$, the condition must hold at equality as otherwise more drivers would join, contradicting the equilibrium, so the result is now established for $0 < \alpha < 1$. For $\alpha = 1$, if it is impossible to strictly satisfy the equilibrium condition, then the result is similarly established.

Suppose otherwise, i.e., for $\alpha = 1$, assume that there exists a wage w' such that $g_1(w') > v$. The strict inequality implies that we can choose $\epsilon > 0$ such that $g_1(w' - \epsilon) > v$. It is clear from Eq. (15) that the platform's expected profit is strictly decreasing in w for fixed $\alpha > 0$ and q, which combined with the optimality of $q(\cdot, \cdot)$ gives us that

$$\Pi(w' - \epsilon, 1, q(w' - \epsilon, 1)) \geq \Pi(w' - \epsilon, 1, q(1, w')) > \Pi(w', 1, q(1, w')).$$

Therefore, any wage w' with $g_1(w') > v$ is strictly sub-optimal. As shown above, it is possible to satisfy the condition at equality, and it therefore must be optimal to do so. Finally, setting $w > r$ can never be globally optimal for the platform, so without loss of optimality we can assume that the platform satisfies the condition at equality in this case.
□

Lemma 8 **(Modified Game: Driver Equilibrium Holds with Equality)**. *In the modified game, consider* $0 < \alpha \leq 1$ *and* q*, and suppose that it is possible to satisfy the equilibrium condition* (11) *for such* α*, i.e., that there exists* w *such that* $w\gamma(\alpha, q) \geq v$*. For the given* α *and* q*, it is optimal for the platform to exactly satisfy the equilibrium condition, i.e., to choose a wage such that* (EQ) *holds with equality.*

Proof. Follows by an argument analogous to (but simpler than) the proof of Lemma 7. □

Lemma 9 **(Modified Game: Wage Exists for Any Human Joining Fraction)**. *In the modified game, for any human joining fraction* $0 \leq \alpha \leq 1$ *and deployment quantity* q *with* $F(q) < 1$*, a wage* $\hat{w}(\alpha, q)$ *exists that satisfies Eq.* (11)*, i.e., with* $\hat{w}(\alpha, q)\gamma(\alpha, q) = v$*.*

Proof. If $F(q) < 1$, then we have $\gamma(\alpha, q) > 0$ for any $0 \leq \alpha \leq 1$ by Eq. (5). Moreover, differentiating Eq. (5) reveals that $\gamma(\alpha, q)$ is decreasing in α for fixed q (this contrasts with the original game in which the deployment quantity responds to the wage and human participation level). Therefore, we have that $\hat{w}(\alpha, q) = v/\gamma(\alpha, q)$, which is unique and well-defined because $\gamma(\alpha, q) > 0$. □