Buy Now or Keep Renting? A Modular Estimation Framework for Renter Decisions in the Rent-to-Own Business

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Problem definition: Rent-to-own (RTO) firms rent products in exchange for a periodic fee and offer the already-rented products for purchase at buyout prices to their renters. Prediction of buyout demand requires a decision model that accurately captures the renters' decision-making process and their ownership and rental utilities for a product, which together determine their willingness to pay buyout prices in different periods. **Methodology/results:** We develop a new, modular framework that cleanly separates utility estimation from solving the renter's decision problem. We build several renter decision models, reflecting different degrees of sophistication in decision making (strategic, myopic, or one-step-look-ahead) and alertness to purchase offers. Analytically, we show that renters with higher utilities target earlier purchases to avoid abandonment of the rental without ownership because of their higher opportunity cost. We also show that renters who are more attentive (i.e., more likely to consider each buyout offer) target later purchases because of their higher continuation surplus. For each model, we estimate a nonparametric utility distribution along with the probabilities of abandoning the rental, neglecting the buyout offer and considering the buyout offer. Finally, using transaction data from an RTO firm, we compare the estimation performance of the different renter decision models and perform counterfactual analyses. **Managerial implications:** Our framework and results illuminate the renter decision-making process, enabling improved demand prediction. The one-step-look-ahead model with limited alertness fits best to our data, suggesting respectively that the firm's renters have a short decision horizon and that they may neglect some buyout offers. Additionally, counterfactual analyses reveal the profit impact of alternative pricing policies and communication strategies. In particular, we find that relative to its current strategy, the firm should increase its mattress buyout prices and emphasize (downplay) the buyout offers to washer/dryer (mattress) renters.

Key words: Finite mixture model, strategic/myopic and alert/inert renter, dynamic structural estimation, likelihood

1 Introduction

In the multibillion-dollar *rent-to-own* (RTO) business, a firm first rents products to customers (renters) and then offers them the option of purchasing the already-rented products. Appliances (refrigerators, washing machines), electronics (TVs, laptops) and furniture (mattresses, sofas) are common RTO products. In addition to periodically collecting rental fees from renters, an RTO firm offers them the option at various times during the rental to buy the product outright for different quoted prices. The renters' willingness to pay is related to their decision-making process, as well as to the utility that they derive from owning (ownership utility) and renting (rental utility) the product. In more traditional retail and e-commerce settings, much research has been conducted to understand the customer decision-making process and predict demand (e.g., Li et al. 2014, Moon et al. 2018). In RTO as in these other settings, an accurate model of customer decision making is critical for effective pricing, inventory management, and other operational activities. Nevertheless, little is understood about RTO renters' decision making, which is complex due to the changing buyout prices and the repeated rent-vs.-buy decisions made by each renter throughout his rental. The present work provides a new estimation framework as well as analytical and empirical results that peel back the curtain on RTO renter decisions.

The annual revenue of the U.S. RTO business is \$8.2 billion, and 30 thousand RTO stores are serving approximately 4 million renters (May 2017). RTO firms include Masters (masterslease.com) and Premier (premierrents.com), as well as the two bigger players, Aaron's and Rent-A-Center. Each renter pays a rental fee once per period (e.g., a week or month). The renter can own the rented item either through a *payoff*, i.e., after renting for a certain duration, referred to as the *term*, or upon paying a periodically quoted price, called the *buyout* price. In practice, the path of buyout prices is determined by the firm at rental initiation. While these prices may be optimized dynamically during a rental, this is not a standard practice, as confirmed by our industry collaborator. Instead, the path of buyout prices is usually fixed in advance, and typically, the price drops by a fraction of the rental fee from one period to the next. This convention reflects the attractiveness of "rent to own" in that a piece of the rental fee is "allocated" toward future ownership of the product. Renters can terminate the rental at any time without penalty. These features distinguish RTO from a fixed-duration lease with early termination penalties and from an installment plan with a commitment to buy.

RTO renters fall mainly into two categories. One category is the asset-limited, income-constrained (ALIC) renters, who earn low to moderate incomes –more than 95% earn below $50k annually (APRO 2015, Mittelstaedt et al. 2007)– and cannot afford retail prices. ALIC renters may lack the ability to continue their rentals or to consider a purchase. To cater to them, some RTO firms approve rentals with renter-reported personal income, without a third-party credit report. Hence, individuals that would not be approved for a purchase on credit gain the opportunity to rent first to own later. The second category of renters has temporary needs or a desire for flexibility. They may be able to afford the retail prices for outright ownership but prefer not to purchase upfront. For example, a young professional who temporarily resides in a city may rent furniture or parents of a musically inclined child can rent a quality instrument. The flexibility of an RTO arrangement is especially valuable in the latter example as the need for the instrument may change. If the child sticks with the instrument, then the parents can pay it to term or purchase it early. Otherwise, they can terminate the rental with no penalty.

Cost-based pricing is prevalent in many industries (e.g., Chen et al. 2022) as well as at RTO firms. However, cost based-pricing, which ignores utilities, may yield prices below what renters are willing to pay. An alternative approach is value-based pricing (Villas-Boas 2009), which aims to capture more of the renter surplus (ownership utility in excess of the buyout price) and requires knowledge of his decision-making process and his ownership and rental utilities. Indeed, through rejections (or acceptances) of the offered buyout prices during the rental agreement, a renter provides repeated signals to the firm about his decision making and utilities. But, unlike in data-rich contexts like e-commerce, RTO firms often possess minimal data about an individual renter other than his history of rental payments and rejections during the rental. Rental agreements are often initiated in person based on self-reported credit information and transacted with cash payments. These practices hinder the firm from discovering other explanatory variables about individual renters. The data-poor context complicates demand estimation, which must be conducted with only transaction history (unlike data-rich contexts, which also have user profiles or browsing history). In addition, the use of explanatory variables, especially demographic factors, to price discriminate falls into a legal and ethical gray area (Ramasastry 2005) and regardless would likely engender strong customer resentment. For example, Amazon sparked an uproar when, in what it claims was a randomized price experiment, a customer found that he was offered a different price for the same item after deleting his cookies. Whether or not price discrimination indeed occurred, the controversy made news (Ramasastry 2005, Useem 2017) and had lasting adverse effects that require continued mitigation (Cooprider 2022). Consequently, it is reasonable to assume that prices must not be directly tied to individual-specific characteristics. Thus, even if an RTO firm possessed explanatory variables, it arguably could not use them to set prices, making such variables fruitless to consider in estimation. So, absent a detailed picture of each individual, a reasonable goal for an RTO firm is to characterize decision making and utility in the overall *population* of renters. We provide tools, results and insights for exactly this endeavor.

The RTO business differs from traditional retail settings in multiple ways that have modeling implications. First, RTO firms serve many customers with limited financial means. Thus, a renter may terminate an agreement or reject a buyout offer due to an unexpected expense, unemployment, or a lack of mental bandwidth to evaluate the offer. Accordingly, our models incorporate three renter statuses, realized probabilistically in each period: *abandoning the rental* (an absorbing status), *neglecting the purchase offer* and *considering the purchase offer*. Another difference is the number of signals received from a single renter. In retail settings, a customer typically faces a single price for a product, and the signal is whether he buys the product at that price. By contrast, an RTO renter sends many signals about his decision making and utility in the form of his responses to different buyout prices during the rental. Also, the rejection signals in particular are directly observed and more credible in the RTO business, whereas in retail they may be absent and inferred. Finally, retail prices often lack variation to cover a range of utilities, but the offered buyout prices by RTO firms for a single product span a much wider range (tens of dollars to a thousand in our data). This confluence of factors, absent in a conventional retail setting, is endemic to the RTO business, which requires a new estimation framework.

In the present work, we develop a novel, modular framework to estimate ownership and rental utilities under various renter decision models. Under each model, we solve the renter's decision problem and obtain structural insights that reveal key drivers of renter decisions; besides, we concisely represent the solution as a *buy set* that contains the periods in which the renter would choose to purchase, if able. Crucially, we can express the likelihood in terms of the buy sets themselves without reference to the renters' underlying decision-making process: hence the *modularity* of our framework. Using transaction data for washer, dryer and mattress rentals from our collaborating RTO firm –referred to as *our RTO firm* firm– we compare estimation performance under different decision models to illuminate how RTO renters make decisions, estimating the joint ownership and rental utility distribution under each model.

Our models span two dimensions: sophistication and alertness of renters. Regarding sophistication, it is not obvious a priori to what extent renters are forward-looking or how well they internalize randomness (say, of abandoning their rentals without obtaining ownership). So, we develop decision models to capture a range of sophistication, from fully strategic renters who correctly capture rent-buy trade off and the future evolution of their status in a dynamic program, to completely myopic renters who ignore the future entirely and purchase when their current purchase surplus exceeds that of not purchasing. Regarding alertness, renters may not have the mental bandwidth or financial means to consider a purchase in every period. If they are busy or strapped for cash, they may maintain the status quo by continuing to rent, postponing consideration of a purchase. An *inert* renter (i.e., one with inertia) with some probability continues the rental in a given period without considering a purchase, ignoring the buyout offer. An *alert* renter always considers a purchase; that is, in each period he evaluates the buyout price and decides whether or not to purchase according to a decision rule based on his sophistication.

Spanning the two dimensions above, we consider six distinct renter decision models. We first consider multiple models involving *strategic* renters, i.e., forward-looking renters who optimize their purchase period. Reflecting a common practice in the RTO business of reducing the price by a fraction of the rental payment in each period, we assume a strategic renter can correctly anticipate future buyout prices, and thus he factors the price path into his decision-making. Our strategic renter models include a strategic and alert renter (SAR) who makes a purchase decision by solving a dynamic program in which he factors in all future buyout prices; a strategic and optimistic renter (SOR) who makes purchase decisions similar to a SAR but does not internalize his inertia toward purchase offers; a strategic and inert renter (SIR) who is realistic about his inertia and finally a strategic and inert one-step-look-ahead renter (SIOR) who only factors in the next period's buyout price in his current purchase decision. In addition to strategic renters, we consider *myopic* renters. We consider both a myopic and alert renter (MAR) and a myopic and inert renter (MIR). For each of our six models, we analytically characterize the solution to the renter's decision problem, i.e., the resulting buy set. These solutions inform RTO firms on how their renters decide to purchase under different decision models.

We represent the renter population as a mixture of segments within each of which renter utilities are homogeneous. We use our structural results to obtain the renters' buy sets under each segment for a given decision model. Buy sets serve as inputs to our estimation procedure, which follows an iterative scheme similar to, e.g., Arcidiacono and Miller (2011) to compute the probabilities of each utility segment. Since we consider two renter attributes, rental and ownership utility, we can cover the utility space with a manageable number of segments, each requiring limited computations. This facilitates efficient nonparametric estimation. In addition, we estimate the probabilities associated with renter statuses. We evaluate each of our renter decision models on transaction data for washers, dryers and mattresses from our RTO firm. Our results reveal rich new insights into how real RTO renters make rent vs. buy decisions during their rentals, as well as their value for the rented products, all of which helps demand prediction. We also perform counterfactual analyses and reveal how directional changes in buyout price paths or renters' probability of considering a purchase affect profits.

Literature Survey: Our work is closely related to the literature on Discrete Choice Dynamic Programming (DCDP) structural estimation models. These models tend to be computationally complex as the solution procedure to each underlying dynamic program is nested within a likelihood maximization routine. Specifically, parameters of a DCDP model can be estimated under some assumptions via the nested fixed point algorithm of Rust (1987, 1994). To alleviate the complexity, various estimation methodologies are used in DCDP contexts and seminal papers include Hotz and Miller (1993) that develops an estimation method based on Conditional Choice Probabilities (CCP) and Keane and Wolpin (1994) that approximates the DP solution; see Aguirregabiria and Mira (2010) for a survey. Recent research papers operationalize approximation schemes (Eisenhauer 2019) and analyze identification of model primitives (Abbring and Daljord 2020). Permanent unobserved heterogeneity of renter utilities in our models fails one of Rust's critical assumptions. Arcidiacono and Jones (2003) and Arcidiacono and Miller (2011) extend the CCP-based methodologies to accommodate finite mixture models that incorporate permanent unobserved heterogeneity among decision makers; likewise, we adopt a finite mixture DP modeling approach to handle the heterogeneity among renters (Aguirregabiria and Mira 2010, p.54). DCDP models have been applied to diverse fields of management; examples include Huang et al. (2015), Bray et al. (2019), Wagner and Martínez-de Albéniz (2020), Souyris et al. (2022).

Our study is among the recent structural estimation papers in operations management. Yu et al. (2017), Akşin et al. (2017) and Hathaway et al. (2022) develop and estimate parameters of structural models for customer decisions in a call center. Li et al. (2014) provide a structural framework to estimate the fraction of strategic customers in the airline industry. Moon et al. (2018) estimate price monitoring cost of strategic customers and discuss its implications on pricing strategy. Emadi and Staats (2020) consider a structural optimal stopping model to study attrition decisions of a firm's employees. He et al. (2021) form a structural model to predict demand on bike-sharing platforms. Guajardo (2019) empirically studies renters' product usage of solar lamp rentals in a developing country; the context mirrors RTO with limited enforceability of rental terms and absence of third-party credit reports.

We briefly discuss the prevalent methods of utility estimation in the literature. A well-known method of utility estimation via survey data is conjoint analysis, which determines how individuals relatively value different features of a product (for a survey, see Agarwal et al. 2015). Via sales transaction data, mixture of multinomial logit (MMNL) models estimate customers' utility of each product in a multi-product setting. These models treat customers as heterogeneous and belonging to different segments (similar to our renter segments) and assume homogeneous utilities within each segment. Different algorithms are used to find the maximum likelihood estimates of the model parameters (Newman et al. 2014, Gallego et al. 2015, Abdallah and Vulcano 2021).

Contributions: To our knowledge, our study is the first to develop structural utility estimation models to understand the decisions of RTO renters. We develop a novel maximum likelihood framework to estimate renter ownership and rental utilities. Our work both builds on and is distinct from standard approaches for estimating dynamic decision processes. Our models capture the key features of the RTO business while also facilitating tractable estimation. In contrast to a standard dynamic structural estimation approach relying on repeated nested fixed point computations, our procedure is computationally efficient as it entails solving a renter's decision problem only once under each segment, with the solutions feeding the estimation directly. Moreover, maximization of the likelihood is agnostic to how the solutions are generated, so our procedure can be applied to any conceivable renter decision-making model under our framework. Our results reveal structural insights about renter decision making that are valuable both for RTO firms and for the future academic study of this business.

As described, a renter's decision-making process produces a buy set. This set can include multiple periods, the earliest of which may not witness a purchase in models with inert renters. Buy sets create modularity by cleanly separating the utility estimation on one hand, from the analysis of renter decisions on the other. The modularity facilitates study of decision models with diverse features. In particular, our consideration of renter sophistication and alertness is novel. The probability of considering a purchase, in addition to improving estimation performance, also has a concrete interpretation.

For each model, we analytically solve for renter decisions. In the models with strategic and inert renters, we show that the higher a renter's ownership utility, the earlier he intends a purchase. The reason is that, with some probability of abandoning the agreement in each period, postponing a purchase entails a risk of losing the ownership utility altogether. The higher this utility, the costlier this loss is; thus, a renter with a higher ownership utility prefers to purchase earlier to mitigate this risk, even if it means paying a higher price. We also show that a renter with a higher purchase consideration probability intends to purchase later. This higher probability makes it more likely that he can purchase in the future, which makes postponing the purchase relatively more attractive. Despite this intention, a renter with a higher purchase consideration probability may still purchase earlier.

Estimation with the real-life RTO data reveals that the inert renter models fit better than those with alert renters. The inert models have the advantage of explaining renter actions by either economic (rental and ownership utilities) or status-related factors. Surprisingly, the models with fully strategic renters do not fit well to our data. Instead, the model with inert one-step-look-ahead renters fits the best for all product types, followed closely by the model with myopic and inert renters. This could suggest that RTO renters are capable of somewhat sophisticated reasoning but are not sufficiently confident in their future circumstances to plan far into the future. Instead, they resort to a near-myopic decision-making process with a limited horizon. Strikingly, this finding suggests that a renter's probability of accepting a given buyout price at a particular period in his agreement may be largely insensitive to the future prices that he will be offered; this decoupling can simplify demand prediction, which should also help with the determination of buyout prices. In short, the inert one-step-look-ahead model best describes our firm's renters; however, all of our models are of independent interest for different settings.

Our nonparametric approach permits arbitrary dependence between ownership and rental utilities. Interestingly, in our data, we do not find significant correlation between estimated ownership and rental utilities. For this finding, we provide an explanation related to the firm's uncertainty about the renter's duration of product use. We also find that mattress renters consider a purchase with a higher probability compared to those of washers and of dryers. This could reflect the absence of an outside alternative or the frequent (everyday) use of mattresses, which perhaps directs renters' attention to purchase offers.

Our framework and renter decision models, as well as our estimation results and insights, are valuable to both academicians and RTO practitioners. For academicians, our work opens a new line of research on RTO renter decision models and utility and, just as important, provides a flexible framework for estimation that is adaptable to related settings. For RTO practitioners, we provide tools for estimation and insights about RTO renters, their decision-making processes, and their utilities, all of which help inform the setting of buyout prices. Finally, for firms that are deciding between renting and selling their products (Mantena et al. 2012 review such decisions) or those that engage in both concurrently by maintaining separate rental and sales inventories (Altug and Ceryan 2022), we pave the road for them to make a comparative evaluation of the RTO business model that combines the benefits of renting and selling, earning small fees from rentals and large sums from buyouts.

The paper is organized as follows. In §2, we introduce our modular framework for estimation. In §3 and §4, respectively, we present the strategic and myopic decision-making models. We evaluate these models on real-world data from our RTO firm and present counterfactual analyses in §5, and in §6, we provide suggestions to practitioners along with concluding remarks. Proofs are in Appendix A.

2 A Modular Estimation Framework for RTO Renter Decisions

We consider an RTO renter who can be in one of three statuses: neglecting the purchase option, abandoning the rental or considering the purchase option. *Neglecting* the purchase option maintains the status quo and requires the renter neither to evaluate alternatives nor to take an action in addition to paying the rent. Hence, it is likely to be observed when a busy renter with limited mental bandwidth continues to rent by passively resorting to this simple default option. *Abandoning* the rental leaves the renter without the rented item and signals a drastic change in his need for this item or ability to rent it. Abandonment can be caused by the renter's relocation or change in employment, which strongly alter needs and abilities. It reflects exogenous factors (outside options and constraints) unknown to and unpredictable by the RTO firm. The status of *considering* the purchase option describes a renter that plans to use the rented item for a long duration. The accumulation of rent payments over such a duration can be large enough, compared to the buyout price, to justify the renter's purchase to own the item. This justification requires careful consideration and financial evaluation of rental and purchase alternatives, and sophisticated renters must also assess the probabilities of different future status paths. Hence, only renters with sufficient mental bandwidth and financial means consider a purchase.

Renter status can be viewed from a renter's perspective. A renter easily knows whether he abandons the rental agreement, which occurs when he has no need for the rented item, no ability to accommodate it or no financial ability to pay the rent. A renter that does not abandon needs the rented item but may not have the mental bandwidth or the financial means to consider a purchase, in which case he resorts to the status quo of neglecting the purchase option. To overcome the inertia of this status quo, he needs both the bandwidth and the financial means, with which he can consider a purchase. Although the renter statuses are explained in terms of a renter's mental bandwidth and financial means, they are intended to reflect any events (e.g., graduation, marriage or relocation) that might shape the renter's overall disposition regarding considering a purchase, preferring to only rent or abandoning the rental. The renter statuses are dynamic, mutually exclusive and can be captured via probabilities by the RTO firm. The probabilities of neglecting the purchase option, considering a purchase and abandoning the rental at any time are respectively denoted by ρ_r, ρ_p, and $1-\rho_r-\rho_p$. Since renter statuses are relevant at the times of buyout offers which are separated by a week or more, we assume that the realizations of renter status are independent over time.

Our renter decision framework involves periods (e.g., weeks or months) and two events in each period. First, the renter's status is realized, and then contingent on it, he chooses his action. If the status is abandoning, he returns the product regardless of his utility. If it is neglecting, he continues the rental; if it is considering, he decides between continuing the rental or purchasing the product.

Our analysis includes a rental utility, i.e., the utility a renter gets from renting the item over a single period. Pertaining to a single period of use, the rental utility of an item is less variable than the ownership utility of that item, as the latter utility encompasses additional uncertainty for the firm related to how long the renter plans to use the item. Moreover, ownership comes with the convenience of avoiding (physical and virtual) store visits for rental fee payments and also induces a feeling of satisfaction and peace of mind (stability and security). These intangible elements of convenience, satisfaction and peace of mind amplify the firm's uncertainty about its renters' ownership utility. On the other hand, single-period rental utility can be suitably represented deterministically or as a random variable with a narrow range. For instance, rental utility for washers can be anchored around laundromat fees.

RTO firms are more interested in buyout price optimization than rental fee optimization, as they have more room to maneuver when pricing buyouts. This is because the rental fee of an item is publicly available, and benchmarked to the RTO market; in contrast, the sequence of buyout prices for an item is not posted publicly. Although we estimate both rental and ownership utilities, we focus on the ownership utility and abbreviate it to "utility," where no confusion arises.

Renters' ownership and rental utilities are realized upon rental initiation; they are independent and identically distributed across renters and remain constant throughout the rental. The reason for this modeling choice is that the lifetimes of the durable products (washer/dryer/mattress) in our data are long, and their main functionality is unlikely to change during the rental; thus, the renter's utility is relatively stable. We aim to estimate the joint distribution of the ownership utility U and the rental utility W, as well as the status probabilities ρ_r, ρ_p.

2.1 Agreements and Outcomes

Each RTO agreement covers a nominal term of T, the number of periods for which the renter must rent the item to gain ownership without any extra payment. In a particular rental agreement, the time index t counts the number of periods since the beginning of that agreement. In each period t, the renter may purchase the item at price p_t, referred to as the *buyout price*. This price is set by the firm and changes from one period to another depending on the firm's strategy. The buyout price in every period of an agreement is no less than the rental fee; otherwise, the firm loses revenue. In addition, the price in period T cannot be greater than the rental fee; otherwise, the buyout offer is rejected. These two relationships are confirmed by the buyout prices in our data. According to our RTO firm, it is not standard practice in the RTO business to dynamically optimize the buyout prices. So for estimation, it is appropriate to assume that the price path for each agreement is fixed in advance. The renter can terminate the agreement in any period without a penalty by returning the product. An agreement,

Table 1: Important notations.

Inputs	$\mathcal{A}, \mathcal{P}$ and $\mathcal{C}$	Sets of abandon, purchase and payoff agreements
	t_i	Duration of agreement i
	p_t^i and r^i	Buyout price in period t and rental fee of agreement i
Outputs	$\mathcal{B}_X^i(k)$	Buy set of the renter in agreement i in model X under segment k
	ρ_r and ρ_p	Probabilities that the renter's status is neglecting and considering the purchase option
	π_k	Probability that a renter belongs to segment k; $\pi_k = \mathrm{P}((U, W) = (u_k, w_k))$

based on its outcome, can be classified as exactly one of the following types:

- *Purchase* agreement that ends prior to its term when the renter pays the buyout price.
- *Payoff* agreement that ends at its term with the renter gaining ownership.
- *Abandon* agreement that ends when the renter ceases the rental without ownership.

We denote the rental fee with r. When needed, we append the agreement index i as a superscript to r and p_t to indicate that they vary across agreements. We may think of the time between when the renter takes the product home and when he is offered the first buyout price as period zero. RTO transactions yield for each agreement i the rental fee r^i, the buyout price p_t^i in period t, the term T_i and the duration $t_i \geq 1$. The duration of an abandon agreement or a purchase agreement is shorter than its term. If the agreement duration is equal to the term of the agreement, then in the last rental period, the renter owns the product by paying an amount equal to the rental fee. The data can compactly be represented according to the agreement types. The agreements can be partitioned into sets $\mathcal{A}$, $\mathcal{P}$ and $\mathcal{C}$ which respectively have the outcomes of abandon, purchase and payoff (by "Completing" the agreement). In addition to its outcome, the data pertaining to agreement i is T_i, t_i, r^i and $(p_1^i, \ldots, p_{T_i}^i)$.

We have obtained real-life transaction data for washer, dryer and mattress rentals from our RTO firm. The descriptive summary of the data –referred to as *our data* hereafter– appears in Table 2 of §5. The washers, dryers and mattresses in our data are all of the same quality and functionality. The buyout prices in our data cover a wide range of values from several hundred dollars to tens of dollars.

Given a data set, we estimate the status probabilities and the joint distribution of ownership and rental utilities with likelihood maximization. U and W respectively denote the ownership utility and rental utility –nonnegative discrete random variables whose joint probability mass function is represented by π_k; see Table 1. With a sequence of buyout prices, many specifications of the renter decision-making process are possible, as this process can reflect different degrees of sophistication. More sophisticated renters may consider more distant future periods and better account for their random statuses. Next, we present a new, unifying framework for RTO ownership utility that succinctly expresses the likelihood in terms of the *output* of a renter's decision-making process, while remaining agnostic to the nature of that process. This framework greatly simplifies the presentation, segregates the process of solving the renter's decision problem from the utility estimation itself, and facilitates comparisons across specifications.

2.2 Generic Likelihood Function

In our framework, a renter's purchase preferences are described by a buy set $\mathcal{B}$, whose members are the indices of the periods in which he prefers to make a purchase if his status is considering. The composition of the buy set depends on the renter's decision-making process, as well as his rental and ownership utilities and status probabilities. However, the modularity of our framework permits us in this section to construct the likelihood function with the buy set and status probabilities as primitives. The buy set's determination under different decision-making models is the subject of §3 and §4, and we will compare the estimation performance for the different models on our firm's data in §5.1.

A renter in a period may abandon the agreement, accept the buyout price, or continue to rent by paying the rental fee. An abandon or a purchase action of a renter ends the agreement, while paying the rental fee extends the agreement to the next period. So, a renter's action in a period is classified as *abandon, purchase*, or *continue*.

Consider a renter with the buy set $\mathcal{B}$. In what follows, we derive the probabilities with which he purchases, abandons, and continues in period $t \geq 1$, conditional on continuing the rental up to period $t-1$. To indicate abandon, purchase and continue actions in period t, we respectively use binary variables $\mathbb{I}_{a,t}$, $\mathbb{I}_{p,t}$ and $\mathbb{I}_{c,t}$. The purchase action occurs if and only if the renter status is considering and $t \in \mathcal{B}$. This is a compound event with probability (wp) of $\rho_p \mathbb{I}_{t\in\mathcal{B}}$. The continue action reflects either of two distinct events. First, if the renter status is neglecting, which occurs wp ρ_r, then he continues the rental. Second, if the renter status is considering, which occurs wp ρ_p, then he continues the rental if and only if $t \notin \mathcal{B}$. Thus, the total probability of continuing the rental in period t is $\rho_r + \rho_p \mathbb{I}_{t\notin\mathcal{B}} = \rho_r^{\mathbb{I}_{t\in\mathcal{B}}} \rho^{\mathbb{I}_{t\notin\mathcal{B}}}$. The abandon action occurs if and only if the renter's status is abandoning and hence occurs wp $1-\rho$. We use these conditional probabilities to find the unconditional probability for each of the renter actions in period t. With $[a:b]$ denoting the set of integers including the integers a, b and those in between, let $\mathcal{B}^t = \mathcal{B} \cap [1:t]$ represent the truncated buy set at t which includes the periods in the buy set prior to period t, and let $\mathcal{B}^0 = \emptyset$. The unconditional probability of a purchase in period t is

$$\mathrm{P}(\mathbb{I}_{p,t}=1) = \mathrm{P}(\mathbb{I}_{c,t-1}=1)\rho_p \mathbb{I}_{t\in\mathcal{B}} = \mathrm{P}(\mathbb{I}_{c,t-2}=1)\rho_r^{\mathbb{I}_{t-1\in\mathcal{B}}} \rho^{\mathbb{I}_{t-1\notin\mathcal{B}}} \rho_p \mathbb{I}_{t\in\mathcal{B}} = \cdots = \rho_r^{|\mathcal{B}^{t-1}|} \rho^{t-1-|\mathcal{B}^{t-1}|} \rho_p \mathbb{I}_{t\in\mathcal{B}}. \tag{1}$$

Similarly, for continue and abandon actions, respectively, we have

$$\mathrm{P}(\mathbb{I}_{c,t}=1) = \rho_r^{|\mathcal{B}^{t-1}|} \rho^{t-1-|\mathcal{B}^{t-1}|} (\rho_r + \rho_p \mathbb{I}_{t\notin\mathcal{B}}) = \rho_r^{|\mathcal{B}^t|} \rho^{t-|\mathcal{B}^t|} \text{ and } \mathrm{P}(\mathbb{I}_{a,t}=1) = \rho_r^{|\mathcal{B}^{t-1}|} \rho^{t-1-|\mathcal{B}^{t-1}|}(1-\rho).$$

Note that if the renter does not continue the rental up to period $t-1$ (i.e., if he purchases or abandons prior to period t), then there is no action to be taken in period t. Thus, the unconditional probabilities

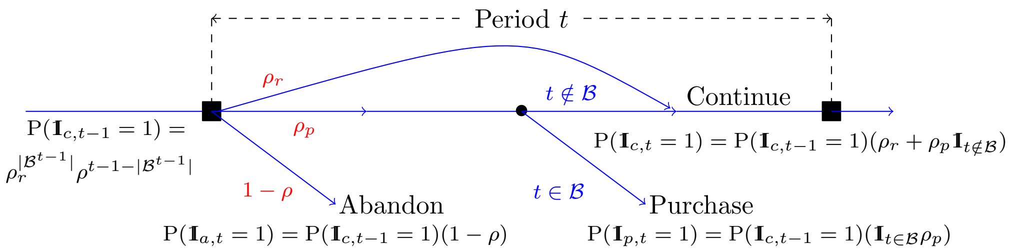


Figure 1: Abandon, purchase and continue action probabilities for a generic renter in period $t < T$.

of purchase, abandon and continue actions in period t sum up to the unconditional probability of continuing up to period $t-1$. The sequence of events and actions in period $t < T$ appears in Figure 1. In period T, continue and purchase actions merge and their aggregate probability becomes ρ.

Suppose the population of renters comprises K distinct segments. The firm does not directly observe the segment of each renter. A renter belongs to exactly one of the K segments, and the renters in each segment have identical buy sets for a given agreement. Consider agreement i which concludes in period t_i, and let $\mathcal{B}^i(k)$ and $\mathcal{B}^{i,t}(k)$ respectively denote the buy set and the truncated buy set at period t for the renter in this agreement if he belongs to segment k. The probabilities of continue, abandon and purchase actions, discussed in and around (1), can be combined into a single expression which yields the likelihood of agreement i based on its outcome, conditional on the renter belonging to segment k:

$$l_i(\mathcal{B}^i(k), \rho_r, \rho_p) \;=\; \left[\rho_r^{|\mathcal{B}^{i,t_i-1}(k)|}\rho^{t_i-1-|\mathcal{B}^{i,t_i-1}(k)|}\right](1-\rho)^{\mathbb{I}_{i\in\mathcal{A}}}\left(\rho_p \mathbb{I}_{t_i\in\mathcal{B}^i(k)}\right)^{\mathbb{I}_{i\in\mathcal{P}}}\rho^{\mathbb{I}_{i\in\mathcal{C}}}. \tag{2}$$

In (2), the term inside square brackets represents the probability of continuing up to period $t_i - 1$, and the other terms reflect the probability of each renter action in the final period of the agreement. The last action of a payoff agreement (associated with $\mathcal{C}$ in (2)) is indistinguishable between continue and purchase, as both actions result in ownership of the item by the renter through identical payment amounts. These indistinguishable actions are treated as missing data, and their probabilities are aggregated: that is, period T of a payoff agreement contributes ρ to the likelihood. Although (2) is derived for $\rho_r > 0$, it accommodates the case of $\rho_r = 0$ if we let $0^0 = 1$ as a convention.

We derive the likelihood $\bar{l}_i$ for the outcome of agreement i by first evaluating the likelihood in (2) for segment k and then mixing the resulting likelihoods according to the probability vector $\boldsymbol{\pi}$. Specifically, $\boldsymbol{\pi} = (\pi_1, \ldots, \pi_K)$, where π_k is the probability that a renter belongs to segment k. Then

$$\bar{l}_i(\boldsymbol{\pi}; \mathscr{B}^i, \rho_r, \rho_p) = \sum_{k=1}^{K} \pi_k l_i\big(\mathcal{B}^i(k), \rho_r, \rho_p\big),$$

where $\mathscr{B}^i = \{\mathcal{B}^i(k) : k \in [1:K]\}$ denotes the collection of buy sets for the renter in agreement i as this renter is considered to be in each of the segments. Accordingly, the likelihood of the data set is

$$L(\boldsymbol{\pi}; \mathscr{B}, \rho_r, \rho_p) = \prod_{i\in\mathcal{A}\cup\mathcal{P}\cup\mathcal{C}} \bar{l}_i(\boldsymbol{\pi}; \mathscr{B}^i, \rho_r, \rho_p) = \prod_{i\in\mathcal{A}\cup\mathcal{P}\cup\mathcal{C}} \left(\sum_{k=1}^{K} \pi_k l_i\big(\mathcal{B}^i(k), \rho_r, \rho_p\big)\right), \tag{3}$$

with $\mathscr{B}$ representing the grand collection $\{\mathscr{B}^i\}_{i\in\mathcal{A}\cup\mathcal{P}\cup\mathcal{C}}$ of buy sets. Observe in (3) that the data set achieves a zero likelihood when $\bar{l}_i(\cdot) = 0$ for some i, which occurs if and only if $l_i(\mathcal{B}^i(k)) = 0$ for all $k \in [1 : K]$, i.e., when none of the segments has a buy set to represent the decisions of the renter. In that case, the likelihood becomes zero for all parameter values. In the rest of this section, we consider a data set such that $\bar{l}_i(\cdot) > 0$ for each agreement i.

For a given renter decision-making process, fixing ρ_r and ρ_p fixes the buy sets, which is important for the modularity of our approach. Interestingly, the likelihood depends only on buy sets, status and segment probabilities, and our data. It does not depend directly on the utilities that a renter in a segment obtains or that renter's decision-making process, as it depends on these only through buy sets. That is, the information in utilities and the renter decision-making process is completely captured and passed to the likelihood via buy sets. Conceptually, buy sets bridge the renter decision-making process to the likelihood; the buy sets are computed once for each segment before estimation, lending modularity and efficiency to our framework and allowing us to present the generic likelihood function (3). This likelihood can be maximized with the iterative scheme presented next.

Theorem 1. *For fixed ρ_r and ρ_p, the likelihood function* (3) *is log-concave in $\boldsymbol{\pi}$. Starting at $\boldsymbol{\pi}^0 = (\pi_1^0, \dots, \pi_K^0) \in (0,1)^K$ with $\sum_{k=1}^K \pi_k^0 = 1$, the convergence point, if it exists, of the iterative scheme*

$$\pi_k^{n+1} = \frac{1}{|\mathcal{A}\cup\mathcal{P}\cup\mathcal{C}|} \sum_{i\in\mathcal{A}\cup\mathcal{P}\cup\mathcal{C}} \frac{l_i\big(\mathcal{B}^i(k), \rho_r, \rho_p\big)}{\bar{l}_i(\boldsymbol{\pi}^n; \mathscr{B}^i, \rho_r, \rho_p)} \pi_k^n \quad \textit{for} \quad k \in [1:K],\ n \geq 0 \tag{4}$$

is a maximum likelihood estimator of $\boldsymbol{\pi}$.

At iteration n, the i-th term $\pi_k^n l_i(\cdots)/\bar{l}_i(\cdots)$ in (4) is the conditional probability that the renter in agreement i belongs to segment k. Similar conditional probabilities appear in the iterative scheme of Arcidiacono and Miller (2011). For each renter decision model, buy sets depend discretely on ρ_r and ρ_p. Accordingly, we estimate ρ_r and ρ_p via grid search, i.e, for each pair (ρ_r, ρ_p) on the grid, we apply the iterative scheme (4) to estimate the $\boldsymbol{\pi}$ corresponding to that pair. We report as our estimates the pair and the corresponding $\boldsymbol{\pi}$ that yield the highest likelihood. Although we consider identical ρ_r and ρ_p for all segments, our framework can accommodate different ρ_r and ρ_p for different segments.

In the rest of the paper, renters are segmented wrt their utilities, i.e., renters in segment k all have utility pair (u_k, w_k), which is different from the utility pairs of renters in other segments. For distinct segments i and j, we may have $u_i = u_j$ or $w_i = w_j$, but we cannot have $u_i = u_j$ and $w_i = w_j$. Therefore, a renter belongs to segment k if and only if his utility pair is (u_k, w_k). This one-to-one correspondence allows us to replace $\mathcal{B}(k)$ with $\mathcal{B}(u_k, w_k)$ in the generic likelihood function (3). When analyzing renter decisions, we consider a fixed (arbitrary) segment, so we drop the segment k from our notation, when there is no ambiguity. As our focus is on ownership utility, we choose to suppress w in our buy set notation to write $\mathcal{B}(u)$. For conciseness, we use net (rental) fee $s = r - w$ in our derivations hereafter.

We consider several renter decision models in this paper and derive the corresponding renter buy sets. To distinguish the buy sets generated under each model, we occasionally append a subscript to indicate the model (see Table 1). The models differ along two dimensions. First, we differentiate the models based on the renter sophistication. For example, a *strategic* renter considers the current period and the upcoming periods in his decision making, whereas a *myopic* renter considers only the current period. The second differentiating factor is the consistency of renters having the status of considering a purchase. The *alert renter* models study renters that are never in the status of neglecting a purchase, i.e., they consider a purchase if they do not abandon the agreement. Accordingly, we have $\rho_r = 0$ in the alert models. In contrast, the *inert renter* models allow for renters to neglect a purchase wp $\rho_r > 0$. For ease of presentation, in §3 and §4, we consider a fixed renter segment and drop k from our notation.

3 Strategic Renters

A strategic renter compares the surplus of the current period's purchase with the expected surplus of deferring this purchase, and opts for the purchase if the former surplus weakly exceeds the latter. The purchase surplus is the difference between the renter's ownership utility and the buyout price. A strategic renter knows the status probabilities ρ_r and ρ_p and perfectly anticipates the buyout price path.

3.1 Strategic and Alert Renter (SAR) Model

A *strategic* and *alert renter (SAR)* always possesses the mental bandwidth and the financial means to consider a purchase if he does not abandon the agreement. For a SAR, $\rho_r = 0$, and $\rho = \rho_p$. Given that a SAR's status is not abandoning, it is certainly considering, as he is alert to the purchase offers.

A SAR makes a purchase in a period if and only if the purchase surplus is at least as much as the continuation surplus in that period. For the SAR with utility u, the purchase surplus in period t is $u - p_t$ and the continuation surplus is $\rho v_{t+1}(u) - s$. This continuation surplus is the value that the renter associates with forgoing a purchase in period t but retaining the future buyout options by incurring the net fee s. The function $v_t(u)$ is the value-to-go function after renter status in period t is realized as considering, and it is found from the following dynamic program (DP):

$$v_t(u) \quad = \quad \max\{u - p_t, \rho v_{t+1}(u) - s\} \;\text{ for } t \in [1 : T-1],\; v_T(u) = u - p_T. \tag{5}$$

After incurring the net fee to continue the rental in a period, a SAR makes another decision in the next period if his status is considering, which occurs wp ρ. Recall that $p_T = r$, and $p_t \geq r$ for $t \in [1 : T-1]$. Upon rental fee payment in the final period, the renter owns the item. This implies

$v_T(u) = \max\{u - p_T, u - r\} = u - p_T$, confirming that each SAR gains ownership in the final period whether he makes a purchase or continues the rental. The sequence of events and actions for the SAR is obtained by setting $\rho_r = 0$ in Figure 1. The specialized illustration appears in Appendix C.1.

We characterize the solution to a SAR's problem in the following proposition.

Proposition 1. *(i) The SAR value-to-go function in* (5) *can be computed with the nonrecursive formula*

$$v_t(u) = \max_{n\in[t:T]} \left\{\rho^{n-t}(u - p_n) - s\sum_{j=1}^{n-t}\rho^{j-1}\right\} \quad \textit{for} \quad t \in [1:T]. \tag{6}$$

(ii) A SAR's purchase in period $t \in [1 : T-1]$ *is optimal if and only if his utility satisfies* $u \in [u^t, u^{t-1})$*, where utility thresholds* $u^0 \geq u^1 \geq \cdots \geq u^{T-1}$ *are given as* $u^0 = \infty$ *and*

$$u^t \quad = \quad \min_{i\in[0:t]} \left\{ \max_{n\in[i+1:T]} \frac{p_i - \rho^{n-i}p_n - s\sum_{j=1}^{n-i}\rho^{j-1}}{1-\rho^{n-t}} \right\} \quad \textit{for} \quad t \in [1 : T-1]. \tag{7}$$

Proposition 1(i) provides a nonrecursive solution to the DP in (5), which enables us to calculate the SAR's buy set efficiently. To do so, for each period t, we find $v_t(u)$ from (6). If $v_t(u) = u - p_t$, then $t \in \mathcal{B}_{sa}(u)$, where $\mathcal{B}_{sa}(u)$ is the buy set of the SAR with utility u; otherwise, the SAR continues the rental in period t conditional on not abandoning the rental.

As the status of a SAR is always considering unless abandoning, he either makes a purchase in the earliest period in his buy set or abandons the rental by that period. The right-hand side of (6) is the largest of the expected purchase surpluses from period t to T. In particular, $v_1(u)$ is the largest expected purchase surplus that a SAR can gain during the agreement. In computing $v_1(u)$, the SAR identifies $\tau_s(u)$, the earliest period when a purchase is optimal (see also the proof of Proposition 1(ii)):

$$\tau_s(u) \quad = \quad \min\left\{ \arg\max_{n\in[1:T]} \left\{\rho^{n-1}(u - p_n) - s\sum_{j=1}^{n-1}\rho^{j-1}\right\} \right\}. \tag{8}$$

This earliest period is the minimal element of $\mathcal{B}_{sa}(u)$, i.e., we have $\tau_s(u) = \min\{t : t \in \mathcal{B}_{sa}(u)\}$. The renter then targets $\tau_s(u)$ for a purchase; an agreement duration never exceeds $\tau_s(u)$ for the SAR with utility u. In particular, the SAR never prefers to postpone a purchase to a future period with a higher buyout price, as the expected purchase surplus inside the argmax in (8) decreases in both n and p_n.

The purchase strategy of a SAR in Proposition 1(ii) enables us to infer his utility range by observing the period of his purchase, abandon or payoff action. A SAR who purchases in period t must have a utility $u \in [u^t, u^{t-1})$. If $u < u^t$, then the SAR with utility u would prefer a later period for a purchase, and if $u \geq u^{t-1}$, then he would have purchased earlier. In addition, a SAR who abandons the agreement in period t has a utility $u < u^{t-1}$; otherwise, he would have purchased in period $t-1$ or earlier. For the same reason, a SAR in a payoff agreement has a utility $u < u^{T-1}$. The SAR model is not fully compatible with the framework of Rust (1994), hence our alternative approach using a finite mixture model; see Appendix B for a detailed discussion of the connection and incompatibility.

With the inferred utility via Proposition 1(ii), we narrow down the segments a SAR can belong to. Consider agreement i that concludes in period t_i. The utility thresholds in (7) depend on agreement-specific components such as term, buyout prices and rental fee. Let u_i^t be the threshold for agreement i in period t. If i is a purchase agreement, the SAR can only belong to segments k with $u_k \in [u_i^{t_i}, u_i^{t_i-1})$. Similarly, if i is an abandon or a payoff agreement, the SAR can only belong to segments k with $u_k < u_i^{t_i-1}$. The remaining segments will have zero contribution to the likelihood for this agreement. Through their impact on (3), the sets of possible segments for different agreements strongly influence the maximum likelihood estimator for the probability vector $\boldsymbol{\pi}$.

That the SAR model implies a unique possible purchase period for a given utility is useful for efficient calculation of thresholds, but it creates problems for estimation with some data sets. Specifically, a limitation of the SAR model is that it is possible that some plausible actions cannot be explained by any nonnegative utility. So, although the model cleanly captures important elements of renter decision making, this limitation motivates consideration of more general and flexible renter decision models.

3.2 Strategic and Inert Renter (SIR) Model

We now consider a strategic renter who is inert to purchase offers; that is, his status is considering wp $\rho_p < \rho$, neglecting wp $\rho_r = \rho - \rho_p$ and abandoning wp $1-\rho$. A reasonable question is whether the renter is aware of his own inertia. Here, we consider a strategic renter who realizes that in each period, with positive probability his status will be neglecting. Hence, this renter is inert in responding to purchase offers. We refer to such a renter as a *strategic* and *inert renter (SIR)*. We also perform estimation for the alternative case with a *strategic* and *optimistic renter (SOR)*, who does not realize that he may have the neglecting status; see Appendix B for details on this model. As hinted above, the neglecting status plays an important role in explaining some renters' actions. Because in any period a renter may be in the neglecting status and therefore reject any buyout price, every feasible sequence of actions has positive probability in the SIR model with appropriate utility segments. Thus, the likelihood (3) is strictly positive for any $\rho_r, \rho_p > 0$ and the corresponding buy sets, and the model is "saturated" in the sense of Aguirregabiria and Mira (2010). This saturation is critical for the iterative scheme (4). By contrast in the SAR model ($\rho_r = 0$), some feasible actions have zero probability (e.g., purchasing at a price p after rejecting a price $p' < p$). If such actions occur in a data set, then the offending agreements must be removed, or the estimation will be crippled because they will zero out the likelihood. Finally, not only does the neglecting status facilitate estimation, but its inclusion also enriches the estimation output by the addition of a concrete and interpretable quantity, namely the probability that a renter will neglect a purchase offer entirely regardless of the buyout price.

Now, consider a SIR with utility u. In period t, wp $1-\rho$, the SIR's status is abandoning, in which case the rental terminates without ownership. With probability ρ_r, his status is neglecting, in which case he continues the rental and gains the expected continuation surplus $v_{t+1}(u)-s$. With the remaining probability ρ_p, his status is considering, in which case he compares the purchase surplus $u-p_t$ with the expected continuation surplus. As such, the SIR solves the following DP:

$$v_t(u) \quad = \quad \rho_r\big(v_{t+1}(u)-s\big)+\rho_p \max\big\{u-p_t, v_{t+1}(u)-s\big\} \text{ for } t\in[1:T-1],\ v_T(u)=\rho(u-p_T). \tag{9}$$

The terminal condition in (9) reflects that both purchase and continue actions yield the same value for $v_T(u)$ since $p_T = r$, so in period T, the SIR gains ownership unless he abandons. Figure 1 shows the sequence of events and actions for the SIR.

We denote the value-to-go functions of the SAR in (5) and of the SIR in (9) with the same v_t with some abuse of notation. This creates no ambiguity, as only the buy sets of each model are required to represent the corresponding likelihood function (3), and subscripts of buy sets indicate their associated models. The SIR model generalizes the SAR model; letting $\rho_r = 0$ in (9) induces buy sets identical to those of a SAR based on (5) (see Appendix C.2 for details). The value-to-go functions (5) and (9) differ with regard to when they are evaluated. The former is evaluated in the middle of a period for a renter that has not abandoned, whereas the latter is evaluated at the beginning of a period when abandonment is still a possibility. This difference facilitates analytical arguments but does not affect the tradeoff between purchase and continue actions in the SAR or SIR model.

After solving the DP in (9), a SIR knows the periods $t \in [1 : T-1]$ for which $u-p_t \geq v_{t+1}(u)-s$. These periods constitute the SIR's buy set $\mathcal{B}_{si}(u)$; that is, we have

$$t\in\mathcal{B}_{si}(u) \iff u-p_t\geq v_{t+1}(u)-s. \tag{10}$$

When two SIRs with identical abandoning status probabilities are considered, the one with a higher (lower) considering status probability is called *more (less) attentive*. Given the equal abandoning status probabilities, the more attentive SIR has a lower neglecting status probability. Aided by this terminology, we can characterize the buy sets in the SIR model by utility thresholds and relative degree of attentiveness. These buy sets expand as the utility rises or attentiveness diminishes, as formalized next.

Proposition 2. *For period $t \in [1 : T-1]$, there exists a utility threshold $\tilde{u}^t$ such that $t\in\mathcal{B}_{si}(u)$ if and only if $u \geq \tilde{u}^t$. Furthermore, for a fixed buyout price path and net fee:*

(i) For two SIRs with different ownership utilities, the buy set of the SIR with lower utility is a subset of that of the other.

(ii) For two SIRs from the same segment, the buy set of the more attentive SIR is a subset of that of the other. If these SIRs face identical status realizations, then the more attentive SIR purchases later.

Proposition 2 first characterizes the periods in which a purchase is *intended* by a SIR. Hence, its conclusion is weaker than that of its counterpart Proposition 1(ii) for a SAR, which determines the purchase period with certainty (barring earlier abandonment). This unique possible purchase period is a consequence of a SAR's alertness, which also yields upper utility limits; for an alert renter, rejection of a buyout offer can only be explained by a utility that is not high enough to justify accepting the offered price. In contrast, because a SIR's rejection of a purchase offer may instead reflect his neglecting status, such rejection does not imply a hard upper bound on his utility.

Points (i) and (ii) of Proposition 2 convey structural insights about two different aspects of RTO renters. Point (i) relates to economic considerations and strategic decisions, showing that a renter with higher utility intends to purchase earlier than one with lower utility. Intuitively, the higher the utility is, the higher the opportunity cost of ending the rental without ownership, and thus the stronger the incentive to reduce the risk of abandonment by targeting an earlier purchase. Point (ii) pertains to status and attentiveness. Its first part is about the renters' intentions and demonstrates that a more attentive SIR intends to purchase later in his agreement than a less attentive one. The more attentive a renter is, the more likely he will be able to entertain future purchase offers, and thus the higher his continuation surplus (the RHS of the inequality in (10)). The higher continuation surplus shrinks the buy set as it becomes better in some periods to wait for future purchase opportunities. By the second part of point (ii), given identical status realizations, the smaller buy set of the more attentive SIR implies that he will purchase later than the less attentive one, which also results in a purchase at a lower price if buyout prices decrease over time. However, although the less attentive SIR targets an earlier purchase, the more attentive SIR may still end up purchasing earlier because he is more likely in each period to consider a purchase. For instance, the less attentive renter's status may turn out to be neglecting in the early periods of his buy set, while the more attentive renter's status may be considering in a period early in his own.

3.3 Strategic, Inert, and One-Step-Look-Ahead Renter (SIOR)

Thus far, we have considered fully strategic renters who account for all of the future purchase opportunities when making a current purchase decision. However, the DP in (9) is not trivial to solve, and a typical renter is not trained to solve such problems. To simplify the problem (or for other possible reasons such as a difficulty in anticipating future buyout prices), a forward-looking, inert renter may consider only a limited horizon in his decisions. The simplest version of a forward-looking renter is one who considers only the subsequent period in addition to the current one. Continuing our study of inert renters, we refer to such a renter as a *strategic, inert* and *one-step-look-ahead renter (SIOR)*. We study the SIOR to demonstrate the case of a renter with a limited decision horizon. However, we emphasize that our modular framework could easily accommodate other horizon lengths; all that would be required is to compute the associated buy sets, which can be accomplished similarly to the development below.

To decide whether to purchase in period $t < T$, a SIOR with utility u compares his current purchase surplus only with the expected surplus of targeting a purchase in period $t+1$. If he makes a purchase in period t, his surplus is $u - p_t$. Otherwise, he incurs the net fee s to continue the rental to the next period. When contemplating a planned purchase in period $t+1$, the SIOR correctly accounts for the probability of each status. With probability $1 - \rho_r - \rho_p$, he abandons the rental in period $t+1$ for an incremental cost of s (the net fee for continuing the rental in period t). With probability ρ_r, he is in the neglecting status in period $t+1$, in which case he continues the rental further and incurs $2s$ in net fees to do so (the net fees for periods t and $t+1$; recall that he ignores future periods beyond $t+1$). With probability ρ_p, he is in the considering status in period $t+1$ and makes a purchase, for an incremental payoff of $-s+u-p_{t+1}$, where again s is the net fee in period t. Combining these scenarios, we obtain the expected surplus of targeting a purchase in period $t+1$ as $(1-\rho_r-\rho_p)(-s)+\rho_r(-2s)+\rho_p(-s+u-p_{t+1})$, which simplifies to $\rho_p(u - p_{t+1}) - (1+\rho_r)s$ and pertains to two periods –we call it *two-period expected surplus*. Period t belongs to the SIOR's buy set $\mathcal{B}_o(u)$ if and only if the period-t purchase surplus exceeds the two-period expected surplus, i.e., if and only if $u - p_t \geq \rho_p(u - p_{t+1}) - (1+\rho_r)s$. In the last period T, the SIOR owns the rented item unless he abandons.

The RHS of the inequality above increases in ρ_p when ρ remains constant (i.e. ρ_r decreases by the same amount). Analogously to Proposition 2 for a SIR, the two-period expected surplus is higher for a more attentive SIOR compared to a less attentive one, everything else equal. Accordingly, conditional on identical status realizations, the more attentive SIOR will purchase later.

4 Myopic Renter Models: Alert (MAR) and Inert (MIR)

In this section, we study a *myopic* renter, i.e., one who considers only the current period in his decisions. We derive his buy set under both the alert and inert models.

We first study a myopic renter who always considers purchase offers, unless he abandons. In other words, the renter's status is either considering or abandoning (never neglecting). Recall from §2 that this implies $\rho_r = 0$ and $\rho = \rho_p$. We refer to such a renter as a *myopic* and *alert renter (MAR)*.

A MAR disregards payments and purchase opportunities beyond the current period. For a MAR with utility u, the period-t purchase surplus is $u - p_t$, and the continuation surplus is $-s$. Therefore, period t belongs to the MAR's buy set $\mathcal{B}_{ma}(u)$ if and only if $u \geq p_t - s$. The illustration of sequence of events and actions for the MAR is obtained by setting $\rho_r = 0$ in Figure 1 (see Appendix C.3). Since the MAR is alert to purchase offers, like a SAR (§3), he definitely makes a purchase in the earliest period of his buy set, unless he abandons earlier. This earliest period is $\tau_m(u) := \min\{j : j \in \mathcal{B}_{ma}(u)\} = \min\{j : u \geq p_j - s\}$.

The composition of the MAR's buy set is independent of the probability ρ as the MAR does not consider his future statuses in making a current purchase decision. Moreover, the MAR's binary status of abandoning or considering is fully revealed by whether he abandons. These points imply that the likelihood of the MAR model is separable in ρ and $\boldsymbol{\pi}$, which enables us to find the MLE for ρ analytically.

Proposition 3. *The MLE of $1 - \rho$ in the MAR model is the ratio of the total number of abandon agreements to the summation of all agreements' durations.*

The MLE of $1-\rho$ for the MAR model is the fraction of abandon actions out of the total opportunities for abandonment; that is, it is the sample proportion of abandon actions.

By definition, the buyout price in period $\tau_m(u)$ is lower than all preceding buyout prices because it is the earliest period when the difference between the buyout price and the net fee is no greater than u. The minimum price $\min_{j \leq t} p_j$ by period t occurs in period t if, for example, the buyout prices are nonincreasing over time, i.e., $p_1 \geq p_2 \geq \cdots \geq p_t$. Conversely, if $p_{j+1} \geq p_j$ for some $j \leq t-1$, then the renter's rejection of the price p_{j+1} provides no additional information beyond that conveyed by his rejection of p_j (i.e., it does not alter the likelihood). A MAR who rejects p_j will certainly also reject the higher price p_{j+1}. This limits the MAR model's flexibility to explain some purchase decisions. Inspired by §3, to increase the flexibility we next augment the model by incorporating the neglecting status.

In contrast to a MAR, we now consider a renter whose status can be either considering or neglecting if not abandoning. Such a renter is inert to purchase offers during his rental. We model this renter with $\rho_r > 0$ and refer to him as a *myopic* and *inert renter (MIR).*

A MIR's purchase criterion is the same as a MAR's, i.e., they have identical buy sets. Specifically, period t belongs to the buy set of a MIR with utility u if and only if $u \geq p_t - s$. Unlike a MAR, the rental for a MIR may extend beyond the earliest period of the buy set, if his status then is neglecting. In this case, he continues the rental and makes a purchase in the earliest future period in his buy set in which his status is considering. Refer to Figure 1 for the sequence of events and actions for a MIR.

A MAR is alert to purchase offers, and his continue actions are always due to his low utility; in contrast, a MIR's continue actions can reflect either his neglecting status or his low utility. This tangling of status and utility in the event of a continue action renders estimation of status probability and utility distribution inseparable in the MIR model, unlike the MAR model.

Like the SIR model, the MIR model is saturated with proper segments and can explain any data. However, because ρ_r and ρ_p are estimated numerically in the MIR model, it is less efficient to estimate than the MAR model. If a manager believes that ρ_r is relatively close to zero, then she might be advised

Table 2: Descriptive statistics for our data sets. Term and monetary values are in weeks and dollars.

	Percentages of agreement outcomes		Averages over purchase agreements		Averages over all agreements	
Product	abandon	purchase	term	purchase price	term	weekly revenue
Washer	73.0%	23.8%	58.5	152.6	84.6	11.3
Dryer	71.0%	25.0%	57.8	136.6	86.1	12.3
Mattress	49.8%	37.1%	45.6	104.8	55.1	17.0

to adopt the MAR model despite discarding a small number of agreements as outliers; the estimators should be similar and the computation itself is faster.

5 Real-life Application for an RTO Firm

We now apply our estimation framework to a real-life RTO data set. We first present the estimation results for our data of washers, dryers and mattresses, under each of the renter decision models. Then, we perform counterfactual analyses to explore whether altering buyout price paths or influencing renter attentiveness would benefit our firm. The results reveal the relative suitability of different decision models and (joint) distribution of utilities for each of the three products, which is useful for managers.

For each product, our data includes the transactional history of all agreements initiated within a time window (see Appendix B for details). For our washer, dryer and mattress data, Table 2 reports the percentages of agreement outcomes, the averages of their terms and weekly revenues (the phrase used in practice for a renter's periodic payment) and the average purchase prices. Statistics for washer and dryer data are similar. In contrast to washer and dryer agreements, mattress agreements are less likely to terminate with abandonment, and they have a shorter average term and a higher average rental fee. Also, mattress purchase prices average below those of the others.

5.1 Estimation Results

We estimate $\boldsymbol{\pi}$, ρ_r and ρ_p for each of the strategic and myopic models developed in §3-4, with our washer, dryer and mattress data. We compare the models with the estimation criterion of loglikelihood. It is possible within our framework to endogenize the renter decision model by estimating a mixture distribution over the models. However, this would require additional parameters, which could lead to overfitting, and hence we choose to estimate the models separately and compare their individual fits.

We segment customers wrt their ownership and rental utilities. For washers and dryers, we consider 84 segments resulting from the Cartesian product of $\mathbf{U} = [25 : 300, 25] \cup [350 : 600, 50] \cup \{800, 1000\} \cup\{16000\}$ and $\mathbf{W} = \{0, 5, 10, 15\}$ measured in dollars, where $[a : b, c]$ denotes the sequence that starts with a, ends in b, and has increments of c. We choose the utility segments to cover a large range of utilities, with a finer grid for ownership utility over the more plausible interval. We later explain why we consider a utility level above a thousand dollars in $\mathbf{U}$.

Table 3: Estimation results of alert models for washers (utilities in dollars).

Model	Loglikelihood	ρ	Ownership utility U mean	st.dev.	1^{st}quartile	median	3^{rd}quartile	Rental utility W mean	st.dev.
SAR	$-10,171$	94%	157.4	123.2	38.1	108.1	232.8	11.2	5.1
MAR	$-10,744$	95.6%	129.1	113.1	26.4	71.1	209.9	8.1	5.8
SOR	$-9,737$	94%	168.6	113.6	44.2	175.5	235.2	9.8	5.1

For status probabilities, we first consider $\rho = \rho_r + \rho_p$ from $[70\% : 98\%, 4\%]$ and then for each such ρ, we consider ρ_r from $[10\% : \rho, 4\%]$. We consider $\rho \geq 70\%$; otherwise, too many agreements would end in abandonment wrt the summary statistics in Table 2. For each of the 148 instances of (ρ_r, ρ_p), we estimate the segment probabilities $\{\pi_k : k \in [1 : |\mathbf{U} \times \mathbf{W}|]\}$ to maximize the loglikelihood.

We first present the estimation results of the alert renter (SAR, SOR and MAR) models for the washer data. As stated in §3.2 and §4, the decisions of some renters do not fit the SAR or MAR model. Namely, renters who rejected one buyout price but decided to purchase later at a higher buyout price cannot be MARs. For instance, consider a renter who rejects the price 100 but later purchases at 150. Under the MAR model, the rejection implies $u + s < 100$ whereas the purchase implies $u + s \geq 150$, which is a contradiction. In the SAR model, a strategic renter who repeatedly postpones a purchase must have a low ownership utility, and some purchases in our data can only be explained by negative ownership utilities, which are implausible. We estimate the alert models on the reduced data of roughly 83% of agreements that create neither contradictions nor implausibilities.

Alert models have $(\rho_r, \rho_p) = (0, \rho)$. The ρ value that yields the highest loglikelihood in the SAR model is 94%. Proposition 3 gives the MLE of ρ in the MAR model as 95.6%. Recall that a SOR's buy set with status probability (ρ_r, ρ_p) is identical to a SAR's with status probability $(0, \rho_r + \rho_p)$. On par with SAR and MAR, the SOR estimate is $\rho = \rho_r + \rho_p = 94\%$, with $\rho_r = 10\%$. On our data, the SAR, SOR and MAR models all assign a very low probability to the abandoning status; renters are unlikely to abandon in each week and are much more likely to have considering or (for SOR) neglecting status.

The estimation results for the alert models appear in Table 3. The SOR model performs the best with respectively 4.3% and 9.4% higher loglikelihood compared to those of the SAR and MAR models. The SOR model takes the initial step toward the richer inert models by allowing renters to be in the neglecting status, although they ignore such a possibility when making purchase decisions.

Unlike the alert models, inert models (SIR, SIOR and MIR) enable estimation with full data; inclusion of neglecting status in these models guarantees saturation with appropriate segments. Table 4 reports the estimation results for the inert models. In all inert models, the estimated ρ value is 98%, slightly exceeding its counterparts in the alert models. To this probability, however, ρ_r contributes significantly more than ρ_p, manifesting the importance of the neglecting status.

The inert model utility estimates can be expected to be larger than the alert model estimates. This is because the inert models have the capability of attributing a rejected buyout offer to the neglecting

Table 4: Estimation results of inert models for washers (utilities in dollars).

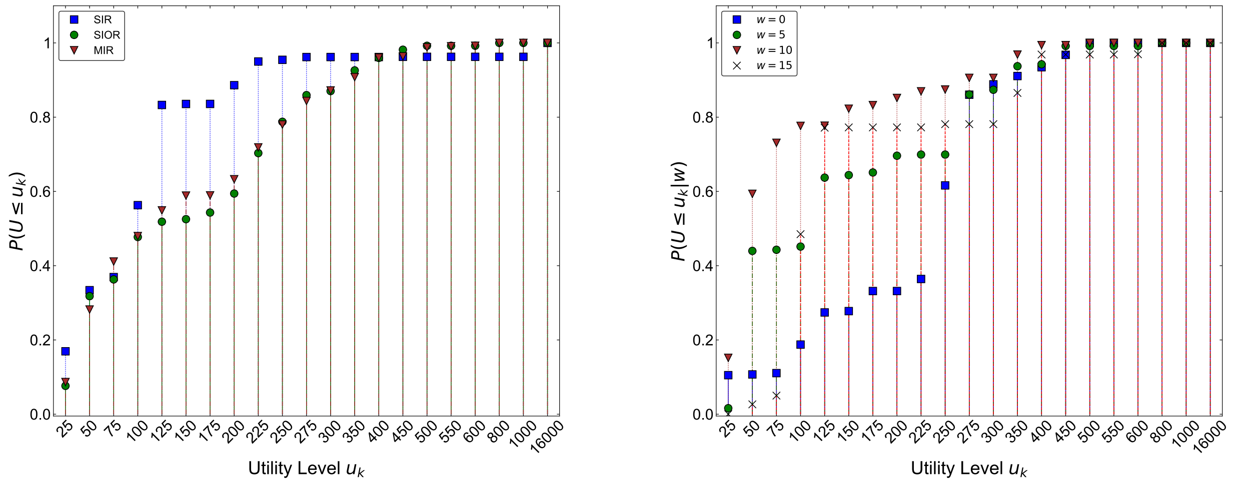
Model	Loglikelihood	(ρ_r, ρ_p)	Ownership utility U mean	st.dev.	1st quartile	median	3rd quartile	Rental utility W mean	st.dev.
SIR	$-13,837$	$(94\%, 4\%)$	696.2	3021	37.2	91.9	117.3	4.2	3.2
SIOR	$-13,461$	$(86\%, 12\%)$	171.3	132.4	43.0	113.8	238.9	6.8	5.3
MIR	$-13,483$	$(86\%, 12\%)$	168.4	136.9	45.9	107.3	237.8	5.5	4.6

status, while a rejection in the alert models can only be due to the renter's low ownership utility.

The mean of estimated U via the SIR model is the highest among the inert models, because the purchase surplus of a SIR in the purchase period must surpass the purchase surplus in all the future periods; the criterion for a purchase is less stringent in the SIOR and MIR models. Moreover, the estimated ρ_p in the SIR model is one third of the other inert models. Attributing a higher probability to the neglecting status to explain purchase rejections, the SIR model estimates a higher ownership utility compared to the SIOR and MIR models. The standard deviation of estimated U by the SIR model is very high, because about 4% of purchase decisions can be explained only by ownership utilities significantly above a thousand dollars. Surprisingly, the SIR model despite its sophisticated decision-making process fails to convincingly explain renter purchases and rejections, achieving a relatively poor loglikelihood value. Indeed, fitting the SIR model to renter actions leads to a questionable utility distribution ranging beyond a thousand dollars.

The SIOR model fits the best to the washer data and achieves a loglikelihood that is 2.7% higher than for the SIR model. This suggests that washer renters use a limited horizon in their purchase decisions. As the SIOR and MIR models achieve almost identical loglikelihoods, with SIOR having a slight edge, washer renters' decision-making process can be considered nearly myopic if not myopic. Figure 2(a) illustrates the estimated cumulative marginal probabilities for U in the inert models. The estimated U in the SIOR and MIR models are close to each other. Such distributional proximity and comparable performance suggest that the MIR model, which is faster to estimate, can be an effective substitute for the SIOR model if the computation time is a manager's concern. Otherwise, SIOR is the best fitting model and is the focus of the remaining discussion.

Figure 2(b) illustrates the estimated cumulative probabilities for U, conditional on a particular rental utility for the SIOR model. The figure suggests minimal correlation between U and W; the ownership utility conditional on rental utility of 0 has larger probabilities associated with high utilities over \$50 – \$275 than those of the ownership utility conditional on rental utility of \$5. However, when rental utility increases from \$10 to \$15, the conditional ownership utility increases stochastically. In fact, the correlation between estimated U and W is small at -0.09. To help explain this finding, consider a renter with random utility pair (U, W) who purchases the product and plans to use it for a certain duration. The firm represents this duration by the random variable L. One possible explanation for the



(a) cumulative probabilities for U

(b) SIOR conditional cumulative probabilities for U

Figure 2: Estimated ownership utility (in dollars) in the inert models.

small correlation between U and W despite their potential dependence is a relatively high coefficient of variation for L, as formalized by the following proposition.

Proposition 4. *If the ownership utility is $U = LW$ for independent L and W, then the correlation between U and W decreases as the coefficient of variation of L increases.*

Table 5 reports the estimation results of the inert models for dryers, considering the same segments and status probabilities as for washers. The SIOR model at $(\rho_r, \rho_p) = (86\%, 12\%)$ achieves the best fit with a respectively 3% and 0.09% higher loglikelihood than those of the SIR and MIR models. The correlation between estimated U and W for dryers is 0.37; slightly above the corresponding correlation in the case of washers, but still minor. The mean of estimated U for dryers is \$149.2, which is 13% lower than for washers. In contrast, the mean of estimated W for dryers is slightly higher than for washers.

In addition to the washers and dryers, we perform our estimation procedures for mattresses over the same ownership utility levels and grid for status probabilities. For rental utility, we consider more levels and set $\mathbf{W} = [0 : 35, 5]$. The SIOR model again turns out to be the best performing model. Similar to washers and dryers, the estimated ρ is 98%, signifying a small probability for renters to abandon the agreement from one period to the next. In contrast to washers and dryers, the estimated ρ_p is high at 54%, implying that mattress renters consider a purchase with higher probability. The means of estimated U and W are respectively \$80.3 and \$13.4; compared to washers and dryers, the mean of estimated U for mattresses drops, whereas the expected W nearly doubles. Lastly, the correlation between estimated U and W is -0.14, very minor as is the case for washers and dryers.

Our estimation results are consistent with the summary statistics of our washer, dryer and mattress data in Table 2, in which mattress agreements have lower purchase prices and higher rents. We also

Table 5: Estimation results of inert models for dryers (utilities in dollars).

Model	loglikelihood	(ρ_r, ρ_p)	Ownership utility U					Rental utility W	
			mean	st.dev.	1stquartile	median	3rdquartile	mean	st.dev.
SIR	$-11,681$	(94%, 4%)	851.2	3,286	19.5	117.3	244.6	4.4	3.8
SIOR	$-11,284$	(86%, 12%)	149.2	129.8	30.7	99.4	217.1	7.2	5.6
MIR	$-11,294$	(86%, 12%)	159.1	131.9	49.9	87.3	225.8	7.9	5.9

observe that the means of estimated W for washers, dryers and mattresses are respectively 40%, 42% and 21% lower than averages of their rental fees over all agreements.

5.2 Counterfactual Analyses

Armed with the estimated status probabilities and joint probability mass function from the best-performing SIOR model, we first conduct a counterfactual analysis to assess the impact of alternative pricing strategies for our RTO firm. As the primary goal of this paper is estimation, we do not attempt price optimization. Instead, we take the firm's existing pricing policy as a benchmark and analyze the effect of directional changes to obtain qualitative insights. In addition, we perform a second counterfactual analysis on the considering status probability ρ_p. The firm could potentially influence this probability by highlighting or downplaying the purchase option in its communications with customers. If the firm aims to increase the considering status probability, it could even reach out directly to customers to remind them about the purchase option.

Each agreement in our data represents an individual realization of the rental and ownership utilities, and a sequence of realized renter statuses – out of a space of possible realizations. So, to obtain a benchmark for comparing alternative price paths, we perform Monte Carlo simulation to replicate each agreement multiple times. Inputs to the simulation are each agreement's term, weekly revenue, buyout price path, status and utility segment probabilities and buy sets for each utility segment. We use the simulation output to compute aggregate performance measures for the benchmark (i.e., current) price path. We then follow the same procedure to compute performance measures for the alternative price paths, which requires computing the associated buy sets.

To get each alternative price path, we take the existing price path and multiply its prices by a constant factor $\omega_p \in \{0.7, 1.3\}$. Under each price path that we consider (including the benchmark of $\omega_p = 1$), we replicate each agreement 100 times, where for each replication we randomly draw the utility segment and the path of status realizations. For each replication of each agreement, we use the realized status path and the buy set for the realized utility segment to assign an action to each period. Note that we couple the utility segment and status realizations across price paths. We then determine for each replicated agreement the profit rate, which is the sum of rental payments received and the purchase price (if any) minus the firm's book value of the item (if purchased or paid off) divided by the agreement's duration. These performance measures are reported for the simulated data sets in Table 6.

Table 6: Statistics under each ω_p. Term and monetary values are in weeks and dollars.

		Percentages of agreement outcomes		Averages over purchase agreements		Averages over all agreements
Product	ω_p	abandon	purchase	term	purchase price	profit rate
Washer	0.7	63.2%	30.8%	66.3	172.7	12.49
	1	68.4%	23.1%	62.2	184.9	12.55
	1.3	71.0%	18.4%	59.6	190.8	12.50
Dryer	0.7	64.3%	29.5%	67.4	161.1	11.24
	1	69.3%	21.8%	63.2	171.7	11.29
	1.3	71.9%	17.2%	60.4	176.8	11.26
Mattress	0.7	51.2%	32.8%	46.0	102.1	15.08
	1	53.7%	27.5%	46.3	109.0	15.31
	1.3	54.9%	23.8%	45.9	115.7	15.40

Altering buyout prices has two effects on the revenue flow. Higher prices overall lead to more rejected buyout offers, which entails more rental revenue before the (potential) purchase, and the eventual purchase price may still be higher than that of an earlier purchase with a smaller price multiplier. As we see in Table 6, the average purchase price is increasing in ω_p. However, more rejected offers mean a greater chance of abandonment with no purchase revenue; in the table, the higher the price multiplier ω_p, the higher the percentage of agreements ending with abandonment.

For mattresses, the extra rental revenue and higher purchase prices outweigh the loss from extra abandonments, and the profit rate rises as prices rise in Table 6. So, the firm may benefit from increasing mattress buyout prices. However, altering washer and dryer buyout prices could hurt the profit rate.

For profit rate, an additional relevant factor is the *idle time*, i.e., how long a returned item sits in inventory before being rented again. Altering prices could affect the agreement initiation rate and in turn the idle time. Ideally, the profit rate calculation would include the average idle time; unfortunately, our data lacks this information. We leave a full study of an RTO firm's pricing strategy for future work.

Our second counterfactual analysis sheds light on the impact on the revenue flow of influencing the renter's purchase consideration probability. The firm cannot influence the life events which determine the abandoning status probability $1-\rho_p-\rho_r$. However, it can possibly influence ρ_p (and ρ_r by the same amount in the opposite direction). For example, the firm can induce an increase in ρ_p by contacting renters to remind them of the buyout or by emphasizing this option in the store. Alternatively, the firm could downplay the buyout option (inducing a decrease in ρ_p) to collect more rental revenue before a possible purchase. As with the price path analysis, we replicate each agreement 100 times, where for each replication we first draw the utility segment and then determine the outcome under three different sets of status probabilities: the benchmark case of statuses realized with the estimated pair (ρ_r, ρ_p), the case of emphasized buyout option with status realizations according to $(\rho_r - 0.08, \rho_p + 0.08)$ and finally the case of downplayed buyout option, where statuses are realized based on $(\rho_r - 0.08, \rho_p + 0.08)$. We couple segment realizations across different status probability cases. Table 7 reports the results.

From Table 7, higher considering status probabilities lead to higher purchase prices, more purchases

Table 7: Statistics under each ρ_p. Term and monetary values are in weeks and dollars.

		Percentages of agreement outcomes		Averages over purchase agreements		Averages over all agreements
Product	ρ_p	abandon	purchase	term	purchase price	profit rate
Washer	0.04	71.7%	14.2%	63.9	177.0	12.28
	0.12	67.8%	24.6%	61.9	187.0	12.60
	0.20	67.1%	26.2%	61.7	188.9	12.65
Dryer	0.04	72.1%	13.4%	65.4	166.5	11.09
	0.12	68.5%	23.6%	62.8	172.5	11.33
	0.20	67.9%	25.1%	62.5	173.7	11.37
Mattress	0.36	54.4%	26.3%	46.4	106.3	15.35
	0.44	53.5%	27.7%	46.2	111.1	15.28
	0.52	53.2%	28.1%	46.2	113.3	15.25

and fewer abandonments, but also less rental revenue before a purchase because of fewer purchase rejections. As such, emphasizing the buyout option improves the overall purchase revenue but hurts the rental revenue. This creates a tradeoff, and the improving direction is different for different products. Washer/dryer renters are very inattentive to purchase offers (estimated considering status probability of 12%), so local changes in attentiveness have a significant effect on the number of abandonments and purchases (e.g., for washers, an increase from 4% to 12% considering status probability increases the number of purchases from 14.2% to 24.6%). The increase in purchases (and reduction in abandonments) with higher attentiveness outweighs the decreased rental revenue before a potential purchase. So, as seen in the rightmost column of the table, the firm could potentially improve its profit rate by emphasizing the purchase option to washer and dryer renters to increase attentiveness above the estimated 12%. For mattresses, on the other hand, we see that the firm's profit rate is decreasing in attentiveness around the estimated value of 44% considering status probability. Because mattress renters are more attentive to purchase offers to start with, local changes have little effect on the overall number of abandonments and purchases: roughly a single percentage point change from one row to the next in the table. However, changes in attentiveness do affect how *many* purchase offers are rejected before a purchase or abandonment. Higher attentiveness yields fewer rejected offers, which in turn entail earlier purchases and higher average purchase prices, since prices tend to decrease during the agreement. Earlier purchases mean less rental revenue before the purchase, and for mattress renters, this effect dominates, leading to the decreasing profit rate observed in the table. Relative to its current communication strategy, our firm would therefore be advised to downplay the buyout option with its mattress renters.

6 Discussion and Managerial Suggestions

Distinct from standard retailing, the RTO business and its renters have unique features that require a new approach and framework to model them faithfully. First, RTO renters may not always have the financial and/or mental capability to consider purchase offers at every opportunity. Second, their need or ability to rent the item may change suddenly, causing them to cease the rental prematurely.

These two features relate to a renter's *status* at a given time. Third, each renter provides multiple ownership utility signals over the course of his rental, corresponding to the periods when he considers the purchase offers. The rejections (acceptances) of buyout offers reflect economic factors, namely renters' utilities and their decision-making process, the latter of which may take various forms depending on renters' sophistication and alertness. Fourth, RTO transactions are frequently conducted offline and in cash, and agreements are initiated without third-party credit reports. Thus, RTO firms often must operate in a *data-poor* context, with few if any explanatory variables to characterize individual renters. Appropriately capturing these features is a prerequisite for accurate utility estimation of RTO renters, but existing approaches in the literature, typically aimed at traditional retailing, do not fit the RTO business. In this work, we develop a novel, modular estimation framework that is tailored to the RTO business but which also introduces new features that could be adapted to other applications. We then propose and analyze several renter decision models and test their estimation performance under our framework using real data. Our results provide managers with insights that characterize the population of RTO renters. An RTO firm could leverage our approach and findings to improve pricing decisions, including a transition from the traditional cost-based pricing to a more lucrative value-based pricing.

We segment renters based on their (rental and ownership) utilities and estimate the probability of each segment. Importantly, our approach cleanly separates utility estimation from solving a renter's decision problem. This separation is possible because the likelihood under our new framework can be expressed solely in terms of *buy sets*, which represent the periods in which the renter would choose to purchase if able. The notion of buy sets is novel; in addition to its value for the RTO setting, we believe that it could be fruitfully applied in other estimation contexts in which a customer is offered multiple opportunities to purchase the same product at different times and may neglect some of these (e.g., email offers for the same or similar products sent periodically to a retailer's subscriber list). By leveraging the buy set formulation of the likelihood, estimating the utility under a particular renter decision model is as straightforward as computing the buy sets for each renter segment and then maximizing the likelihood in terms of the segment probabilities; the likelihood maximization is completely agnostic to the particular decision model that generated the buy sets. Furthermore, although buy set computation may require solving DPs, our approach has a significant computational advantage over related procedures like nested fixed point because in our case the DPs are solved only once for each renter segment, at the start of estimation. In addition, the modularity of our framework allows us to study various renter decision models without affecting the core estimation procedure.

Under our framework, a renter's status in each period of his agreement is realized from three possibilities –abandoning the rental, neglecting the purchase option, or considering the purchase option. The neglecting status plays a dual role in our framework. First, the actions of some renters are inconsistent with their actively contemplating all of the buyout offers, so the model would not be saturated if we assumed that renters always consider purchase offers. Inclusion of the neglecting status saturates the model by allowing for the rejection of a buyout price to be attributed not to an active decision on the renter's part but rather a passive neglect of the buyout offer, perhaps because the renter lacked the mental bandwidth or financial means to contemplate a purchase at the time. Second, the neglecting status probability reflects an important characteristic of RTO renters. Specifically, it measures how likely renters are not to contemplate a given buyout offer at all, which is clearly relevant for RTO buyout pricing. Thus, like the mean-zero noise employed in Rust's framework (Rust 1994, p.3103), the neglecting status is necessary for saturation, but in our case, the modeling tool that provides saturation also leads to an estimation output with a concrete interpretation. As with buy sets, we hope that other researchers may find this concept useful to adapt to and build on in other contexts.

We consider multiple renter decision-making processes, representing different degrees of sophistication. Renters can be strategic, one-step-look-ahead or myopic in their decision making. Strategic renters compare their current purchase surplus against all future purchase surpluses, one-step-look-ahead renters compare this surplus only with the two-period expected surplus, and myopic renters consider only the current period. In addition, we distinguish renters by the probability that they consider each purchase offer; alert renters always consider purchase offers unless in the abandoning status, whereas inert renters neglect them with positive probability. Different combinations of the decision-making processes and probability of considering a purchase yield the SAR, SOR, SIR, SIOR, MAR and MIR models.

We estimate rental and ownership utilities using each of the models over three different data sets pertaining to washers, dryers and mattresses. Out of all the models and across all data sets, the SIOR model (corresponding to one-step-look-ahead and inert renters) fits the best, with the MIR model a close second. The superior performance of these models suggests that RTO renters do not consider the full future of their agreements in their decision making. Possibly, the difficulty of anticipating buyout prices far in the future may hinder careful evaluation of all upcoming purchase opportunities, leading the renter to opt instead for a limited-horizon comparison.

For both washer and dryer renters, the SIOR model estimates 86% for the neglecting status probability and 12% for that of the considering status. These estimates imply 2% as the estimate for abandoning status probability. The high estimate for neglecting status probability for washers and dryers indicates that many buyout price rejections stem from neglecting the purchase option, rather than low ownership utility or a strategic decision to postpone a purchase. Thus, if the firm wishes to ensure that the renter considers a given price at least once, then it might choose to offer this price in several periods.

Additionally, given the low estimate for the abandoning status probability, the firm can experiment with aggressive buyout prices with limited risk; if the renter has high enough ownership utility, he may accept the offer and if not, the firm most likely will have future opportunities to offer moderate prices.

In contrast to renters of washers and those of dryers, the estimate for neglecting status probability of mattress renters is 44%, and the estimate for considering status is 54%. The higher purchase consideration probability of mattress renters could be due to lack of an alternative for a mattress; in the case of a washer or a dryer, on the other hand, an alternative could be to use a public laundromat.

The mean of estimated rental utility for mattress renters is $13.4, higher than for both washer and dryer renters: $6.8 and $7.2, respectively. The higher expected rental utility can relate to higher usage frequency for mattresses, which may increase awareness of the product and, by extension, of the purchase opportunities for it. For each product, the mean of estimated rental utility is less than the average rental fee (see Table 2). This suggests that renters are willing to "overpay" in rental fees, hoping to gain ownership by a purchase during their agreement or at the end of the term via a payoff. Interestingly, we observe minimal correlation between ownership and rental utilities in each product category. A plausible explanation for this lack of correlation could be a high degree of uncertainty for the firm regarding the duration that a renter plans to use the product after purchase.

The mean of estimated ownership utilities for washer, dryer and mattress, respectively, are $171, $149.2 and $80.3. The average mattress purchase prices are lower than those of washers and those of dryers, which leads to lower utility estimates. Being more likely to consider purchase offers, mattress renters have more incentive to postpone purchases, which can lead to their paying lower purchase prices. The ownership utility distribution of our washer renters can be approximated fairly closely by a mixture of two distributions; a mass at the lowest utility level and a uniform distribution over segments with larger utilities. The mass could represent renters with short-term needs for the product, so they do not seek ownership. A similar approximation can be applied to the utility distribution of dryer renters. Indeed, the estimation results for washers and dryers are close enough that they could be plausibly considered as a single product category for estimation.

Our counterfactual analyses reveal that altering washer and dryer buyout price paths might not benefit our firm. For mattresses, however, higher prices could boost profits. In addition, our firm should emphasize the buyout more for washers and dryers to induce more purchases but downplay it for mattresses to generate more rental revenue before purchases.

For each product category, greater than 90% of the population is estimated to have ownership utility less than $350. Thus, a buyout price above this amount is unlikely to be accepted. On the other hand, roughly 50% of renters are estimated to have utility no less than $75. Given our renters' near-myopic behavior, the manager may delay dropping the price below \$75. Such offers have a good chance of acceptance, so if, for example, the manager targets a total rental revenue before ceding ownership, then she would be advised to wait until her target is reached before reducing the price below this threshold.

Our framework is versatile and efficient but is not without limitations; for instance, as with other finite-mixture models, the computation can be intractable with many segments. Nonetheless, we believe that our framework and models appropriately trade off model fidelity and computational efficiency.

To summarize, we build a completely new, parsimonious and modular framework for the little-understood RTO business. The framework borrows features from dynamic structural estimation approaches, while also introducing elements tailored to the business but with the potential for broader applicability. We employ this framework and our proposed renter decision models on real-world data. Our findings demystify the utilities and sophistication of renters, and in addition to our work's readily apparent value for RTO firms, we hope that it will inspire further research in this exciting area.

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Appendix A Proofs

This section is available from the authors upon request.

Appendix B Supplementary Discussions

Connection to Rust's Framework

We briefly discuss the connection of our models (using the SAR model as a prototype) to other dynamic structural estimation models. For comparison with these models, the SAR DPs for individual segments can be combined to obtain a single DP for all segments. This single DP includes in its state the random utility that is realized in the beginning of an agreement and observed by the renter but remains unobserved for the firm. The firm observes only the event of this renter's abandonment or purchase, either of which ends the agreement, so the observed state of the renter from the firm's perspective is a binary variable reflecting whether the agreement has yet terminated. In the single DP, the purchase surplus is $u_k - p_t$ in period t, where u_k is the utility of a renter in segment k. This surplus can be separated into two pieces: an observed-state dependent part and an unobserved-state dependent part. The utility u_k cannot be observed-state dependent, as the observed state does not distinguish segments. By the process of elimination, it must depend on the unobserved state. So, we can satisfy the additive-separability assumption of Rust (1994) only by incorporating u_k into the unobserved-state dependent part of the surplus. Rust (on p.3103) makes a conditional independence assumption with the following implication: "any serial dependence between [unobserved states of t and $t+1$] is transmitted entirely through the observed state [of $t+1$]." In our context, the utilities in different periods are in the unobserved state and perfectly dependent (indeed, the same), but the observed state is binary and so insufficient to transmit this dependence across periods. Therefore, our utilities fail the conditional independence assumption. Speaking at a high level, these utilities model heterogeneity among renters and remain unobserved, thus infecting the SAR model with *permanent unobserved heterogeneity*, which is one of the four departure avenues from Rust's framework (Aguirregabiria and Mira 2010, p.42). This has motivated our use –in the SAR model and in the other renter decision models that we study– of a finite mixture modeling approach to represent heterogeneous utilities of renter segments.

Strategic and Optimistic Renter (SOR) Model

Here we discuss a strategic renter who optimistically believes he is alert to purchase offers but in reality is not. Specifically, given $\rho > 0$, this renter's buy set (generated from the solution to the DP in (5) from Proposition 1) is identical to that of a SAR with the same utilities whose status is considering wp ρ and abandoning wp $1-\rho$; however, his true status probabilities are considering wp $\rho_p < \rho$, neglecting wp $\rho_r = \rho - \rho_p$ and abandoning wp $1-\rho$, so in reality he possibly does not consider purchase offers in some periods. We refer to such a renter as a *strategic* and *optimistic renter (SOR)*.

A SOR's optimism creates an inconsistency between his plan and reality. Unlike a SAR, a SOR's status is neglecting with positive probability. However, like a SAR, a SOR initially targets the first period in his (identical) buy set for a purchase. If he does not abandon before then and his status in this period is realized as considering, then he purchases the item, just as a SAR would. But if his status is realized as neglecting (which would have zero probability for a SAR), a SOR instead continues the rental and revises his target purchase period to the chronologically next period in his buy set.

This process continues until he either abandons the rental, purchases the item because his status is considering in a period in his buy set, or owns it through payoff.

Data Preparation

Here, we describe the steps required to prepare the raw transaction data for use in our estimation framework. For each agreement, this includes each period's remitted rental fee payment, the term, the rental fee and the accepted buyout price offers. To calculate rejected washer and dryer buyout prices, the firm provided us with their buyout price formula. The formula used for mattresses is the same, although the parameters may vary slightly in the initial several months of the agreement. We use the same parameters in this formula for all products. Additionally, missed payments, catch-up payments and early payments mean the actual payment in a period may differ from the rental fee. A few renters in our data are behind on their payments and continue renting beyond their agreements' terms. For each such renter, we extend the term to cover the actual duration. To track agreement i with term T_i, we use the actual amounts $\tilde{r}_t^i$ and $\tilde{p}_t^i$ paid by the renter in period t for rental and purchase, respectively ($\tilde{p}_t^i = 0$ in the case of rejection). We calculate a *rental balance* for each agreement in each period t after $\tilde{r}_t^i$ is paid by the renter (if he makes a payment, while otherwise a payment of zero is recorded). The rental balance is $R_t^i = \sum_{j=1}^t \tilde{r}_j^i - r^i t$ at the end of period t. A positive (negative) rental balance indicates a renter who is ahead of (behind) the payment schedule.

To assign an (renter) action to each period $t \in [1 : T_i - 1]$ of agreement i, we first determine whether it is terminated in the period by a purchase or an abandonment. The purchase action is assigned to a period in which the buyout price is paid. The abandon action is assigned to the first period with a negative rental balance after which no payment is recorded. Formally, period $t \in [1 : T_i - 1]$ is assigned

$$\text{(i) Purchase action if } \tilde{p}_t^i > 0 \quad \text{or} \quad \text{(ii) Abandon action if } R_t^i < 0 \text{ and } \tilde{p}_\tau^i, \tilde{r}_\tau^i = 0 \ \text{ for } \tau \geq t.$$

If condition (i) or (ii) is satisfied, then the agreement is marked as concluded in period t with a purchase action or an abandon action. If neither condition is satisfied, we assign a continue action to period t.

In the final period T_i, both purchase and continue actions entail equal payments ($p_T^i = r^i$) and lead to product ownership. As such, a renter's recorded payment, i.e., $\tilde{p}_{T_i}^i > 0$ or $\tilde{r}_{T_i}^i > 0$, does not reveal his actual intention regarding purchase or continue action. Thus for period T_i, we first check condition (ii). If (ii) is satisfied, then we assign abandon action to this period. Otherwise, the agreement is marked as a payoff agreement but the action of the final period remains indistinguishable, which is captured by our likelihood (2). Regardless, choosing to assign purchase actions (or continue actions) to all these final periods has negligible impact on estimation with our data.

Appendix C Derivations and Technical Details

C.1 Sequence of Events and Actions for a SAR

Figure 3 is the specialized version of Figure 1 for a SAR.

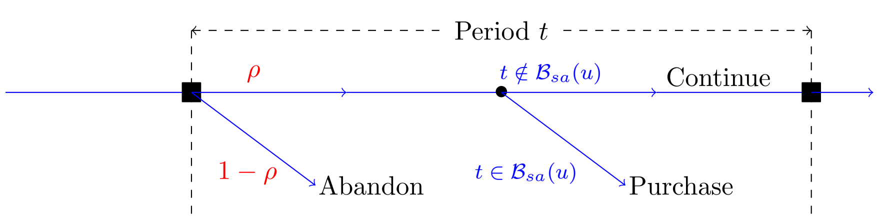

Figure 3: Sequence of events and actions for a SAR with utility u in period $t < T$.

C.2 Equivalence of a SAR to a SIR with $\rho_r = 0$

By setting $\rho_r = 0$ in (9), we have $\rho = \rho_p$, and (9) becomes

$$v_t \quad = \quad \rho \max\{u - p_t, -s + v_{t+1}\} \;\text{ for }\; t \in [1 : T-1], v_T = \rho(u - p_T). \tag{11}$$

We show that a renter's decisions (continue or purchase) implied by (11) are identical to the ones implied by (5) in any period t. To avoid confusion, we represent the value-to-go function in (5) with the letter z. Letting $\hat{z}_t = \rho z_t$, we can re-write (5) as

$$\hat{z}_t = \rho \max\{u - p_t, -s + \hat{z}_{t+1}\} \;\text{ for }\; t \in [1 : T-1], \hat{z}_T = \rho(u - p_T). \tag{12}$$

The renter's decisions resulting from (12) is identical to the one resulting from (5) because the components inside the maximum in (12) are equal to the ones inside the maximum in (5) in any period t. Moreover, (12) is identical to (11) and therefore both induce identical renter's decision in any period t.

C.3 Sequence of Events and Actions for a MAR

The figure below illustrates event and actions for a MAR.

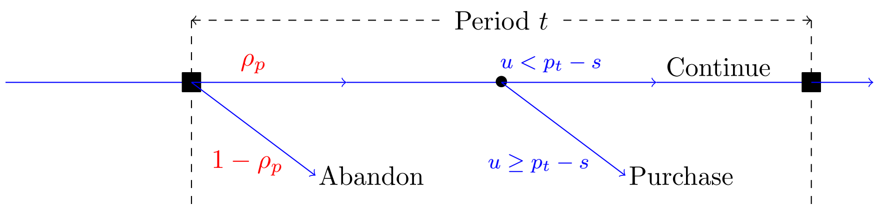

Figure 4: Sequence of events and actions for a MAR with utility u in period $t < T$.