Managing Relationships Between Restaurants and Food Delivery Platforms: Conflict, Contracts, and Coordination

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Abstract

In the restaurant industry, delivery platforms collect customer orders via mobile apps or the web, transmit them to restaurants, and deliver the orders to customers. Platforms claim they provide value to restaurants by expanding their markets, but critics claim they destroy restaurant profitability by taking a percentage of revenues from delivery orders and generating congestion that negatively impacts dine-in customers. We consider these tensions using a stylized model of a restaurant as a congested service system, with a goal of understanding how best to structure relationships between restaurants and platforms to maximize joint profits. First, we analyze the efficacy of the most commonly used contracts, “simple revenue sharing,” in which the platform takes a percentage cut of each delivery order. We show that such contracts fail to coordinate the system because the platform does not internalize the effect of its price on dine-in revenues, setting its price too low and attracting many delivery customers who generate congestion for dine-in customers; the restaurant is forced to respond to this by setting dine-in prices that are too low, which in turn negatively impacts restaurant revenues compared to a centralized system in which both prices are set to maximize joint revenue. We then investigate several proposed mechanisms to improve the relationship between restaurants and platforms and coordinate the system. We show that commission caps, which have been legally mandated in some US states and localities, are ineffective at coordinating the system because they do not solve the structural problem with simple revenue sharing contracts. Allowing the restaurant to set price floors for the delivery platform is similarly incapable of coordination, generating double marginalization that leads to prices that are too high relative to the central optimum. Finally, we propose an alternative, practical coordinating contract that is a variation of simple revenue sharing: for each delivery order, the platform pays the restaurant a fixed, flat fee and a percentage revenue share. For appropriately chosen contract parameters, we show that this contract coordinates the system, protects restaurant margins by ensuring a lower bound on its revenue per delivery order, and allocates revenue with a high degree of flexibility between the restaurant and the platform.

Keywords: on-demand services, delivery platforms, supply chain coordination

1 Introduction

Delivery platforms such as Grubhub, DoorDash, Postmates, and UberEats are a large and rapidly growing segment of the restaurant industry. These platforms are third-party service providers connecting customers and restaurants: they typically combine a convenient and easy-to-use web-based ordering system with the capability to pick up orders from many different restaurants and physically deliver those orders to customers. For customers, delivery platforms generate value by expanding their options for at-home restaurant consumption, conveniently offering delivery via one website or smartphone app from dozens or hundreds of restaurants, many of whom, in the absence of the platform, did not offer any delivery service. For restaurants, delivery platforms generate value by allowing them to enter the delivery market without building their own...
ordering platform or investing in their own delivery drivers, expanding their customer base with minimal
up-front investment and little to no increase in their fixed operating costs. Because of this, platforms often
pitch themselves to restaurants as a source of marketing services to capture new customers and generate
“incremental” revenue, i.e., additional profit on top of their existing dine-in revenue (Meyersohn, 2018). Al-
ready growing at a rapid pace, these platforms became increasingly critical to the restaurant industry during
the COVID-19 pandemic as dine-in service was prohibited or placed under severe capacity limits; even after
the pandemic, it is expected that delivery will retain a more prominent place in the industry than it once
did (Morgan Stanley, 2020).

Despite their seeming advantages, delivery platforms are controversial with restaurant owners (Houck,
2017; Dunn, 2018; Meyersohn, 2018). Among the potential pitfalls of partnering with delivery platforms,
two issues in particular stand out. The first is that the standard contractual relationship offered by most
platforms—including Grubhub, DoorDash, Postmates, and UberEats, the largest delivery services in the
US—resembles simple revenue sharing: revenue on each order is split between the platform and restaurant
according to some pre-negotiated rate. The platform’s share of the revenue, also called a commission, is
typically around 15-30% (Restaurant Engine, 2021), leaving the restaurant with only 70-85% of its normal
revenue on each item sold.1 Given the already thin margins in the restaurant industry, which average 4-5%
or less (Lunden, 2020), relinquishing any share to a third party can drastically reduce—or eliminate—the
profitability of an order.

The second key issue is that a large volume of delivery orders may place a strain on restaurant opera-
tions, adding complexity for staff and deteriorating service for dine-in customers, and possibly harming
the restaurant’s reputation. Restaurant staff are faced with the daunting task of juggling orders from multiple
customer streams, such as dine-in, take-out, and a stream for each delivery platform that the restaurant
supports. Incremental orders coming from the platform can place greater demand on the kitchen—a shared
resource that serves both dine-in and delivery customers—and can lead to increased errors, longer wait times
for customers, and an overall worse impression of service (Buell, 2017; Houck, 2017). In the long-run, this
can cause even loyal dine-in customers to switch to delivery (where, as noted above, they generate less profit
for the restaurant) or simply stop patronizing the restaurant altogether.

The combination of these issues means that not only do incremental orders generated by the delivery
platform earn a lower profit rate than orders generated by dine-in customers, but they may deteriorate
service for dine-in customers, leading to fewer (profitable) dine-in orders and more (unprofitable) delivery
orders. This illustrates a key source of conflict in the relationship between platforms and restaurants, one

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1 Precise terms of contracts between restaurants and platforms are not usually publicly disclosed; however, Restaurant Engine
(2021) describes typical commission rates around 30% for each of the four major US platforms (Grubhub, DoorDash, Postmates,
and UberEats), although this can vary depending on options chosen by the restaurant (e.g., how prominently to display the
restaurant in search results) as well as the restaurant’s negotiating power. More powerful restaurants—such as large national
chains—typically have better negotiating power and enjoy lower commissions, while most small and independent restaurants
pay the default commission rate of ~30%. Besides taking a share of sales revenue, platforms also earn money by charging
delivery fees to customers, charging monthly rental fees to restaurants for in-store tablet and receipt printer hardware, and in
some cases, charging an “activation fee” the first time a restaurant is listed on the platform’s website.
that can lead to a downward spiral for restaurants consisting of both increasing demand and decreasing profit margins. As the owners of two popular New York restaurants put it, “We know for a fact that as delivery increases, our profitability decreases,” and as a result, “sometimes it seems like we’re making food to make [Grubhub subsidiary] Seamless profitable” (Dunn, 2018).

In a case study, Buell (2017) describes a vivid example of precisely this effect: The Paramount, a popular Boston restaurant, finds its restaurant operations out-of-balance after partnering with a delivery platform. This causes worse service (longer waits for food) for dine-in customers, which in turn leads dine-in customers to switch to the delivery platform to avoid the hassle of waiting for their orders at the restaurant, or to take their business elsewhere. This creates a vicious cycle in which a larger and larger fraction of customers come from the platform, further deteriorating service for dine-in customers, and ultimately reduces restaurant profitability since the platform takes a significant share of the revenue from delivery orders; in turn, this has led some restaurants to limit, or even eliminate, their partnerships with delivery platforms (Houck, 2017).

In this paper, we investigate precisely these issues. Our chief goal is to understand how to resolve this potential source of conflict between restaurants and platforms via the use of appropriate contracts that align incentives and maximize the joint revenue of platforms and restaurants, giving customers a service that they value while avoiding the negative effects described above. To accomplish this, we model a restaurant as a congested service system. The restaurant serves two separate channels of demand: dine-in customers and delivery customers. Potential customers for each channel decide whether to purchase from that channel or to exercise an outside option, whose value is normalized to zero. The restaurant has full power over the price in the dine-in channel, while the platform determines the price in the delivery channel. Reflecting the issues described above, the platform keeps a share of the delivery revenue, and incremental orders from the delivery channel generate congestion that is costly for dine-in customers.

We begin by studying the performance of the type of revenue sharing contract that currently prevails in the industry, i.e., a “simple” contract in which revenue is split according to some predetermined rule with no additional transfer payments. We show that these contracts give both the platform and restaurant incentive to set their prices in a way that ignores or even exacerbates the negative interactions between channels. The platform earns money only from deliveries: as a result, it will set its price to maximize the revenues it receives from the delivery channel, ignoring any potential negative impact that the delivery channel may have on dine-in revenue. Consequently, it sets the delivery price too low, attracting more delivery customers than optimal and pushing some congestion-sensitive dine-in customers out of the market. Meanwhile, the restaurant must set the dine-in price lower than centrally optimal to compensate for this effect, eroding margins further on the remaining dine-in customers. Hence, the price in each channel is lower than the centrally optimal price, and margins for both the platform and the restaurant suffer as a result.

We then investigate several potential mechanisms for mitigating these issues and improving the performance of the system: commission caps, price floors, and more advanced contracts. The first mechanism is motivated by recent laws enacted by some state and local governments which “cap” the maximum commission
rate that platforms can charge, often at a much lower level than the prevailing rate (e.g., 15% instead of 30%, Lucas, 2020). We show that, unfortunately, even with a commission cap, the system remains uncoordinated: the form of the contract is still simple revenue sharing, so the platform still ignores the congestion effect of its customers on the dine-in revenue. Commission caps thus help restaurants by allocating them a larger percentage of the system’s profit, but serve as a mere “band-aid” on an inadequate contract, failing to fix the underlying structural issues with simple revenue sharing.

We next consider a modified version of simple revenue sharing in which the restaurant is allowed to set a “price floor” in the delivery channel. Some platforms give menu pricing power to restaurants but retain the power to set customer delivery and service fees; in effect, this allows the restaurant to set a “minimum” price for an order, with the platform potentially raising the final price above this floor. Mathematically, this allows the restaurant to set two different prices: a dine-in price paid by customers and a separate, higher price for delivery orders (which the platform may raise further still). Since simple revenue sharing results in prices that are too low in both channels, ideally, a price floor would yield increased prices that are closer to the central optimum. However, we show that while price floors do allow the restaurant to protect its margins, they can actually be too effective at increasing prices: the equilibrium prices on both channels are not only higher than under simple revenue sharing, but can be even higher than centrally optimal. This means that while margins are higher for both the restaurant and the platform, demand is too low, leaving money on the table for both firms.

Given that these common modifications to simple revenue sharing are ineffective at coordinating the system, we then investigate more advanced contracts. First, we note that a “two-way” revenue sharing contract, in which the platform shares delivery revenue with the restaurant and the restaurant shares dine-in revenue with the platform, coordinates the system and can allocate revenues arbitrarily between the two parties; however, restaurants are unlikely to be willing to share dine-in revenues with the platform, making such a contract impractical. We then propose a more practical coordinating contract, called generalized revenue sharing. Under this contract, for each delivery order, the platform pays the restaurant both a percentage revenue share and a fixed fee; this resembles the type of general revenue sharing contract described in a supply chain context in Cachon & Lariviere (2005), but is novel in the restaurant delivery platform setting. With appropriately chosen parameters, such a contract induces the restaurant to maximize dine-in revenues, while the platform maximizes the incremental revenue generated by delivery, both of which align with the central optimum. It can also achieve a wide (but not total) range of allocations of the delivery revenue between the parties. Additionally, our proposed contract ensures that for each delivery order, the restaurant receives some minimum revenue level which, under certain conditions, is at least as high as the restaurant’s revenue on each dine-in order; thus, generalized revenue sharing helps protect restaurant margins while also increasing aggregate revenue in a way that neither commission caps nor price floors can accomplish.

In sum, our results show that the predominant contract used in practice (simple revenue sharing) is sub-
optimal, hurting restaurants and platforms alike by leaving money on the table compared to a coordinating contract. The inefficiency of this contract type is derived from its inherent inability to force the platform to account for the negative externality it generates on the dine-in channel. Simple adjustments to this type of contract (commission caps and price floors) can help increase restaurant revenue, but cannot correct the structural issues with simple revenue sharing nor achieve the central optimum. By contrast, our proposed contract with a flat fee and a percentage revenue share—which resembles a two-part tariff paid by the platform on each order—alleviates the key sources of conflict between restaurants and platforms, maximizing aggregate revenue and coordinating the system in a way that is relatively straightforward to implement in practice.

2 Related Literature

The literature on food delivery platforms is nascent but growing rapidly. A number of recent papers study operational issues faced by delivery platforms. For instance, Mao et al. (2019) empirically analyze the impact of faster delivery times on customer retention in food delivery platforms, while Liu et al. (2020) study how to assign orders to delivery drivers to achieve on-time delivery. Our focus is on the relationship between a restaurant and a food delivery platform. A few papers in the literature study elements of this relationship. Li & Wang (2021b) empirically evaluate the effect of delivery platforms on restaurants. They also compare the impact on chain restaurants versus independent restaurants, finding that chain restaurants tend to benefit more from platforms than independent restaurants. Oh et al. (2019) study a game-theoretic setting in which multiple restaurants compete for attention on the platform. They consider “sponsored display,” in which restaurants pay to be highlighted by the platform, and “sponsored delivery,” in which restaurants bid commissions that they are willing to pay and the restaurant chooses endogenously which to highlight. They find that the platform can benefit by offering sponsored display and sponsored delivery simultaneously and that many restaurants also benefit from this arrangement.

The closest work to ours in this stream is Chen et al. (2020), who also study contracts between a restaurant and a platform. Their model is an unobservable queueing system in which some customers are “tech savvy” and choose between delivery and dine-in, while others consider only dine-in. Like us, they find that the platform can hurt restaurant profitability, and they find that some contracts perform better than others. They also show that a two-way revenue sharing contract coordinates the system; this commonality is not surprising in that two-way revenue sharing results in both parties’ revenues being proportional to the aggregate revenue. Our results diverge, however, in what other types of contracts coordinate the system. They find that a simple revenue sharing contract in which the restaurant sets a price ceiling for the platform can coordinate. By contrast, in our case, a simple revenue sharing contract does not coordinate. Instead, we propose a new coordinating contract called generalized revenue sharing, in which the platform pays to the restaurant both a flat fee and a percentage revenue share on every order, which coordinates the system.
and has several other desirable properties.

Our work is also related to the broader supply chain contracting literature, in particular work on revenue sharing contracts. Cachon & Lariviere (2005), for example, show that a revenue sharing contract where a retailer shares some of its revenue with its supplier can coordinate the supply chain and assign arbitrary fractions of revenue to each party. See Cachon (2003) and Lariviere (2015) for reviews and perspectives on the supply chain contracting literature. Our work differs from this literature because of the existence of dual sales channels—the delivery channel and the dine-in channel—and the interaction between them.\(^2\)

In addition, our work shares some features with the literature on supplier encroachment. Arya et al. (2007), for example, find that a manufacturer adding a direct channel to sell to customers can benefit both the manufacturer and the retailer in the right circumstances. More recent work in this area (see, e.g., Li et al., 2013, Ha et al., 2015) investigates conditions under which encroachment benefits and hurts each party, and David & Adida (2015) investigate contracts to coordinate a supply chain with encroachment. However, this literature does not account for congestion effects and also differs from ours in who is doing the encroaching. With supplier encroachment, the status quo is for the retailer to be the only party directly selling to customers, while the manufacturer sells only through the retailer. In our setting it is the opposite: the status quo involves the “manufacturer” (restaurant) selling directly to customers, which is then disrupted by the “retailer” (platform) entering the market and encroaching on the manufacturer’s territory.

Finally, our paper is related to the emerging stream of literature on pricing and marketplace innovation. Recent work in this stream mainly focuses on ride-hailing platforms and the relationship between platforms and independent workers, in many cases allowing for strategic consumers (e.g., Banerjee et al., 2016; Cachon et al., 2017; Jacob & Roet-Green, 2021; Afèche et al., 2018; Bai et al., 2019; Benjaafar et al., 2021; Benjaafar et al., 2019; Bimpikis et al., 2019; Taylor, 2018). While our work shares some similarities with this literature in context and modeling choices, we abstract away from individual workers and instead focus on the relationship between the established provider (“restaurant”) and the disruptive firm (“platform”).

3 Model

A restaurant provides service via two channels: a delivery channel that serves customers via a third-party delivery platform, and a dine-in channel that serves customers directly at the restaurant. We next describe the firms that operate each of these channels (§3.1) and the customer populations that visit each channel (§3.2), then provide preliminary results on the optimal customer purchasing decisions (§3.3).

3.1 The Restaurant and the Platform

For simplicity, we assume the restaurant sells a single identical item to each customer seeking service regardless of channel. The restaurant has full control over the price in the dine-in channel, \(p\), and sets that

\(^2\)Further discussion of the relationship between the results in Cachon & Lariviere (2005) and our model is contained in §6.4.
price to maximize its revenue. All revenue from the dine-in channel goes directly to the restaurant. The platform has full control over the price in the delivery channel, \( \theta \), and sets that price to maximize its revenue. Revenue from the delivery channel is split amongst the platform and restaurant according to the terms of their contractual relationship, and the precise form and terms of these contracts may take various forms as described in §§4-6.

The price \( \theta \) is the “all-in” price for delivery channel customers, including the price of the food as well as any service or delivery fees. In practice, the delivery channel price can be set either entirely by the platform or by a combination of the restaurant and the platform (e.g., the restaurant may set the menu price of the food while the platform may set service and delivery fees). For most of our analysis, we consider the former scenario, i.e., the platform has full pricing power in the delivery channel; this is reasonable because, in many cases, even if the restaurant sets the menu price of the food, the platform has the power to both add fees and offer discounts and promotions that could effectively give it full pricing power.\(^3\) However, in §6.2, we consider the latter scenario, where the restaurant sets the price of the food in the delivery channel, effectively establishing a “price floor” on top of which the platform may raise the price further by adding service and delivery fees.

Motivated by practice at most restaurants, both channels are served via a shared kitchen; hence, demand from either channel generates congestion in this shared resource, which can in turn lead to delays and poor service as described in the introduction. The demand placed on the kitchen is thus the sum of demands from each of the channels. We ignore congestion or delays generated by either the dining room capacity (for the dine-in channel) or delivery capacity (for the delivery channel). Each of these sources of congestion only affects a single channel at a time, and therefore has straightforward consequences on system performance; we focus instead on the more interesting shared step in the process that affects both channels simultaneously, i.e., the kitchen.

We also assume that the capacity of the kitchen is fixed and exogenous, meaning congestion is an increasing function of the total volume of customers that patronize the restaurant from either channel (more on the functional form of this congestion term will be discussed below). We make the assumption of fixed capacity because, in practice, during peak hours (i.e., lunch and dinner) restaurants are often fully staffed and kitchen capacity is limited by physical space; in the short- or medium-term, it will be difficult to expand capacity further. Moreover, even if staffing is a bottleneck in the kitchen, hiring quality kitchen staff at restaurants can be costly, slow, and challenging (Deliso, 2021). Lastly, as we will show, the presence of the delivery channel reduces the restaurant’s margins; it can be challenging to justify capacity expansion as margins deteriorate. Hence, a first order concern will be to coordinate the system (i.e., maximize restaurant and platform profits via contract design) and protect the restaurant’s margins, after which capacity expansion might be considered.

\(^3\)For example, if the restaurant sets a price of $10 for its food, the platform can set a service or delivery fee to raise the final price above $10, or can offer a promotional discount (e.g., 25% off, $5 off, free delivery, etc.) to lower the final price below $10.
3.2 Customer Segments

The customer population has total size normalized to one and consists of two customer types. *Dine-in* customers only consider using the dine-in restaurant channel (i.e., they ignore the delivery channel). These customers comprise a fraction $\alpha$ of the total market, and are denoted by the subscript $R$ for “restaurant” customers. *Delivery* customers only consider using the delivery channel (i.e., they ignore the dine-in restaurant channel). These customers comprise a fraction $1 - \alpha$ of the total market, are denoted by the subscript $H$ for “home,” since their consumption (usually) occurs at their homes. We assume $0 < \alpha < 1$ to avoid trivial cases where there is only one customer type and hence all traffic flows through a single channel. Upon arrival, customers choose whether to purchase via their desired channel, or not to purchase at all; that is, customers do not consider switching between channels. Customer types are thus exogenous and derive from an earlier stage customer decision process, reflective of the fact that customer preferences for dine-in or delivery often result from factors outside of our model (e.g., a customer may be ill or tired and strongly prefer delivery, or may be planning a special evening out and strongly prefer to dine-in at the restaurant).

Both dine-in and delivery customers are heterogeneous in their valuation for service on their respective channels. Dine-in customers’ valuations $V_R$ are drawn from a continuous distribution with cumulative distribution function (CDF) $F_R$ on $[0, \bar{v}_R]$, with $\bar{v}_R > 0$ and corresponding complementary CDF $\bar{F}_R$ (i.e., $\bar{F}_R(\cdot) = 1 - F_R(\cdot)$). Delivery customers’ valuations $V_H$ are drawn from a continuous distribution with CDF $F_H$ on $[0, \bar{v}_H]$, with $\bar{v}_H > 0$ and complementary CDF $\bar{F}_H$. To provide clean insights, these distributions are assumed to be uniform. The range of customer valuations on each channel, parameterized by $\bar{v}_R$ and $\bar{v}_H$, reflects not only the heterogeneity within a channel but also how dine-in customer valuations compare to those for delivery customers. For instance, if $\bar{v}_R < \bar{v}_H$, then delivery customers tend to have higher valuations than dine-in customers (or vice versa if $\bar{v}_R > \bar{v}_H$). The relative value of these parameters plays a key role in determining the equilibrium prices under different contracts.

As described above, greater demand placed on the shared resource of the kitchen can increase delays experienced by customers in both channels. In accordance with this, we assume that customers experience a linear congestion cost proportional to the total volume of customers who purchase across both channels. Specifically, let $c_t$ be the marginal waiting cost for customers of type $t \in \{H, R\}$, and suppose that the total mass of customers who purchase equals $y$. We assume that the congestion cost incurred by customers is equal to $c_t y$. A linear cost function captures the fact that customer utility is decreasing in the volume of customers who use the resource, while still maintaining tractability. It is appropriate if, for example, customers arrive faster than they can be processed by the kitchen, a reasonable assumption during peak hours for restaurants, forming a queue that is proportional in length to the number of arrivals.\(^4\)

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\(^4\)To see this, suppose $\lambda$ customers arrive uniformly over a time window of 1 hour. Suppose further the kitchen is capable of deterministically processing $\mu < \lambda$ customers per hour. Then, by the end of the 1 hour arrival window, $L = \lambda - \mu$ customers have accumulated in a queue. On average, $L/2 = (\lambda - \mu)/2$ customers are waiting in this queue. From Little’s Law, since customers are departing the system at rate $\mu$, the average wait time in the queue is thus $W = L/\mu = (\lambda/\mu - 1)/2$ hours. The average waiting cost for a type $t$ customer is thus $c_t W = c_t (\lambda/\mu - 1)/2$, i.e., it is a linear function of the total volume of customers who seek service, $\lambda$. 

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Table 1. Description of channels and customer segments.

<table>
<thead>
<tr>
<th>Channel/Customer Segment</th>
<th>Pricing Control</th>
<th>Fraction of Demand</th>
<th>Valuation</th>
<th>Congestion Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dine-In</td>
<td>Restaurant</td>
<td>$\alpha$</td>
<td>$U[0, \bar{v}_R]$</td>
<td>$c_R(\alpha \gamma_R + (1 - \alpha) \gamma_H)$</td>
</tr>
<tr>
<td>Delivery</td>
<td>Platform</td>
<td>$1 - \alpha$</td>
<td>$U[0, \bar{v}_H]$</td>
<td>0</td>
</tr>
</tbody>
</table>

While, in practice, congestion costs could apply to both customer segments, it is also true that dine-in customers should be more sensitive to congestion than delivery customers, since delivery customers wait for their food in relatively comfortable locations (e.g., their homes or hotel rooms), while dine-in customers wait in less comfortable locations (e.g., the restaurant’s waiting area or on the street). Moreover, most delivery platforms allow customers to preorder food for arrival at a specific time, allowing them to avoid (or mostly avoid) waits altogether. Hence, delivery customers should have lower marginal waiting costs than dine-in customers, i.e., $c_H < c_R$. In most of our analysis, we consider one particular extreme case of such heterogeneous waiting costs: dine-in customers incur a strictly positive marginal cost ($c_R > 0$), while delivery customers incur zero cost ($c_H = 0$). This assumption captures that dine-in customers are more sensitive to congestion than delivery customers in a tractable way that significantly simplifies our analysis; in the conclusion and §C of the online supplement, we discuss the case in which delivery customers have positive marginal waiting costs, i.e., $0 < c_H < c_R$.

Let $\gamma_H$ be the fraction of delivery customers who purchase and let $\gamma_R$ be the fraction of dine-in customers who purchase. Given $\alpha$, $\gamma_R$, and $\gamma_H$, the dine-in volume is $\alpha \gamma_R$, and the delivery volume is $(1 - \alpha) \gamma_H$. Hence, the total number of customers who purchase is $\alpha \gamma_R + (1 - \alpha) \gamma_H$, and the congestion cost incurred by dine-in customers is $c_R(\alpha \gamma_R + (1 - \alpha) \gamma_H)$. Customer and channel characteristics are summarized in Table 1. Customers from both segments are assumed to purchase if they have non-negative utility. The sequence of events is as follows. First, the restaurant publicly announces the dine-in price $p$. Then, the platform announces the delivery price $\theta$. Finally, customers make their purchase decisions, with customers for both channels making their choices simultaneously.

### 3.3 Preliminary Results

We first derive the customer equilibrium purchase volume on each channel for given prices (we will discuss the revenue functions for different contract structures in later sections). The utility for a delivery customer with realized valuation $v_H$ who decides to purchase is

$$U_H(\theta; v_H) = v_H - \theta.$$

We solve for the delivery customer equilibrium by finding a threshold valuation above which customer utility is positive.
Lemma 1 (Delivery Equilibrium). The equilibrium delivery purchase fraction $\hat{\gamma}_H$ is given by

$$\hat{\gamma}_H = \bar{F}_H(\theta) = \left[ \frac{\bar{v}_H - \theta}{\bar{v}_H} \right]^+.$$  

Proof. Proofs of analytical results can be found in §A of the online supplement. □

In other words, the fraction of delivery customers who purchase is exactly the fraction whose valuation is larger than the delivery price $\theta$, namely $\bar{F}_H(\theta)$. On the other hand, the utility for a dine-in customer involves both the dine-in price and the congestion cost, which depends on the purchase volume. If a fraction $\gamma_H$ of delivery customers purchase, and a fraction $\gamma_R$ of dine-in customers purchase, then the purchase utility of a dine-in customer with realized valuation $v_R$ is

$$U_R(p; v_R, \gamma_R, \gamma_H) = v_R - p - c_R(\alpha \gamma_R + (1 - \alpha) \gamma_H).$$  

(1)

Using the equilibrium delivery purchase fraction, we can next compute the equilibrium dine-in purchase fraction.

Lemma 2 (Dine-in Equilibrium). The equilibrium dine-in purchase fraction $\hat{\gamma}_R$ is given by

$$\hat{\gamma}_R = \left[ \frac{\bar{v}_R - p - c_R(1 - \alpha) \bar{F}_H(\theta)}{\bar{v}_R + c_R \alpha} \right]^+.$$  

(2)

Intuitively, the dine-in purchase fraction is decreasing in the dine-in price $p$. It is also increasing in the delivery price $\theta$: the higher the delivery price, the fewer delivery customers purchase and the less congestion there is in the system, and therefore the more dine-in customers will be willing to purchase. Importantly, the customer equilibria of Lemmas 1 and 2 hold for any given dine-in and delivery prices, regardless of the contract (or lack thereof) between the restaurant and the delivery platform that precipitated those prices.

4 Simple Revenue Sharing

Under a simple revenue-sharing contract, the restaurant keeps all dine-in revenue, and the platform shares with the restaurant an agreed fraction of the revenue from delivery orders. We apply the adjective “simple” to distinguish this from other forms of revenue sharing that will be discussed later. We denote by $a$ the share of the revenue from delivery customers that the restaurant receives. Under simple revenue sharing, if the platform charges a customer-facing price $\theta$, then the restaurant’s revenue for each delivery customer is $a\theta$, and the platform’s revenue per delivery customer is $(1 - a)\theta$.

The platform sets the delivery price $\theta$ after observing the dine-in price $p$. Given the percentage revenue share, however, and because delivery customers are not congestion-sensitive, the platform’s revenue is not a
function of the dine-in price under simple revenue sharing. The platform’s revenue is given by

\[ z_{P, RS}(\theta) = (1 - a) \theta (1 - \alpha) \bar{F}_H(\theta). \]  

(3)

Solving the platform’s optimization problem gives the optimal delivery price \( \theta_{RS} \) in the following proposition.

**Proposition 1** (Optimal Delivery Price: Simple Revenue Sharing). *Under simple revenue sharing, the platform’s optimal price \( \theta_{RS} \) is given by

\[ \theta_{RS} = \frac{\bar{F}_H(\theta_{RS})}{f_H(\theta_{RS})} \iff \theta_{RS} = \frac{\bar{v}_H}{2}. \]  

(4)

Because the platform’s optimal price does not adjust to the dine-in price, the restaurant can treat the platform’s price (and therefore also the restaurant’s total revenue from delivery customers, given the fixed share \( a \)) as constant. The restaurant’s revenue can then be written as

\[ z_{R, RS}(p) = p \alpha \gamma_R + a \theta_{RS}(1 - \alpha) \bar{F}_H(\theta_{RS}) 
= p \alpha \left[ \frac{\bar{v}_R - p - c_R(1 - \alpha) \bar{F}_H(\theta_{RS})}{\bar{v}_R + c_R \alpha} \right]^+ + a \theta_{RS}(1 - \alpha) \bar{F}_H(\theta_{RS}). \]  

(5)

As mentioned, the second term above, which denotes the restaurant’s revenue from delivery customers, is constant in the dine-in price \( p \) and drops out when differentiating. When optimizing its price, then, the restaurant ignores delivery revenue and maximizes dine-in revenue. The delivery price still plays a role because it determines the delivery purchase volume, which in turn influences the dine-in purchase fraction through the congestion cost incurred by dine-in customers. Hence, the restaurant’s problem is to maximize dine-in revenue, given the fixed level of congestion created by delivery customers and the endogenous level of congestion from dine-in customers. The solution to this optimization problem is given next.

**Proposition 2** (Optimal Restaurant Price: Simple Revenue Sharing). *Under simple revenue sharing, the restaurant’s optimal price \( p_{RS} \) is

\[ p_{RS} = \left[ \frac{\bar{v}_R}{2} - \frac{c_R(1 - \alpha) \bar{F}_H(\theta_{RS})}{2} \right]^+. \]

Moreover, if the bracketed term above is negative, then there will be no dine-in demand regardless of the dine-in price.

Succinctly, either (i) the delivery congestion is enough that the restaurant cannot convince any dine-in customers to purchase, in which case the price is irrelevant, or (ii) the dine-in revenue is a concave function of \( p \). Observe also that the slopes of this price with respect to the parameters are intuitive. The dine-in price \( p_{RS} \) is naturally increasing in the upper bound on the dine-in valuation \( \bar{v}_R \). It is also increasing in the dine-in fraction \( \alpha \): the higher the fraction of dine-in customers, the lower the total delivery purchase volume.
(1 − α)\tilde{F}_H(\theta) and therefore the less congestion in the system from delivery customers, increasing dine-in customers’ willingness to pay and allowing the restaurant to charge a higher price. For a similar reason, namely that a higher delivery price also reduces (1 − α)\tilde{F}_H(\theta), the dine-in price is increasing in the delivery price \theta_{RS} (\theta_{RS} is constant for given parameters, but it could increase or decrease as a result of a change in the distribution \tilde{F}_H). On the other hand, the dine-in price is decreasing in the congestion cost \epsilon_H: the more dine-in customers care about congestion, the lower the restaurant must set the price in order to induce them to purchase.

5 Centralized System and Revenue Comparison

Having derived the (decentralized) optimal prices under a simple revenue sharing contract, in this section, we study a central planner’s problem who controls both prices and maximizes aggregate revenue. In turn, this will allow us to illustrate whether and how the decentralized system with simple revenue sharing contracts generates inefficiencies, and to discuss the magnitude of the revenue loss under simple revenue sharing compared to the central optimum.

5.1 Centralized Revenue

The aggregate revenue for prices p and \theta and purchase fractions \gamma_R and \gamma_H is

\[ Z(p, \theta) = p\alpha\gamma_R + \theta(1 - \alpha)\gamma_H, \quad (6) \]

where the dine-in revenue is \( p\alpha\gamma_R \) and the delivery revenue is \( \theta(1 - \alpha)\gamma_H \). By Lemmas 1 and 2, we can substitute the equilibrium purchase fractions \( \hat{\gamma}_R \) and \( \hat{\gamma}_H \) into equation (6) to give

\[ Z(p, \theta) = p\alpha\left[\frac{\bar{v}_R - p - \epsilon_R(1 - \alpha)\tilde{F}_H(\theta)}{\bar{v}_R + \epsilon_R\alpha}\right]^+ + \theta(1 - \alpha)\tilde{F}_H(\theta). \quad (7) \]

Figure 1 is a three-dimensional plot of the aggregate revenue (7) against the dine-in and delivery prices. Observe that above a threshold price on one channel, increasing or decreasing the price on that channel does not change the aggregate revenue. The reason is that for high enough prices on a channel, that channel’s customers are priced out of the market completely. We also see that the threshold dine-in price at which \( \hat{\gamma}_R \) reaches zero, which corresponds to the point where the revenue becomes constant in the dine-in price, is increasing in the delivery price. As the delivery price increases, the “corner” of the surface occurs at higher dine-in prices. The reason is that the higher the delivery price, the smaller the delivery purchase volume \( \hat{\gamma}_H \), which implies a lower congestion cost for dine-in customers and therefore an increased willingness to pay. So, for higher delivery prices, the dine-in price can be set higher while still recruiting some dine-in customers. The converse holds as well: if the delivery price is low enough, then so many delivery customers will purchase that there will be no dine-in volume no matter the dine-in price (see the discussion surrounding Proposition
2). Similar observations can be made about the delivery customers as the delivery price increases, although the price at which delivery customers are priced out of the market does not depend on the dine-in price.

We can also see that the optimal solution in the example involves serving customers in both channels: the optimal dine-in and delivery prices are below $\bar{v}_R$ and $\bar{v}_H$, respectively. Next, we give a sufficient condition under which it is optimal to operate both the dine-in and delivery channels, as in the example of the figure.

**Lemma 3** (Sufficient Condition to Operate Both Channels). If $c_R(1 - \alpha)/4 < \bar{v}_R/2 < \bar{v}_H$, then it is centrally optimal to operate both channels.

First, note that the parameters in Figure 1 satisfy the condition in the lemma (we have $c_R(1 - \alpha)/4 = 5 < 12 = \bar{v}_R/2 < 30 = \bar{v}_H$). Intuitively, for it to be optimal to operate both channels, the average dine-in valuation $\bar{v}_R/2$ should be “moderate” relative to the other parameters. If it is too low relative to the congestion cost and the delivery fraction, then dine-in customers may not be worth serving because they will not be willing to pay high enough prices given the congestion from delivery customers. On the other hand, if the average dine-in valuation is more than twice the average delivery valuation, then it may be optimal to price delivery customers out of the market altogether, to prevent congestion from crowding out the more lucrative dine-in customers. Under the assumptions of Lemma 3, we can restrict our attention to prices with strictly positive purchase fractions on both channels, implying that the bracketed term for $\hat{\gamma}_R$ in equation (7) is strictly positive.

Next, we solve for the centrally optimal prices under the condition that it is centrally optimal to operate both channels (Lemma 3). Taking partial derivatives of equation (7) with respect to $p$ and $\theta$ yields a pair of first-order conditions, the unique simultaneous solution of which gives the centrally optimal prices:
Proposition 3 (Centrally Optimal Prices). Under the assumptions of Lemma 3, the unique optimal price pair \((p^*, \theta^*)\) that maximizes aggregate revenue uniquely solves the system

\[
p^* = \frac{\bar{v}_R}{2} - \frac{c_R(1 - \alpha)\bar{F}_H(\theta^*)}{2} \tag{8}
\]

and

\[
\theta^* = \frac{\bar{v}_H}{2} + \frac{p^*}{2} \left( \frac{c_R\alpha}{\bar{v}_R + c_R\alpha} \right). \tag{9}
\]

The optimal prices \((p^*, \theta^*)\) can be expressed in closed form, but the formulas are complicated and conditions (8) and (9) are more insightful. Condition (8) for the dine-in price is exactly the same as in simple revenue sharing (replacing \(\theta_{RS}\) with \(\theta^*\)). Specifically, the dine-in price is optimally set to maximize dine-in revenue, given the level of congestion from delivery customers. However, unlike under simple revenue sharing, the delivery price in (9) accounts for the negative externality (and thus reduced willingness to pay) imposed on dine-in customers by congestion from delivery customers, raising the delivery price (by an amount equal to the second term of (9)) to compensate. Observe that the second term of (9) is increasing in \(c_R\) and \(\alpha\), i.e., as the negative externality imposed on the dine-in channel becomes more significant, the delivery price is adjusted further from the optimal value under simple revenue sharing (given in Proposition 1).

Examination of these conditions immediately leads to an important corollary of Proposition 3:

Corollary 1 (Delivery Price Too Low Under Simple Revenue Sharing). Under the assumptions of Lemma 3, we have \(\theta_{RS} < \theta^*\).

The result follows from inspection of equations (4) and (9). Under simple revenue sharing, the delivery price is set to maximize delivery revenue irrespective of the impact on congestion and, by extension, on dine-in revenue. By contrast, at the central optimum, the delivery price is set to trade off delivery and dine-in revenue and maximize their sum, meaning that it is set higher than \(\theta_{RS}\) to alleviate some of the congestion in the system and make room for more dine-in customers. We will see later that there is a problematic imbalance between the channels under simple revenue sharing that harms the aggregate revenue; the low delivery price attracts more delivery customers than centrally optimal, which pushes congestion-sensitive dine-in customers out of the market.

Also, because the delivery platform attracts too many customers under simple revenue sharing, the restaurant is forced to lower its dine-in price relative to the central optimum. Let \(\gamma_{RS}^R\) be the equilibrium dine-in purchase volume under simple revenue sharing and \(\gamma_{opt}^R\) the dine-in purchase volume at the central optimum. Then we have the following corollary to Proposition 3:

Corollary 2 (Dine-In Suffers Under Simple Revenue Sharing). Under the assumptions of Lemma 3, we have (i) \(p_{RS} < p^*\) and (ii) \(\gamma_{RS}^R < \gamma_{opt}^R\).

Corollary 2 shows that simple revenue sharing hurts restaurants compared to the central optimum in two ways: it yields less volume and a lower price. The margin on dine-in customers is lower because the
restaurant must drop its price to compensate for the excess congestion from delivery customers. But despite the lower price, the excess congestion still crowds out enough dine-in customers to yield a lower dine-in volume than the central solution. This result is consistent with the real-world examples cited in Buell (2017) and Dunn (2018) of platforms hurting restaurants’ profitability in multiple ways. We investigate this issue further in the next subsection as we compare revenue between the centralized system and a decentralized system with a simple revenue sharing contract.

5.2 Revenue Comparison

To understand when the issues identified in the previous section result in the greatest forfeited revenues, in this section we consider several specific illustrative examples. Figure 2 plots the aggregate revenue, dine-in purchase volume, and delivery purchase volume against the dine-in fraction $\alpha$. Figure 3 plots the equilibrium channel prices. Note that these figures do not depend on the revenue share $a$ because both parties’ prices are the same regardless of the delivery split under simple revenue sharing.

In the left panel of Figure 2, for extreme values of the dine-in fraction $\alpha$, the revenue lost under simple revenue sharing is negligible, i.e., the aggregate revenue curve is close to that for the central optimum. The reason is that if one of the channels has very few potential customers, then this channel has little impact on the aggregate revenue, so the sub-optimal delivery price does not significantly impact the system. For very small $\alpha$, there are so few dine-in customers that there is little dine-in revenue to lose from the excess congestion generated by extra delivery customers. For very large $\alpha$, there are so few delivery customers that the extra delivery volume does not cause enough congestion to seriously impact dine-in customers. Intermediate $\alpha$, on the other hand, is a “worst case scenario” for the restaurant and the platform: there are many potential customers on both channels, and hence they can have a significant impact on one another. It is in this regime where the aggregate revenue suffers most.

Specifically, for intermediate $\alpha$, the congestion effects are significant enough to seriously impact dine-in
customer decisions, and there are enough dine-in customers that this impact has a substantial effect on firm revenues. In other words, the too-low delivery price results in significantly more delivery volume than optimal, and there are a substantial number of dine-in customers to be displaced by these delivery customers. Thus, the too-low delivery price (right panel of Figure 3) creates an imbalance between the channels relative to the optimal solution, leading to too many delivery customers and not enough dine-in customers. This can be seen in the right two panels of Figure 2, which depict the channel purchase volumes against $\alpha$. As $\alpha$ increases from zero, the difference between dine-in purchase volume for the centralized system and for revenue sharing increases, and is largest for moderate $\alpha$. While the reduction in volume might appear small in magnitude in the figure, it is exacerbated by the difference in prices: not only are there too few dine-in customers, the revenue earned from each is also too low (Figure 3). The combined effect can result in a significant loss in dine-in revenue for the restaurant.

Also note that under simple revenue sharing, as the dine-in fraction $\alpha$ increases, the delivery price (right panel of Figure 3) and delivery purchase fraction do not change, but because $1 - \alpha$ is decreasing, the delivery purchase volume decreases (see right panel of Figure 2). This means that as $\alpha$ increases, congestion from delivery customers reduces, alleviating some of the downward pressure on the dine-in price. The result is that the dine-in price is increasing in $\alpha$ under simple revenue sharing (left panel of Figure 3).

While the delivery price is independent of $\alpha$ under simple revenue sharing, in the centralized system, the delivery price increases with $\alpha$. This happens because the central planner seeks to deter delivery customers from generating congestion that negatively affects dine-in customers, and as $\alpha$ increases and there are more dine-in customers, the central planner has stronger incentives to reduce delivery volume. This immediately leads to the observation that the difference between the delivery price at the central optimum and under simple revenue sharing increases with $\alpha$, as can be seen in Figure 3 (right panel). As a result, for large values of $\alpha$, the delivery price under simple revenue sharing is much lower than centrally optimal; however, the fact that large $\alpha$ corresponds to very few delivery customers means that the revenue loss from an artificially low
delivery price is relatively low in this regime (left panel of Figure 2). Hence, as noted previously, the system has the most to lose under simple revenue sharing when \( \alpha \) is intermediate and the firms face a mixture of customers from both channels.

Interestingly, when \( \alpha \) is close to zero, the aggregate revenue under simple revenue sharing is negligibly different from the central optimum. This suggests that if there is a demand shock, as during the COVID-19 pandemic, that immediately pushes \( \alpha \) to or near zero, the inefficiency of simple revenue sharing contracts (at least in terms of aggregate revenue) is minimized. Put differently, if there are no dine-in customers and the system operates as a ghost kitchen, a simple revenue sharing contract can perform quite well. Indeed, during the height of pandemic-related closings, restaurateurs were grateful for the presence of delivery platforms (albeit begrudgingly) as one of their only ways to generate revenue. However, as society emerges from the pandemic (and hence \( \alpha \) increases), our results imply that the inefficiency of simple revenue sharing contracts will intensify, perhaps leading to the kind of conflict that has emerged between platforms and restaurants as they jockey over commissions and profitability in a post-pandemic landscape (Elejalde-Ruiz, 2021).

Beyond these specific examples, we have also conducted a comprehensive numerical study, considering 30,400 parameter combinations (not counting variations in the revenue share \( a \), which do not affect the central optimum) and measuring the aggregate revenue under both the centralized solution and simple revenue sharing. We consider values of \( \alpha \) from .05 to .95, values of \( c_R \) from 0 to 30, values of \( \bar{v}_R \) from 0 to 40, and values of \( \bar{v}_H \) from 0 to 40. The percentage revenue loss under revenue sharing ranges from 0% to 26.2%, with an average of 1.1% and a median of .2%. Because the restaurant industry has very small margins, on the order of 4-5% of sales or even less (Lunden, 2020), even a single-digit percentage reduction in the top line can have an outsize impact on the bottom line. Hence, we conclude that the revenue losses associated with simple revenue sharing can be substantial.

Finally, we remark briefly that not only is simple revenue sharing sub-optimal in the sense that it cannot achieve the centralized optimal revenue, but it can even perform worse than if the platform and restaurant used no contract at all. Absent a formal agreement, the platform can act like a regular customer, paying menu price for each order with no revenue share and then delivering to customers (i.e., essentially placing a take-out order on behalf of each delivery customer). This approach, which has been controversial among restaurateurs due to the fact that platforms can deliver their food without their explicit cooperation, was at one time used by Grubhub and Postmates (Saxena, 2019), but has since been outlawed in some localities such as the state of California (Batey, 2021). We discuss this arrangement in the online supplement (§B), and we show that the aggregate revenue can in fact be higher under this arrangement than under simple revenue sharing. The reason for this is that, by forcing the platform to set the delivery price at least as high as the restaurant’s dine-in price, the restaurant can regain some pricing power in the delivery channel and avoid the the vicious cycle that occurs under simple revenue sharing wherein the platform sets too low a price for delivery, forcing the restaurant to set too low a price for dine-in. However, the no contract scenario can also be significantly worse than simple revenue sharing depending on the contract and problem parameters,
underscoring the need for an alternative (better) contract.

6 Achieving Coordination

We have now observed that significant revenue can be left on the table under simple revenue sharing, the most commonly observed contract in practice. In this section, we consider how to remedy this and coordinate the system. First, we consider two common variations on simple revenue sharing that aim to protect the restaurant: commission caps (§6.1) and price floors (§6.2). Next, we propose two new contracts that generalize simple revenue sharing in different ways, by adding an additional direction of revenue flow from the restaurant to the platform (§6.3) and by adding a fixed payment for each order in addition to a revenue share (§6.4).

6.1 Commission Caps

In an effort to protect restaurant margins, recent regulations in some US cities and states have capped the commission charged by delivery platforms at 15% or similar (Lucas, 2020), initially in response to the COVID-19 pandemic but potentially becoming permanent in some municipalities (Forman, 2021). It is not clear, however, that these caps have their intended effect, namely protecting restaurants and, in particular, small independent restaurants; in fact, recent findings in Li & Wang (2021a) have already shown that such commission caps do not help vulnerable independent restaurants as much as larger chain restaurants. Below, we discuss the implications of commission caps in the context of our model.

Commission caps are a special case of simple revenue sharing in which the commission $1 - a$ paid to the platform for delivery orders is limited below a maximum value. It is convenient to parameterize this cap through the minimum share $a$ that must go to the restaurant. Because Corollaries 1 and 2 hold for any revenue share $a$, we have an immediate result regarding the prices under simple revenue sharing with a commission cap.

**Corollary 3 (Prices Too Low Under Commission Cap).** For any commission cap $a < 1$, under the assumptions of Lemma 3, the equilibrium prices on both channels are strictly less than centrally optimal, and the equilibrium dine-in volume is strictly less than centrally optimal.

In other words, because the underlying contract is still simple revenue sharing, both parties still price too low under a commission cap, and the restaurant still sees less dine-in volume than optimal; as a result, even though the goal of the cap is to protect restaurant margins, restaurant (and aggregate) revenue will still be lower than in a centralized system, i.e., money is still left on the table. Thus, while commission caps are a well-intentioned effort to protect restaurant margins, they fail to fix the underlying problems with a simple revenue sharing contract, namely that the platform does not account for the congestion effects of delivery customers on dine-in revenue. Consequently, commission caps cannot help the coordination problem.
6.2 Delivery Price Floor

Motivated by the failure of commission caps to coordinate the system, we next consider another variation of simple revenue sharing that introduces a potential remedy to the problem of delivery prices that are too low: allowing the restaurant to set a price floor for orders placed on the platform. Indeed, in practice, the platform sometimes allows the restaurant to set the menu price for delivery orders (on top of which the platform adds its own fees): this fundamentally changes the structure of simple revenue sharing contracts by allowing the restaurant to set two separate menu prices, one charged to dine-in customers and another (likely higher) price charged to delivery customers. This arrangement amounts to the restaurant setting a price floor for the platform, and given that the dine-in and delivery prices are too low under simple revenue sharing, this appears to be a solution that may help achieve coordination.

Specifically, suppose that the restaurant sets a price floor $\theta$, and the platform must choose a delivery price $\theta \geq \theta$. Other than the price floor, the setting is equivalent to simple revenue sharing, i.e., the restaurant receives a fraction $\rho$ of delivery revenues.\(^5\) From §4, we know that under simple revenue sharing, the platform’s revenue is unimodal in $\theta$ and that the optimal $\theta$ does not depend on $p$. The platform’s revenue function is the same with a price floor, but it has an additional constraint $\theta \geq \theta$. Thus, with a price floor $\theta$, the equilibrium delivery price will be $\max\{\theta_{RS}, \theta\}$.

Let us consider the restaurant’s problem under simple revenue sharing, supposing that the restaurant can choose both customer-facing prices, i.e., $p$ and $\theta$ (we will see that this is equivalent to the actual price floor setting because the restaurant always sets $\theta > \theta_{RS}$). The restaurant sets both prices and receives all of the dine-in revenue and a fraction $\rho$ of the delivery revenue. The restaurant’s revenue function is the same as equation (5), but now $\theta$ is a decision variable, i.e., we have

$$z_{R,PF}(p, \theta) = \rho \left[ \frac{\bar{v}_R - p - c_R(1 - \alpha)\bar{F}_H(\theta)}{\bar{v}_R + c_R\alpha} \right]^+ + \rho(1 - \alpha)\bar{F}_H(\theta). \quad (10)$$

Assuming $\hat{\gamma}_R > 0$, differentiating with respect to $p$ gives

$$\frac{\partial z_{R,PF}}{\partial p} = \frac{\alpha}{\bar{v}_R + c_R\alpha} \left( \bar{v}_R - 2p - c_R(1 - \alpha)\bar{F}_H(\theta) \right),$$

which is the same FOC (8) as in the central planner’s problem. Similarly, for $\hat{\gamma}_R > 0$, differentiating with respect to $\theta$ gives

$$\frac{\partial z_{R,PF}}{\partial \theta} = (1 - \alpha) \left( \alpha \bar{F}_H(\theta) - \left( \rho \theta - \frac{pc_R\alpha}{\bar{v}_R + c_R\alpha} \right) \bar{F}_H(\theta) \right)$$

and the corresponding FOC

$$\frac{pc_R\alpha}{\rho(\bar{v}_R + c_R\alpha)} = \theta - \frac{\bar{F}_H(\theta)}{\bar{F}_H(\theta)}.$$

\(^5\)We note that, typically in practice, the platform only splits revenue of the menu price with the restaurant, i.e., the platform keeps 100% of the customer delivery fees. In other words, for a delivery fee $f$, menu price $\theta'$, and restaurant revenue share $a'$, platform revenue is $f + (1 - a')\theta'$ while restaurant revenue is $a'\theta'$. For a given $a'$, the platform chooses $f$ while the restaurant chooses $\theta'$. Our formulation is equivalent to this if we define a new menu price $\theta \equiv f + \theta'$ and a new revenue share $a \equiv a'\theta'/(f + \theta')$, i.e., for any $f$, $a'$, and $\theta'$, we can provide a $\theta$ and $a$ such that our model results in equivalent outcomes.
Note that the left hand side of this latter FOC differs from both the central planner’s problem (equation 9) and from the platform’s FOC under simple revenue sharing (equation 4). Under simple revenue sharing with no price floor, the platform’s FOC has zero on the LHS, while the central planner’s problem has \( p(c_R\alpha)/(\bar{v}_R + c_R\alpha) \). The solution to this FOC will be larger than \( \theta_{RS} \) because the LHS is positive. Because the delivery price under simple revenue sharing is too low, an increase is needed to improve the aggregate revenue. However, rather than merely nudging the delivery price towards the central optimum, for some parameters, the price floor actually sends it too far: because the LHS is divided by \( a \leq 1 \), the delivery price with a price floor is larger than the centrally optimal delivery price for a given dine-in price.

The result is that the price floor has two unintended consequences, as shown in the following proposition. Denote the equilibrium prices under revenue sharing with a price floor by \( p_{PF} \) and \( \theta_{PF} \). Then we have:

**Proposition 4** (Prices Too High with Price Floor). For \( 0 \leq a < 1 \), if \( c_R(1-\alpha)/2 < \bar{v}_R/2 < a\bar{v}_H \), then both channels operate in equilibrium, but we have \( \theta_{PF} > \theta^* \) and \( p_{PF} > p^* \).

The restaurant receives only \( a\theta \) for each delivery order, so it values delivery orders less than the system values them. Thus, the restaurant will care more about reducing congestion than would a central planner who considers the full revenue \( \theta \) on each delivery order. So, for dine-in valuations in the appropriate range, the restaurant then sets the delivery price floor not only higher than the equilibrium price \( \theta_{RS} \) from simple revenue sharing, but even higher than the centrally optimal \( \theta^* \). Moreover, because the delivery price is high, there is less congestion, reducing the downward pressure on the dine-in price, and we also end up with \( p_{PF} > p^* > p_{RS} \). Therefore, with simple revenue sharing as the starting point, if we allow the restaurant to impose a delivery price floor, not only does the delivery price end up higher than the central optimal, but the dine-in price does as well. This implies that, for valuation distributions satisfying the conditions in Proposition 4, under revenue sharing with a price floor, customers in both channels pay higher prices than under simple revenue sharing or at the central optimum.

Also important is that under revenue sharing with a price floor, the restaurant is able to implement its first-best outcome: the optimal delivery price \( \theta_{PF} \) for the restaurant’s revenue is larger than \( \theta_{RS} \), so when the restaurant sets \( \theta = \theta_{PF} \), the platform sets \( \theta = \max\{\theta_{RS}, \theta_{PF}\} = \theta_{PF} \). Therefore, given the revenue sharing split \( a \), the equilibrium under a price floor gives the restaurant the maximum revenue that it could possibly achieve for any prices. The price floor can then be thought of as giving pricing power over to the restaurant. If \( a = 1 \), then unsurprisingly, the restaurant sets the same prices as a central planner would. But if the revenue share is strictly less than 1, the restaurant undervalues delivery orders, which results in a failure to achieve coordination. Hence, allowing the restaurant to set a price floor for the delivery channel does give it the power to protect its margins, although it is perhaps too effective in doing so; the result is that the restaurant maximizes its own revenue to the detriment of the entire system, leaving aggregate revenue on the table.
6.3 Two-Way Revenue Sharing

Neither commission caps nor price floors solve the coordination problem. In this subsection, we present a simple contract that generalizes simple revenue sharing and does coordinate the system. We call this contract “two-way revenue sharing” because it involves each party sharing revenue with the other from its respective channel (dine-in or delivery). Let $b$ be the fraction of dine-in revenue that the restaurant keeps for itself (so the platform receives a fraction $1 - b$ of the dine-in revenue). Similarly, let $a$ be the fraction of delivery revenue that the platform cedes to the restaurant (so the platform keeps a fraction $1 - a$ of the delivery revenue). Observe that simple revenue sharing is recovered by setting $b = 1$.

The restaurant’s revenue under two-way revenue sharing, $z_{R,2W}(p, \theta)$, is

$$z_{R,2W}(p, \theta) = b p \alpha \hat{\gamma}_R + a \theta (1 - \alpha) \hat{\gamma}_H.$$ 

The platform’s revenue under two-way revenue sharing is similarly given by

$$z_{P,2W}(p, \theta) = (1 - b) p \alpha \hat{\gamma}_R + (1 - a) \theta (1 - \alpha) \hat{\gamma}_H. \quad (11)$$

 Appropriately chosen fractions $b$ and $a$ induce the restaurant and platform to choose the centrally optimal prices and permit arbitrary allocations of the revenue between the two parties, as the following proposition shows.

**Proposition 5** (Two-Way Revenue Sharing Coordinates). For $0 < c < 1$, under a contract that sets $a = b = c$ (i.e., each party’s share of the dine-in revenue is the same as his share of the delivery revenue), in equilibrium, the centrally optimal aggregate revenue is achieved. Moreover, the revenue can be allocated arbitrarily through the choice of $c$.

Thus it would seem that the solution to the coordination problem is given in the form of two-way revenue sharing. However, to implement such a contract, the restaurant would have to give a fraction of its dine-in revenue to the platform. Even given the growing market power of delivery platforms, restaurants are unlikely to share dine-in revenue that the platform had no part in generating. Moreover, this contract suffers from a verifiability problem: the restaurant has no incentive to accurately disclose its dine-in revenue to the platform, i.e., it has incentive to under-report dine-in revenue (note that the reverse cannot happen, i.e., the platform cannot under-report delivery volume or revenue to the restaurant, since the restaurant prepares each meal that the platform delivers). Nonetheless, Proposition 5 demonstrates that a simple contract can coordinate the system. We turn next to devising a similarly simple—but far more practical—coordinating contract, without the problematic requirement of shared dine-in revenue.
6.4 Generalized Revenue Sharing

At the central optimum, as noted earlier, the dine-in price should be set to maximize dine-in revenue irrespective of the delivery revenue (given the congestion from delivery customers), as it does under simple revenue sharing. Meanwhile, the delivery price must optimally trade off incremental delivery revenue with the congestion externality that delivery customers impose on dine-in customers. A well-designed (and practical) coordinating contract should thus incentivize the restaurant to maximize dine-in revenue, while also inducing the platform to appropriately account for the congestion that its customers create; in addition, such a contract should preferably have the ability to protect restaurant revenue from delivery orders (i.e., to ensure such revenue is not too low) and to allocate revenue between the restaurant and platform with a high degree of flexibility. In this section, we propose just such a contract, which we call generalized revenue sharing, and demonstrate that it accomplishes all of the above goals.

In a generalized revenue sharing contract, the platform pays a flat fee $\tau$ to the restaurant per order, as well as a percentage $a$ of the delivery price $\theta$. This contract resembles the revenue-sharing contract in Cachon & Lariviere (2005), but applied here to a service delivery (rather than supply chain) context. As a result of our different context, the contract terms must be constructed more deliberately in our setting than in Cachon & Lariviere (2005), as will be seen below. Under this contract, the platform’s revenue does not depend on the dine-in price $p$. The platform’s revenue $z_{P,G}$ can be written as

$$z_{P,G}(p, \theta) = (1 - a)\theta - \tau \tilde{F}_H(\theta)(1 - \alpha).$$

(12)

Even though platform revenue does not directly include the dine-in congestion cost, an appropriately chosen fixed fee per order can achieve the same effect and induce the platform to choose the centrally optimal delivery price. Recall that $p^*$ is the centrally optimal dine-in price and $\theta^*$ is the centrally optimal delivery price. Throughout this subsection, we make the same assumptions as in Lemma 3. Under these conditions, our first result shows how generalized revenue sharing (with a careful choice of the fixed fee $\tau$) can coordinate the pricing decision of the platform:

**Lemma 4 (Platform Sets Centrally Optimal Price).** For $0 \leq a < 1$, if

$$\tau = (1 - a)p^* \left( \frac{c_R \alpha}{v_R + c_R \alpha} \right),$$

(13)

then in equilibrium, the platform sets $\theta = \theta^*$.

This contract directly incentivizes the platform to charge the optimal delivery price $\theta^*$: the fixed fee per order is designed to align the solution of the platform’s FOC with the centrally optimal delivery price. Observe that the expression for $\tau$ is similar to the second term of condition (9) in Proposition (3): in essence, the fixed fee serves to “charge” the platform for each delivery an amount that encapsulates the cost of the externality it imposes on dine-in customers.
Using the required contract terms from Lemma 4, we next show that the restaurant sets the centrally optimal price \( p = p^* \). Given \( \tau \) as defined in (13) and anticipating that the platform will set \( \theta = \theta^* \), the restaurant’s revenue is

\[
z_{R,G}(p, \theta^*) = p \alpha \left[ \frac{\bar{v}_R - p - c_R(1 - \alpha)\bar{F}_H(\theta^*)}{\bar{v}_R + c_R\alpha} \right]^+ + (a\theta^* + \tau)\bar{F}_H(\theta^*)(1 - \alpha).
\]  

(14)

Because the platform’s price will be \( \theta^* \) regardless of the dine-in price, and because \( \tau \) and \( a \) are given, the second term in this expression is independent of \( p \). So, the restaurant sets its price to maximize the total dine-in revenue (the first term in (14)), which leads to the coordinated outcome, as the following proposition shows:

**Proposition 6 (Coordination Achieved).** If \( \tau \) satisfies equation (13) for \( 0 \leq a < 1 \), then in equilibrium, the restaurant sets \( p = p^* \). Therefore, both parties set the centrally optimal prices, and this contract coordinates the system.

Proposition 6 shows that a coordinating contract exists that does not share dine-in revenue: a continuum of generalized revenue sharing contracts accomplishes this goal. Note that our contract terms differ from those in Cachon & Lariviere (2005), who also include a fixed payment and a fractional revenue share. While their contract terms can be written as a linear function of the model primitives, our contract must be constructed with a fixed payment that depends on the optimal dine-in price (\( p^* \)) that solves the central planner’s problem. The reason for this difference is that there are two distinct sales channels in our model. Without sharing dine-in revenue (something we sought to avoid), inducing the platform to properly account for the externality its channel generates on the dine-in channel requires careful calibration of the contract terms in a way that depends on the desired centrally optimal equilibrium outcome (\( p^* \)).

As discussed previously, it is also desirable for a coordinating contract to protect restaurant margins by ensuring revenue from the delivery channel is not too low. A valuable insight along these lines can be deduced from Proposition 6, with implications for contracts and for how policymakers should regulate delivery platforms.

**Proposition 7 (Restaurant Earnings on Delivery Orders).** Under a coordinating generalized revenue sharing contract: (i) for each delivery order, the restaurant receives at least \( p^*(c_R\alpha) / (\bar{v}_R + c_R\alpha) \), and (ii) if \( \bar{v}_R < \bar{v}_H \) and \( a \geq \bar{v}_R / \bar{v}_H \), then the amount that the restaurant earns for each delivery order is at least as large as the dine-in price, i.e., \( a\theta^* + \tau \geq p^* \).

Proposition 7 gives a lower bound on the revenue per order that a restaurant can expect under a coordinated generalized revenue sharing contract, and shows that the restaurant’s margin on delivery orders compared to its margin on dine-in orders cannot be reduced by more than a maximum factor (part (i) of the proposition). In part (ii), it also shows that for a range of parameter values, generalized revenue sharing not only coordinates the system, but it also ensures a higher revenue per order to the restaurant on delivery orders than on dine-in orders. In such cases, the restaurant’s margins increase on both types of orders.
relative to simple revenue sharing: the equilibrium dine-in price is higher under generalized revenue sharing, increasing dine-in margins, and the transfer payment that the restaurant receives on delivery orders is higher still. Moreover, the dine-in volume is also higher under generalized revenue sharing (see Corollary 2); thus, the dual negative effects of simple revenue sharing, namely less dine-in volume at a lower price, are fully eradicated under generalized revenue sharing. In an industry with many small, independent restaurants with little bargaining power, these are highly desirable features, protecting restaurant revenue (like commission caps or price floors) while also coordinating the system.

As discussed in §6.1, government-imposed commission caps, while well-intentioned, do not address the coordination problem faced by restaurants and platforms. Capping commissions at some positive percentage implicitly validates a business model wherein restaurants must pay a commission to platforms. Compared to our coordinating contract, these commission caps do not go far enough to protect restaurants, and revenue is left on the table as a result. Even with a commission cap, the platform sets its price too low under simple revenue sharing and attracts excess delivery customers (Corollary 3), which forces down the dine-in price and pushes dine-in customers out of the market, hurting both the restaurant’s margins and the aggregate revenue. An alternative policy in-line with our proposed contract might require the platform to pay both a fixed fee and a revenue share.

Note that under generalized revenue sharing, the form of the coordinating contract is valid even for \( a = 0 \). This observation provides another valuable insight: the right wholesale price contract coordinates the system.

**Proposition 8** (Existence of Coordinating Wholesale Price). A wholesale price contract in which the platform pays \( p^*(c_R\alpha)/(\bar{v}_R + c_R\alpha) \) to the restaurant for each order achieves the centrally optimal aggregate revenue.

The result follows by letting \( a = 0 \) in Proposition 6. As a special case of our generalized revenue sharing contract, this specific wholesale price contract has no flexibility in revenue allocation between the platform and restaurant, i.e., it achieves only a single revenue allocation. In the interest of simplicity, it is nonetheless valuable that a properly designed price-only contract can coordinate the system. However, if we allow \( a > 0 \), as the next result shows, generalized revenue sharing also has the flexibility to achieve a wide range of allocations of the delivery revenue, which further illustrates the benefits of this contract form:

**Proposition 9** (Flexible Allocation). A coordinating generalized revenue sharing contract can allocate to the platform any fraction between 0 and \( \delta \) of the delivery revenue, where \( \delta \geq 1 - (\bar{v}_R/\bar{v}_H)(c_R\alpha)/(\bar{v}_R + c_R\alpha) \). Moreover, if \( c_R\alpha \leq \bar{v}_R \leq \bar{v}_H \), then \( \delta \geq 1/2 \).

Thus, the generalized revenue sharing contract is quite flexible. In particular, when dine-in customers and delivery customers value the respective services similarly and highly enough, the contract can achieve anywhere from a 50-50 split of delivery revenue (at one extreme, a wholesale price contract with \( a = 0 \)) to full allocation to the restaurant (the other extreme with \( a \approx 1 \)). While the realized contract terms are
a matter of negotiation between restaurant and platform, our contract has the flexibility to accommodate
a wide range of relationships and bargaining scenarios while also achieving coordination. Moreover, under
a coordinating generalized revenue sharing contract, the minimum restaurant revenue per delivery order
guaranteed by Proposition 7 can give peace of mind to a restaurant negotiating with a larger and more
powerful delivery platform, since the negative impact on their margins is limited.

In short, generalized revenue sharing simultaneously protects the restaurant and increases aggregate
revenue, while ensuring a wide range of revenue allocations are possible. The existence of a simple, practical
coordinating contract is good news for restaurants and platforms alike. Restaurants can be assured of a
minimum revenue per delivery order regardless of the contract terms, ensuring their participation in the
platform, while platforms can take advantage of the increased overall revenue. Moreover, policymakers
interested in regulating this market can take note. After commission caps were instituted in some cities
in 2020, platforms disputed them with legal arguments and spirited lobbying efforts, and Uber Eats even
tacked on an additional fee for customers to offset lost commissions (Carson, 2020; Brown, 2020). Regulations
based on a coordinating contract could be more acceptable to platforms: if the overall revenue increases,
then capped commissions based on generalized revenue sharing should be easier for platforms to accept.

7 Conclusion

In this paper, we have considered how to best structure relationships between food delivery platforms and
the restaurants with whom they partner. Given their rapid growth and their popularity with customers,
delivery platforms have come to occupy an important place in the food delivery supply chain. However, their
impact on restaurants is controversial at best: some restaurateurs suspect that delivery platforms actively
destroy their profits, a belief that has led to restaurants delisting from platforms and threatens the long-term
viability of the delivery platform business model.

Our results show that the most common contractual relationship between platforms and restaurants—a
simple, one-way revenue sharing agreement—has inherent flaws. It cannot coordinate the system, and can
reduce the profitability of restaurants, many of whom already experience very thin margins. Thus, while
simple revenue sharing is often effective in supply chains for physical goods, it is less effective in food
delivery supply chains due to the interaction between sales channels arising from congestion effects. While
there are doubtless many additional factors that may play a role in the profitability of delivery platforms
for restaurants, our analysis shows that there is a fundamental issue related to the interaction between
channels that results in delivery negatively impacting restaurant margins. Moreover, our model illustrates
that this effect occurs because of the use of a popular contract type (simple revenue sharing) that cannot
properly incorporate the negative externality that the delivery channel imposes on the dine-in channel. This
supports the popular view (Houck, 2017, Dunn, 2018, Meyersohn, 2018), and the situation in the case study
discussed by Buell (2017), that delivery platforms may not benefit restaurants, and partnering with a third-
party delivery service can lead to a vicious cycle in which service quality and profitability at the restaurant deteriorate despite the increase in volume.

Efforts to remedy this shortcoming using either commission caps or price floors are partially effective (in the sense that they can improve restaurant margins) but cannot achieve coordination; however, our proposed generalized revenue sharing contract can coordinate the system, in addition to protecting the restaurant revenues and flexibly allocating delivery revenue. Hence, we conclude that this contract form, which is not much more complicated to implement than simple revenue sharing, potentially has many benefits. It is also worth noting that a fully centralized delivery model, in which the restaurant offers its own delivery service, would also naturally achieve coordination (but may be costly).

We note here that throughout our analysis, we assumed that congestion was costly for dine-in customers (i.e., $c_R > 0$) but costless for delivery customers ($c_H = 0$). This assumption greatly simplifies the analysis and allows us to derive clean insights; however, in §C of the online supplement, we numerically relax this assumption to consider the case where delivery customers experience positive waiting costs, i.e., $0 < c_H < c_R$. We find that, as one might expect, simple revenue sharing continues to be incapable of achieving coordination in this case, since the platform still fails to account for the negative externality it generates for dine-in customers; hence, our insights about the detrimental effects of simple revenue sharing persist. While it can be shown that the generalized revenue sharing contract proposed in Proposition 6 (i.e., assuming $c_H = 0$) does not always coordinate when $c_H > 0$, we numerically observe that it performs exceptionally well, even when using contract parameters optimized for the $c_H = 0$ case as in the proposition: in 2,700 parameter combinations, we find that the average revenue loss relative to the central optimum is just 0.006% (compared to an average loss of 1.2% with simple revenue sharing) while the maximum loss is 0.3% (compared to a maximum of 11.1% with simple revenue sharing). Generalized revenue sharing performs extremely well because the restaurant, in receiving a share of the delivery revenue, already has a strong incentive to account for the negative externality it generates for the delivery channel; consequently, we conclude that our key insights are robust to this assumption, and even under the more general setting in which both delivery and dine-in customers are congestion sensitive, the generalized revenue sharing contract we propose exhibits excellent performance.

Given the relative novelty of this business model and the rapidity with which it is evolving, there are many aspects of the relationship between platforms and restaurants that warrant further study. Future work on this topic could incorporate other factors that may influence the profitability of a delivery platform or the restaurants that partner with it, such as vehicle routing and delivery driver incentives, competition between restaurants on platforms, and competition between platforms for the partnership of restaurants. It would also be interesting to consider how restaurants should respond to the presence of a platform by adjusting or redesigning kitchen capacity: for example, should a restaurant add more shared capacity or should it distribute orders between dedicated dine-in capacity and dedicated delivery capacity (i.e., a so-called “ghost kitchen” model, see Hawley, 2020). While this is a longer-term decision—requiring, for instance, moving to a
new location or physically expanding or re-organizing facilities at the original location—this question could be explored by modeling the restaurant’s capacity choice under different contractual arrangements, i.e., by letting the restaurant invest to increase $\mu$ in Footnote 4. Finally, while we focused solely on congestion caused by (shared) kitchen capacity, it may also be interesting to consider the interaction of different types of capacity in the system, e.g., seating capacity (which only affects dine-in customers) and delivery capacity (which only affects delivery customers).

As delivery platforms continue to grow in popularity, it will become increasingly important for such platforms to effectively manage relationships with restaurants in order to avoid conflict with their key partners. Our work illustrates some of the important issues that can arise in these relationships, and offers a contractual way to alleviate them and improve coordination of the food delivery supply chain.

References


Online Supplement to
“Managing Relationships Between Restaurants and Food Delivery Platforms: Conflict, Contracts, and Coordination”

A Proofs

**Proof of Lemma 1.** The only cost to delivery customers is the purchase price, so the fraction that purchases is exactly the fraction of customers whose valuation exceeds the delivery price \( \theta \), namely \( \bar{F}_H(\theta) \).

**Proof of Lemma 2.** It is optimal for a given dine-in customer to purchase if and only if \( UR(p, \theta; \gamma_R, \hat{\gamma}_H) \geq 0 \). By Lemma 1, we can replace \( \hat{\gamma}_H \) with \( \bar{F}_H(\theta) \). So, by equation (1), a dine-in customer with a realized valuation \( v_R \) should purchase if and only if

\[
v_R - p - c_R(1 - \alpha) \bar{F}_H(\theta) \geq 0, \tag{A.1}
\]

then the system can support some dine-in customers. Clearly, for any purchase fraction \( \gamma_R \), if a customer with valuation \( v_R \) purchases, then it must also be optimal for a customer with valuation \( v' _R > v_R \) to purchase. Hence, the equilibrium has a threshold structure in which customers with valuations above some threshold purchase, and the others do not. From equation (1), then, our requirement for equilibrium is to find a threshold valuation \( v_R \) that solves the equation \( v_R = p + c_R \bar{F}_R(v_R) + c_R(1 - \alpha) \bar{F}_H(\theta) \). This equation has a strictly positive solution because the left-hand side (LHS) is smaller than the right-hand side (RHS) for \( v_R = 0 \), while the LHS is larger than the RHS for \( v_R = \bar{v}_R \) by equation (A.1) and the fact that \( \bar{F}_R(\bar{v}_R) = 0 \). The solution is unique because the LHS is increasing and the RHS is decreasing in \( v_R \), and the purchase fraction \( \hat{\gamma}_R = \bar{F}_R(\bar{v}_R) \) will be strictly between 0 and 1 because the threshold \( \bar{v}_R \) is strictly between 0 and \( v_R \). Substituting the uniform complementary CDF \( \bar{F}_R(v_R) = 1 - \bar{v}_R/\bar{v}_H \) and isolating \( \bar{v}_R \) gives the equilibrium threshold \( \bar{v}_R \), yielding the corresponding purchase fraction \( \hat{\gamma}_R = \bar{F}_R(\bar{v}_R) \) given in equation (2).

**Proof of Proposition 1.** Differentiating the platform’s revenue in equation (3), we have

\[
z'_{P,RS}(\theta) = (1 - \alpha)(1 - \alpha) \left[ \bar{F}_H(\theta) - \theta f_H(\theta) \right],
\]

yielding the FOC \( \theta = \frac{\bar{F}_H(\theta)}{f_H(\theta)} \). The uniform density \( f_H(\theta) = 1/\bar{v}_H \) and complementary CDF \( \bar{F}_H(\theta) = 1 - \theta/\bar{v}_H \) imply that \( \bar{F}_H(\theta)/f_H(\theta) = \bar{v}_H - \theta \). Substituting into the FOC and isolating \( \theta \) then gives the unique solution
to the FOC provided in the statement of the proposition. Moreover, we have

\[ z'_{P,NC}(0) = (1 - a)(1 - \alpha)\bar{F}_H(0) > 0. \]

Thus, the function is strictly increasing at \( \theta = 0 \), implying that the unique solution to the FOC is a local maximum. Because the FOC has a unique solution, the derivative crosses zero exactly once, so the solution to the FOC is the global maximizer of the function.

**Proof of Proposition 2.** We have two cases, corresponding to whether achieving positive dine-in volume is feasible, given the level of congestion from delivery customers.

**Case 1:** \( \bar{v}_R - c_R(1 - \alpha)\bar{F}_H(\theta_{RS}) \leq 0 \). In this case, there will be no dine-in purchases regardless of price, and without loss of optimality, we let \( p_{RS} = 0 \).

**Case 2:** \( \bar{v}_R - c_R(1 - \alpha)\bar{F}_H(\theta_{RS}) > 0 \). In this case, strictly positive dine-in revenue is achievable. We can thus restrict our attention to prices with strictly positive dine-in volume, i.e., \( p < \bar{v}_R - c_R(1 - \alpha)\bar{F}_H(\theta_{RS}) \).

On this interval, differentiating the restaurant’s revenue (5) gives

\[ z'_{R,RS}(p) = \frac{\alpha}{\bar{v}_R + c_R\alpha} \left( \bar{v}_R - 2p - c_R(1 - \alpha)\bar{F}_H(\theta_{RS}) \right) \]

Differentiating a second time gives \( z''_{R,RS}(p) = \frac{-2\alpha}{\bar{v}_R + c_R\alpha} < 0 \). So, the restaurant’s revenue is concave in \( p \) on the relevant interval, and the first-order condition is sufficient for a global maximum. The FOC is

\[ p_{RS} = \frac{\bar{v}_R - c_R(1 - \alpha)\bar{F}_H(\theta_{RS})}{2} < \bar{v}_R - c_R(1 - \alpha)\bar{F}_H(\theta_{RS}), \]

(A.2)

implying that the solution to the FOC falls into the appropriate interval and is the globally optimal price. This FOC is replicated in the statement of the proposition, and the condition for the solution to be positive is equivalent to the hypothesis of this case. If the solution is negative, then Case 1 holds instead, and no dine-in volume is achievable for any price.

**Proof of Lemma 3.** Setting \( \theta = \theta_{RS} \) maximizes delivery revenues (this can be seen by noting that the platform’s revenue under simple revenue sharing is directly proportional to the total delivery revenue, independent of the dine-in price). Under our assumption that \( \bar{v}_R > c_R(1 - \alpha)\bar{F}_H(\theta_{RS}) = c_R(1 - \alpha)/2 \), there exists a dine-in price \( p > 0 \) such that some dine-in customers are willing to purchase with the congestion from delivery customers induced by delivery price \( \theta_{RS} \), which will achieve the same delivery revenue along with positive dine-in revenue. So, operating both channels dominates operating just the delivery channel, and any prices resulting in zero dine-in volume are strictly sub-optimal.

Similarly, it is straightforward to show that the optimal dine-in price for a system with no delivery customers is \( p = \bar{v}_R/2 \), so the maximum possible dine-in revenue that can be achieved is \( Z(\bar{v}_R/2, \bar{v}_H) \) (when
\( \theta = \bar{v}_H \), we have \( \hat{\gamma}_H = \bar{F}_H(\bar{v}_H) = 0 \). The partial derivative of the aggregate revenue with respect to \( \theta \), for values of \( \theta \) in which the dine-in channel is active, is

\[
\frac{\partial Z}{\partial \theta} = (1 - \alpha) \left[ f_H(\theta) \left( \frac{c_R \alpha p}{\bar{v}_R + c_R \alpha} - \theta \right) + \bar{F}_H(\theta) \right].
\] (A.3)

For \( p = \bar{v}_R/2 \) and substituting the density \( f_H \) and complementary CDF \( \bar{F}_H \), the corresponding FOC, which has a unique solution, is \( \theta = \frac{\bar{v}_H}{2} + \frac{1}{2} \left( \frac{\bar{v}_H}{2} \right) \left( \frac{c_R \alpha}{\bar{v}_R + c_R \alpha} \right) < \bar{v}_H \), where the inequality holds by our assumption that \( \bar{v}_R/2 < \bar{v}_H \). At a delivery price \( \theta = \bar{v}_H \) and dine-in price \( p = \bar{v}_R/2 \), no delivery customers will purchase, and positive dine-in volume will be achieved (this corresponds to the optimal solution of the dine-in-only system). Since the unique solution to the FOC is strictly less than \( \bar{v}_H \), and because the derivative is moving from positive to negative at this solution (the quantity in equation A.3 is strictly positive at \( \theta = 0 \)), the partial derivative \( \partial Z/\partial \theta \) is strictly negative at \( \theta = \bar{v}_H \) for \( p = \bar{v}_R/2 \).

Strictly speaking, the expression for the partial derivative given in (A.3) does not apply at \( p = \bar{v}_R/2 \) and \( \theta = \bar{v}_H \), as the function is not differentiable at this point because delivery volume exactly reaches zero there, creating a kink. So, we define the single-variable function \( Z_{\bar{v}_R/2}(\theta) \), which is the aggregate revenue as a function of \( \theta \), given a dine-in price of \( \bar{v}_R/2 \). This function coincides with \( Z \) for all price pairs with \( p = \bar{v}_R/2 \). The function \( Z_{\bar{v}_R/2} \) is left-differentiable for \( \theta \leq \bar{v}_H \), and the left derivative is equal to the expression on the RHS of equation (A.3), with the appropriate prices substituted. By the argument in the preceding paragraph, this left derivative is strictly negative at \( \theta = \bar{v}_H \). Hence, there exists \( \epsilon > 0 \) such that \( Z(\bar{v}_R/2, \bar{v}_H) < Z_{\bar{v}_R/2}(\bar{v}_H - \epsilon) = Z(\bar{v}_R/2, \bar{v}_H - \epsilon) \). Since the prices \( p = \bar{v}_R/2 \) and \( \theta = \bar{v}_H \) achieve the maximum possible dine-in revenue, the above implies that we can achieve strictly higher aggregate revenue by operating both channels than by operating only dine-in. Hence, any prices resulting in zero delivery volume are strictly sub-optimal.

Having established that it is strictly sub-optimal to operate either channel without the other, we conclude that it is centrally optimal to make both channels active.

**Proof of Proposition 3.** Solving the system (8)-(9) yields the unique solution \((p^*, \theta^*)\). As mentioned, the expressions are in closed form but uninformative and are omitted for brevity, but it is important to note that under the assumptions of Lemma 3, it can be shown that we have \( p^* > 0 \). Because \( p^* > 0 \), we must have \( \theta^* \geq \theta_{RS} \) by comparison of equations (4) and (9).

Substituting from equation (8), and letting \( \hat{\gamma}^{RS}_R \) be the equilibrium dine-in purchase fraction under simple
revenue sharing and $\gamma_R^*$ the corresponding quantity at the central optimum, we have

$$
\gamma_R^* = \frac{[\bar{v}_R - p^* - c_R(1 - \alpha)\bar{F}_H(\theta^*)]^+}{\bar{v}_R + c_R\alpha}
= \frac{1}{2}[\bar{v}_R - c_R(1 - \alpha)\bar{F}_H(\theta^*)]^+
\geq \frac{1}{2[\bar{v}_R + c_R\alpha]}\left[\bar{v}_R - c_R(1 - \alpha)\bar{F}_H(\theta_{RS})\right]^+
\geq \gamma_{RS}^R > 0,
$$

(A.4)

where the weak inequality holds because $\theta^* \geq \theta_{RS}$ and the strict inequality holds by our assumption that $\bar{v}_R > c_R(1 - \alpha)/2 = c_R(1 - \alpha)\bar{F}_H(\theta_{RS})$. Thus, there is strictly positive dine-in volume at the prices $(p^*, \theta^*)$.

Moreover, we have $p^* \leq \bar{v}_R/2 < \bar{v}_H$ by inspection of equation (8) and our assumption that $\bar{v}_R/2 < \bar{v}_H$. From equation (9), we then have

$$
\theta^* = \frac{\bar{v}_H}{2} + \frac{p^*}{2}\left(\frac{c_R\alpha}{\bar{v}_R + c_R\alpha}\right) < \frac{\bar{v}_H}{2} + \frac{\bar{v}_H}{2}\left(\frac{c_R\alpha}{\bar{v}_R + c_R\alpha}\right) < \bar{v}_H,
$$

which implies that $\bar{F}_H(\theta^*) > 0$, i.e., there is strictly positive delivery volume at the prices $(p^*, \theta^*)$. Thus, both channels are active at $(p^*, \theta^*)$, implying that the expressions for the partial derivatives are valid at these prices and that $(p^*, \theta^*)$ is indeed a critical point of the aggregate revenue function.

We can restrict our attention to prices $p \in [0, \bar{v}_R]$ and $\theta \in [0, \bar{v}_H]$ without loss of optimality. Consider any prices $(p, \theta)$ in this rectangle other than the pair $(p^*, \theta^*)$ (here we allow $p = p^*$ or $\theta = \theta^*$, but not both). If one or both channels is not active at $(p, \theta)$, then these prices are strictly sub-optimal by Lemma 3. On the other hand, if both channels are active, then the expressions for the partial derivatives that lead to the FOC’s (8) and (9) are valid in a neighborhood around $(p, \theta)$. That these prices fail to solve one or both of the FOC’s then implies that there is a strictly improving direction for the aggregate revenue, and hence, the price pair cannot be optimal. Thus, all price pairs other than $(p^*, \theta^*)$ are strictly sub-optimal. By the extreme value theorem, the aggregate revenue function $Z$ must achieve a global maximum on the rectangle $[0, \bar{v}_R] \times [0, \bar{v}_H]$ because it is a continuous function on a closed and bounded set. We conclude that the price pair $(p^*, \theta^*)$ uniquely achieves the global maximum of the aggregate revenue.

Proof of Corollary 2. Part (i) follows because the FOC (8) for the dine-in price $p$ in terms of $\theta$ is the same as the restaurant’s FOC for simple revenue sharing (equation A.2); the solution to this FOC is increasing in the delivery price (see Proposition 2 and surrounding discussion); and we have $\theta_{RS} < \theta^*$ by Corollary 1. Note that if the solution to the restaurant’s FOC is negative under simple revenue sharing, then by convention we take $p_{RS} = 0 < p^*$. Part (ii) follows from equation (A.4) in the proof of Proposition 3.
Proof of Proposition 4. After rearranging, the FOC’s (8) and (9) for maximizing the aggregate revenue can be expressed equivalently as
\[ p = \frac{\bar{v}_R}{2} - \frac{c_R(1 - \alpha)\bar{F}_H(\theta)}{2} =: g(\theta) \]  
(A.5)
and
\[ p = \left(\frac{\bar{v}_R + c_R \alpha}{c_R \alpha}\right)(2\theta - \bar{v}_R) =: h(\theta), \]  
(A.6)
respectively. The solution \((p^*, \theta^*)\) to this system of equations can be viewed as the intersection point of two increasing functions \(g(\theta)\) and \(h(\theta)\), as defined above. The function \(g(\theta)\) is linearly increasing in \(\theta\) because \(\bar{F}_H(\theta)\) is linearly decreasing in \(\theta\), while \(h(\theta)\) is linearly increasing in \(\theta\). In the price floor setting in which the restaurant sets both prices, the first FOC is the same as equation A.5. The second FOC becomes
\[ p = a \left(\frac{\bar{v}_R + c_R \alpha}{c_R \alpha}\right)(2\theta - \bar{v}_R) =: \tilde{h}(\theta). \]  
(A.7)
The assumptions in the proposition statement imply that a unique solution \((p_{PF}, \theta_{PF})\) exists to the system (A.5) and (A.7), with \(p_{PF}, \theta_{PF} > 0\); that this solution is the global maximizer of \(z_{R,PF}(p, \theta)\); and that both channels are active at these prices. Note that we have \(h(0) < \tilde{h}(0) < 0 < g(0)\) (where \(g(0) > 0\) by the assumption that \(\bar{v}_R > c_R(1 - \alpha)\)), implying that \(h\) increases to meet \(g\) at \((p^*, \theta^*)\), while \(\tilde{h}\) increases to meet \(g\) at \((\tilde{p}, \tilde{\theta})\).

We have \(g(\theta^*) = p^* = h(\theta^*) > ah(\theta^*) = \tilde{h}(\theta^*) > 0\). Thus, the function \(\tilde{h}\) is still below \(g\) at \(\theta^*\), implying that the intersection point of the increasing functions \(g\) and \(\tilde{h}\) must be at a larger value of \(\theta\) than the intersection of \(g\) and \(h\), i.e., we have \(\theta_{PF} > \theta^*\). Moreover, that the function \(g\) is increasing in \(\theta\) implies that also \(p_{PF} = g(\theta_{PF}) > g(\theta^*) = p^*\).

We note that it is indeed optimal to operate both channels by an analogous result to Lemma 3 for the restaurant’s revenue. In revenue sharing with a price floor, because \(\theta_{PF} > \theta_{RS}\), if the restaurant sets \(\theta = \theta_{PF}\), then the platform’s best response is to set \(\bar{\theta} = \max\{\theta_{RS}, \theta_{PF}\} = \theta_{PF}\). So, the restaurant can implement its own optimal solution by setting \(\bar{\theta} = \theta_{PF}\), which is therefore its equilibrium strategy (along with \(p = p_{PF}\)). Thus, the equilibrium with a simple price floor is the same as the outcome when the restaurant sets both prices, namely \(p = p_{PF}\) and \(\theta = \theta_{PF}\). \(\square\)

Proof of Proposition 5. Let \(Z^*\) be the optimal aggregate revenue, i.e., \(Z^* := \max_{p, \theta} Z(p, \theta)\), and suppose by way of contradiction that an equilibrium \((p, \theta)\) exists with \(Z(p, \theta) < Z^*\).

Case 1: A platform price \(\theta'\) exists such that \(Z(p, \theta') = Z^*\). In this case, we have
\[ z_{P,2W}(p, \theta) = (1 - b)Z(p, \theta) < (1 - b)Z^* = (1 - b)Z(p, \theta') = z_{P,2W}(p, \theta'). \]
Thus, the platform is acting sub-optimally, contradicting the equilibrium assumption.
Case 2: Otherwise. In this case, we have $\max_\theta Z(p, \theta) < Z^*$. But by Case 1, if the restaurant had chosen a price $p'$ such that $Z(p', \theta') = Z^*$ for some $\theta'$, then the platform’s best response would be $\theta'$, giving the restaurant a revenue of $bZ^*$. Because the restaurant’s revenue in the assumed equilibrium $(p, \theta)$ is bounded above by $b\max_\theta Z(p, \theta) < bZ^*$, we conclude that the restaurant could achieve a strictly higher revenue by setting a different price $p'$, again contradicting the equilibrium assumption.

Proof of Lemma 4. Substituting $\tau = (1 - a)c_R \alpha p^*/(\bar{v}_R + c_R \alpha)$ into the platform’s revenue function (12) and differentiating with respect to $\theta$ gives

$$\frac{\partial z_{P,G}}{\partial \theta} = (1 - a)(1 - \alpha)\left(\bar{F}_H(\theta) - \left(\theta - \frac{c_R \alpha p^*}{\bar{v}_R + c_R \alpha}\right)f_H(\theta)\right).$$

Evaluating this expression at $\theta = c_R \alpha p^*/(\bar{v}_R + c_R \alpha)$ (a lower price would earn the platform strictly negative net revenue) gives

$$\frac{\partial z_{P,G}}{\partial \theta} \bigg|_{\theta=p^*} = (1 - a)(1 - \alpha)\bar{F}_H\left(p^*\left(\frac{c_R \alpha}{\bar{v}_R + c_R \alpha}\right)\right) > 0,$$

where the inequality holds because $p^* < \bar{v}_H$ (see the proof of Proposition 3) and the fraction it is multiplied with is less than 1. The FOC has the same solution $\theta^*$ as the central planner’s optimal delivery price (equation 9). Being the unique solution to the FOC and a local maximum (derivative is transitioning from positive to negative by equation A.8), this solution is the global maximizer of the platform’s revenue. We conclude that the platform sets the centrally optimal delivery price $\theta^*$.

Proof of Proposition 6. Differentiating the revenue (14) gives the same FOC as the central optimum, namely equation (8), replacing $\theta$ with $\theta^*$. This is also the same as the restaurant’s FOC under simple revenue sharing (replacing $\theta_{RS}$ with $\theta^*$). Analogous arguments to those in the proof of Proposition 2 imply that the unique solution to this FOC is optimal for the restaurant (and results in positive dine-in volume with delivery price $\theta^*$ by the proof of Proposition 3), and the resulting solution is the centrally optimal restaurant price $p^*$. We conclude that under a generalized revenue sharing contract with $\tau$ that satisfies equation (13) for $0 \leq a < 1$, the equilibrium prices are $(p^*, \theta^*)$ and the maximum possible aggregate revenue is achieved.

Proof of Proposition 7. Under a coordinating generalized revenue sharing contract, the platform’s payment to the restaurant for each delivery order is

$$a\theta^* + (1 - a)p^*\left(\frac{c_R \alpha}{\bar{v}_R + c_R \alpha}\right) = a(\bar{v}_H - \theta^*) + p^*\left(\frac{c_R \alpha}{\bar{v}_R + c_R \alpha}\right).$$

Point (i) follows by inspection of equation (A.9): for any value of $a$, the restaurant’s payment on each delivery order is at least $p^*(c_R \alpha)/(\bar{v}_R + c_R \alpha)$. For (ii), if $\bar{v}_R < \bar{v}_H$, then it can be deduced that $a\theta^* + \tau \geq p^*$ if and
only if
\[
a \geq \bar{v}_R \left( \frac{2\bar{v}_R - c_R(1 - \alpha)}{2\bar{v}_H\bar{v}_R + c_R\alpha(2\bar{v}_H - \bar{v}_R)} \right), \tag{A.10}
\]
If \( \bar{v}_R < \bar{v}_H \), then the RHS of equation (A.10) is bounded above by \( \bar{v}_R/\bar{v}_H \). The inequality therefore holds by our assumption for (ii) that \( a \geq \bar{v}_R/\bar{v}_H \), and the proof is complete.

\textbf{Proof of Proposition 8.} Follows immediately by letting \( a = 0 \) in Proposition 6.

\textbf{Proof of Proposition 9.} An allocation of zero to the platform can be approached by setting \( a \approx 1 \). On the other hand, the maximum allocation that the delivery platform can receive under a coordinating generalized revenue sharing contract is with \( a = 0 \). In that case, the platform’s net revenue per order is \( \theta^* - p^* \left( \frac{c_R\alpha}{\bar{v}_R + c_R\alpha} \right) \). Dividing this quantity by \( \theta^* \) gives the fraction \( \kappa \) of delivery revenue kept by the platform, namely \( \kappa = 1 - \frac{p^*}{\theta^*} \left( \frac{c_R\alpha}{\bar{v}_R + c_R\alpha} \right) \). From equations (8) and (9), respectively, we have \( p^* \leq \bar{v}_R/2 \) and \( \theta^* \geq \bar{v}_H/2 \). Combined with our assumption that \( c_R\alpha \leq \bar{v}_R \leq \bar{v}_H \), these bounds imply
\[
\frac{1}{2} \leq 1 - \frac{c_R\alpha}{\bar{v}_R + c_R\alpha} \leq 1 - \frac{\bar{v}_R/2}{\bar{v}_H/2} \left( \frac{c_R\alpha}{\bar{v}_R + c_R\alpha} \right) \leq 1 - \frac{p^*}{\theta^*} \left( \frac{c_R\alpha}{\bar{v}_R + c_R\alpha} \right) = \kappa,
\]
which completes the proof.

\textbf{B No Contract Case}

Another possible financial arrangement between restaurant and platform is the lack of any formal contract: we call this the “no contract” case. In this case, the platform pays the dine-in menu price for each delivery order, like any other customer, and sets its own markup on top of this price. In this appendix, we briefly discuss this arrangement. The sequence of events is the same as in the rest of the paper, and the equilibrium purchase fractions are again determined from Lemmas 1 and 2. Given a dine-in price \( p \), the platform’s net revenue from charging a delivery price \( \theta \) is given by
\[
z_{P,NC}(p, \theta) = (1 - \alpha)(\theta - p)\hat{\gamma}_H.
\]
The platform clearly should set \( \theta \geq p \), as otherwise it loses money on every order. So, if \( p \geq \bar{v}_H \), then the platform is priced out of the market: any price that achieves positive net revenue for the platform will result in zero delivery volume. If \( p < \bar{v}_H \), then the platform can induce positive volume at a money-making price. In this setup, the restaurant earns \( p \) for each order, whether dine-in or delivery. So, the restaurant’s revenue function is
\[
z_{R,NC}(p, \theta) = p(\alpha\hat{\gamma}_R + (1 - \alpha)\hat{\gamma}_H).
\]
Solving the game by backward induction, we have the equilibrium prices given in the following proposition. The constants $\delta_1$ and $\delta_2$ depend on $\bar{v}_R, \bar{v}_H,$ and $\alpha$.

**Proposition B.1** (No-Contract Equilibrium Prices). If $\bar{v}_R/2 < \bar{v}_H \leq 2\bar{v}_R$ and $c_R \leq \min\{\delta_1, \delta_2\}$, then both channels operate in equilibrium, and the dine-in price $p_{NC}$ and delivery price $\theta_{NC}$ are given by

$$p_{NC} = \frac{\bar{v}_H \bar{v}_R (1 + \alpha)}{2(2\bar{v}_H \alpha + \bar{v}_R (1 - \alpha))} \quad \text{and} \quad \theta_{NC} = \frac{\bar{v}_H + p_{NC}}{2}.$$ 

We omit the proof of Proposition B.1 for brevity. Figure B.1 illustrates that the aggregate revenue without any formal contract can be higher than the aggregate revenue under simple revenue sharing. In the figure, these scenarios are for larger $\alpha$. That the complete absence of a contract can outperform simple revenue sharing again highlights the problems of the latter. However, the no-contract arrangement is not always an improvement over simple revenue sharing, and sometimes can perform much worse, which we observe in the figure for smaller $\alpha$. Neither contract dominates the other, and both are sub-optimal, unlike generalized revenue sharing.

**C Positive Delivery Channel Waiting Costs**

In our base model, we assumed that delivery customers are not sensitive to congestion—i.e., they have zero waiting cost, $c_H = 0$—in order to reflect that they are less congestion-sensitive than dine-in customers in a tractable way. In this appendix, we relax the assumption by numerically considering the case in which delivery customers have a positive waiting cost $c_H > 0$.

First, consider fixed prices $p$ and $\theta$. The equilibrium dine-in threshold $v_R$ and delivery threshold $v_H$
(derived for the base model in Lemmas 1 and 2) are found from the simultaneous solution to the equations

\[ v_R = p + c_R(\alpha \bar{F}_R(v_R) + (1 - \alpha) \bar{F}_H(v_H)) \]  

(C.1)

and

\[ v_H = \theta + c_H(\alpha \bar{F}_R(v_R) + (1 - \alpha) \bar{F}_H(v_H)), \]  

(C.2)

if this solution results in positive volume on both channels. Equations (C.1) and (C.2) form a linear system with two unknowns, and the system has a unique solution that can be found in closed form.

If the solution does not give positive volume on both channels, then in equilibrium at most one channel is active. A dine-in-only equilibrium must have a dine-in threshold that satisfies

\[ v_R = p + c_R \alpha \bar{F}_R(v_R). \]

If, for a dine-in purchase fraction determined by the above threshold, there is no delivery volume, then the dine-in-only equilibrium exists. Analogous arguments reveal conditions for a delivery-only equilibrium to exist. It is possible that multiple equilibria exist (i.e., both channels active, and only one channel active); in our numerical analysis, our selection rule is to begin by searching for a multi-channel equilibrium, then search for a dine-in-only equilibrium if the multi-channel fails to exist, and finally search for a delivery-only equilibrium if both the previous two equilibria fail to exist. If none of the equilibria exist, then we say that the equilibrium purchase fractions are zero on both channels for the given prices.

Aside from the introduction of \( c_H > 0 \), the model is otherwise identical to our base model. To analyze this model, we numerically determined the optimal delivery and dine-in prices under both simple revenue sharing (as in §4) and for a central planner (as in §5), and compared both the prices and aggregate revenue for these two different cases. We also calculated the prices and revenue under generalized revenue sharing with terms set based on the analysis in §6.4 (i.e., the contract terms are determined assuming \( c_H = 0 \)).

Figure C.1 presents the results of this analysis for a representative example. The results suggested in the figure align qualitatively with our previous findings. Namely, (i) simple revenue sharing fails to coordinate because the platform sets the delivery price too low under simple revenue sharing, crowding out dine-in customers and reducing the aggregate revenue, and (ii) generalized revenue sharing achieves near-optimal aggregate revenue. Indeed, in the figure, the outcomes under generalized revenue sharing and the central optimum appear extremely close, perhaps even identical. However, as we next prove via a counterexample, generalized revenue sharing with the parameters from §6.4 may not, in general, perfectly coordinate the system for \( c_H > 0 \):

**Proposition C.1** (Generalized Revenue Sharing with \( c_H > 0 \)). For \( c_H > 0 \), a generalized revenue sharing contract with parameters defined in Proposition 6 (i.e., assuming \( c_H = 0 \)) does not always achieve the maximum aggregate revenue.
Proof. Consider an instance of our model with \( \bar{v}_R = \bar{v}_H = 1 \), \( c_R = c_H = 1 \), and \( \alpha = 1/2 \). In this case, for prices \( p \) for which both channels are active, the simultaneous solution of equations (C.1) and (C.2) gives

\[
\hat{\gamma}_R = \frac{1}{2} + \frac{\theta}{4} - \frac{3p}{4} \quad \text{and} \quad \hat{\gamma}_H = \frac{1}{2} + \frac{p}{4} - \frac{3\theta}{4}.
\]

Substituting these values into the aggregate revenue function and solving the first-order conditions yields the prices \( p^* = \theta^* = 1/2 \), achieving aggregate revenue of 1/4, and indeed both channels are active at these prices with equilibrium purchase fractions \( \hat{\gamma}_R = \hat{\gamma}_H = 1/4 \). Moreover, the function is concave (the Hessian matrix is easily verified to be negative definite), so these prices are indeed optimal among prices that induce both channels to be active. Finally, it is optimal to induce both channels to be active because solving the problem for a single channel yields a maximum revenue of 1/6. Thus, \( p^* = \theta^* = 1/2 \) are the centrally optimal prices for this instance.

Now, consider our generalized revenue sharing contract with the terms found in §6.4. Namely, for a revenue share \( a \), we let the flat payment \( \tau = (1 - a)p^*(c_R\alpha/\bar{v}_R + c_R\alpha) = (1 - a)/6 \). For a dine-in price \( p = p^* = 1/2 \), we have

\[
\hat{\gamma}_R = \frac{1}{2} + \frac{\theta}{4} - \frac{3p}{4} = \frac{1}{8} + \frac{\theta}{4} > 0,
\]

so, in the neighborhood of this dine-in price, there will be positive dine-in volume no matter the delivery price, meaning that the formulas above for \( \hat{\gamma}_R \) and \( \hat{\gamma}_H \) are valid in this neighborhood (we can ignore prices \( \theta \) high enough that there is no delivery volume, as these are clearly sub-optimal for the platform). Substituting \( \hat{\gamma}_R, \hat{\gamma}_H, \) and \( \tau = (1 - a)/6 \) into the platform’s generalized revenue sharing revenue function and differentiating gives the first-order condition \( \theta = 5/12 + p/6 \), which is the platform’s optimal price as a function of the dine-in price and will also yield positive delivery volume for any dine-in price. We then substitute this expression for \( \theta \) into the restaurant’s revenue function under generalized revenue sharing, which eliminates \( \theta \) and yields a function only of the dine-in price \( p \). Differentiating this revenue function for the restaurant

Figure C.1. Revenues and purchase volumes under different contracts (\( \bar{v}_R = 25, \bar{v}_H = 18, c_R = 22, c_H = 5 \))
gives the derivative

\[
\frac{30 - 68p + a(3 + 2p)}{48}.
\]

Then, substituting \( p = 1/2 \) into the derivative gives \(- (1 - a)/12 < 0\), implying that there is a strictly improving direction for the restaurant. Thus, the restaurant will not set \( p = 1/2 \), implying that generalized revenue sharing does not achieve the optimal aggregate revenue.

Because the generalized revenue sharing contract with terms set assuming \( c_H = 0 \) does not always perfectly coordinate, we conducted a comprehensive numerical study to evaluate its performance. We considered 2,700 parameter combinations (21,600 combinations if we account for variations in the revenue share \( a \)) and determined the aggregate revenue under the centralized solution, simple revenue sharing, and generalized revenue sharing using the contract terms from §6.4. We considered values of \( \alpha \) from .1 to .9, values of \( c_R \) from 10 to 30, values of \( c_H \) from 2 to 8, values of \( \bar{v}_R \) from 8 to 40, values of \( \bar{v}_H \) from 8 to 40, and values of the restaurant’s delivery revenue share \( a \) from .6 to .95.

We observed that the percentage revenue loss under simple revenue sharing ranges from 0% to 11.1%, with an average of 1.2% and a median of .6%. By contrast, under generalized revenue sharing, the percentage revenue loss ranges from 0% to 0.3%, with an average of 0.006% and a median of .0013%. Thus, encouragingly, in line with Figure C.1, we see from the comprehensive study that generalized revenue sharing achieves near-optimal performance over a wide range of parameters. These findings suggest that generalized revenue sharing is robust to delivery customers who exhibit some degree of congestion-sensitivity. A key reason for this is that, since the restaurant receives a share of the platform revenue (as well as a fixed fee for each delivery order), the restaurant when setting its price already internalizes some (perhaps even most) of the negative externality that dine-in customers impose on congestion-sensitive delivery customers.