November 11, 2013

Midterm Exam II

CE/EE/TE 3302: Signals and Systems

NOTE: Please, complete the following table and keep record of your assignment number.

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**Exercise 1.** Consider the periodic signal \( x(t) \), which has the following Fourier series coefficients: \( a_1 = \alpha \) and \( a_2 = j \beta \). Both \( \alpha \) and \( \beta \) are positive real constant values. Let \( T \) be the fundamental period of \( x(t) \). Consider signal \( y(t) = \int_{-\infty}^{t} x(\tau - 2T) \, d\tau \).

A) Determine whether \( y(t) \) is a periodic signal, and if so, derive the coefficients of the Fourier series of \( y(t) \), i.e., \( b_k \) [pt. 15].

B) Compute how much power is carried by the second harmonic component of \( y(t) \), i.e., \( P_2 \) [pt. 10].

**Exercise 2.** Consider the continuous-time signal \( x(t) = u(t) e^{-\alpha t} \sin(\omega_0 t) \cos(\omega_0 t) \), where \( u(t) \) is the unit step function.

A) Derive an expression for the Fourier transform of \( x(t) \), i.e., \( X(j\omega) \) [pt. 15].

**Exercise 3.** A continuous-time signal \( x(t) \) has the following spectrum

\[
X(j\omega) = \begin{cases} 
\frac{1 + \cos(\omega T_1)}{2} & |\omega| < \frac{\pi}{T_1} \\
0 & \text{otherwise} 
\end{cases}
\]

where \( T_1 \) is a constant positive real value.

A) Derive an analytical expression for \( x(t) \) [pt. 15].

**Exercise 4.** Consider the continuous-time signal

\[
x(t) = \frac{\sin(Bt)}{\pi t} e^{-jBt}
\]

where \( B \) is a constant positive real value. Signal \( x(t) \) is sent to the input of a LTI system. The output signal of the system is defined as \( y(t) = \frac{d}{dt} x(t) \).

A) Calculate the energy of \( x(t) \) [pt. 10].

B) Compute the spectrum of the output signal, i.e., \( Y(j\omega) \), and calculate the energy of \( y(t) \) [pt. 15].

**Exercise 5.** Consider the continuous-time signal \( x(t) = -te^{\alpha t}u(-t) \), where \( \alpha \) is a real and positive constant. Signal \( x(t) \) is the input signal of a LTI system, which is described by the following differential equation

\[
y(t) = \alpha \frac{d}{dt} x(t) - \frac{d^2}{dt^2} x(t)
\]

where \( y(t) \) is the output signal of the LTI system.

A) Derive an analytical expression for the frequency response of the LTI system, i.e., \( H(j\omega) \), and the spectrum of the output signal, i.e., \( Y(j\omega) \) [pt. 20].