BASIC PROBLEMS

7.21. A signal $x(t)$ with Fourier transform $X(j\omega)$ undergoes impulse-train sampling to generate

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT) \delta(t - nT)$$

where $T = 10^{-4}$. For each of the following sets of constraints on $x(t)$ and/or $X(j\omega)$, does the sampling theorem (see Section 7.1.1) guarantee that $x(t)$ can be recovered exactly from $x_p(t)$?
(a) $X(j\omega) = 0$ for $|\omega| > 5000\pi$
(b) $X(j\omega) = 0$ for $|\omega| > 15000\pi$
(c) $\text{Re}\{X(j\omega)\} = 0$ for $|\omega| > 5000\pi$
(d) $x(t)$ real and $X(j\omega) = 0$ for $\omega > 5000\pi$
(e) $x(t)$ real and $X(j\omega) = 0$ for $\omega < -15000\pi$
(f) $X(j\omega) \ast X(j\omega) = 0$ for $|\omega| > 15000\pi$
(g) $|X(j\omega)| = 0$ for $\omega > 5000\pi$

7.22. The signal $y(t)$ is generated by convolving a band-limited signal $x_1(t)$ with another band-limited signal $x_2(t)$, that is,

$$y(t) = x_1(t) \ast x_2(t)$$

where

$$X_1(j\omega) = 0 \quad \text{for} \ |\omega| > 1000\pi$$
$$X_2(j\omega) = 0 \quad \text{for} \ |\omega| > 2000\pi.$$
\[ y_p(t) = \sum_{n=-\infty}^{\infty} y(nT) \delta(t - nT). \]

Specify the range of values for the sampling period \( T \) which ensures that \( y(t) \) is recoverable from \( y_p(t) \).

7.23. Shown in Figure P7.23 is a system in which the sampling signal is an impulse train with alternating sign. The Fourier transform of the input signal is as indicated in the figure.
(a) For \( \Delta < \pi/(2\omega_M) \), sketch the Fourier transform of \( x_p(t) \) and \( y(t) \).
(b) For \( \Delta < \pi/(2\omega_M) \), determine a system that will recover \( x(t) \) from \( x_p(t) \).
(c) For \( \Delta < \pi/(2\omega_M) \), determine a system that will recover \( x(t) \) from \( y(t) \).
(d) What is the maximum value of \( \Delta \) in relation to \( \omega_M \) for which \( x(t) \) can be recovered from either \( x_p(t) \) or \( y(t) \)?

7.24. Shown in Figure P7.24 is a system in which the input signal is multiplied by a periodic square wave. The period of \( s(t) \) is \( T \). The input signal is band limited with \( |X(\omega)| = 0 \) for \( |\omega| \geq \omega_M \).