**BASIC PROBLEMS**

4.21. Compute the Fourier transform of each of the following signals:

(a) \([e^{-\alpha t} \cos \omega_0 t]u(t), \alpha > 0\]  
(b) \([e^{-\beta t} \sin 2t]\]  
(c) \(x(t) = \begin{cases} 1 + \cos \pi t, & |t| \leq 1 \\ 0, & |t| > 1 \end{cases}\]  
(d) \(\sum_{k=0}^{\infty} \alpha^k \delta(t - kT), |\alpha| < 1\]  
(e) \([re^{-2t} \sin 4t]u(t)\]  
(f) \(\begin{bmatrix} \sin \pi t \\ -\frac{\sin 2\pi(t-1)}{\pi(t-1)} \end{bmatrix}\]  
(g) \(x(t)\) as shown in Figure P4.21(a)  
(h) \(x(t)\) as shown in Figure P4.21(b)  
(i) \(x(t) = \begin{cases} 1 - t^2, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}\)  
(j) \(\sum_{n=-\infty}^{+\infty} e^{-|t-2n|}\)

![Figure P4.21](image)

4.22. Determine the continuous-time signal corresponding to each of the following transforms.

![Figure P4.22](image)
4.25. Let $X(j\omega)$ denote the Fourier transform of the signal $x(t)$ depicted in Figure P4.25.
(a) $X(j\omega)$ can be expressed as $A(j\omega) e^{j\Theta(\omega)}$, where $A(j\omega)$ and $\Theta(\omega)$ are both real-valued. Find $\Theta(j\omega)$.
(b) Find $X(j0)$.
(c) Find $\int_{-\infty}^{\infty} X(j\omega) \, d\omega$.
(d) Evaluate $\int_{-\infty}^{\infty} X(j\omega) \frac{2\sin(\omega)}{\omega} e^{j2\omega} \, d\omega$.
(e) Evaluate $\int_{-\infty}^{\infty} |X(j\omega)|^2 \, d\omega$.
(f) Sketch the inverse Fourier transform of $\Re\{X(j\omega)\}$.
Note: You should perform all these calculations without explicitly evaluating $X(j\omega)$.

![Figure P4.25](image)

4.26. (a) Compute the convolution of each of the following pairs of signals $x(t)$ and $h(t)$ by calculating $X(j\omega)$ and $H(j\omega)$, using the convolution property, and inverse transforming.
(i) $x(t) = te^{-2t}u(t)$, $h(t) = e^{-4t}u(t)$
(ii) $x(t) = te^{-2t}u(t)$, $h(t) = te^{-4t}u(t)$
(iii) $x(t) = e^{-t}u(t)$, $h(t) = e^t u(-t)$
(b) Suppose that $x(t) = e^{-(t-2)}u(t-2)$ and $h(t)$ is as depicted in Figure P4.26. Verify the convolution property for this pair of signals by showing that the Fourier transform of $y(t) = x(t) * h(t)$ equals $H(j\omega)X(j\omega)$.

![Figure P4.26](image)

4.27. Consider the signals

$$x(t) = u(t - 1) - 2u(t - 2) + u(t - 3)$$

and

$$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} x(t - kT),$$
where \( T > 0 \). Let \( a_k \) denote the Fourier series coefficients of \( x(t) \), and let \( X(j\omega) \) denote the Fourier transform of \( x(t) \).
(a) Determine a closed-form expression for \( X(j\omega) \).
(b) Determine an expression for the Fourier coefficients \( a_k \) and verify that \( a_k = \frac{1}{T}X\left(\frac{2\pi k}{T}\right) \).

4.28. (a) Let \( x(t) \) have the Fourier transform \( X(j\omega) \), and let \( p(t) \) be periodic with fundamental frequency \( \omega_0 \) and Fourier series representation
\[
p(t) = \sum_{n=-\infty}^{+\infty} a_n e^{jn\omega_0 t}.
\]
Determine an expression for the Fourier transform of
\[
y(t) = x(t)p(t). \tag{P4.28-1}
\]
(b) Suppose that \( X(j\omega) \) is as depicted in Figure P4.28(a). Sketch the spectrum of \( y(t) \) in eq. (P4.28-1) for each of the following choices of \( p(t) \):
(i) \( p(t) = \cos(t/2) \)
(ii) \( p(t) = \cos t \)
(iii) \( p(t) = \cos 2t \)
(iv) \( p(t) = (\sin t)(\sin 2t) \)
(v) \( p(t) = \cos 2t - \cos t \)
(vi) \( p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - \pi n) \)
(vii) \( p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - 2\pi n) \)
(viii) \( p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - 4\pi n) \)
(ix) \( p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - 2\pi n) - \frac{1}{T} \sum_{n=-\infty}^{+\infty} \delta(t - \pi n) \)
(x) \( p(t) \) is the periodic square wave shown in Figure P4.28(b).

![Figure P4.28](image-url)
4.30. Suppose $g(t) = x(t) \cos t$ and the Fourier transform of the $g(t)$ is

$$G(j\omega) = \begin{cases} 1, & |\omega| \leq 2 \\ 0, & \text{otherwise} \end{cases}.$$  

(a) Determine $x(t)$.

(b) Specify the Fourier transform $X_1(j\omega)$ of a signal $x_1(t)$ such that

$$g(t) = x_1(t) \cos \left(\frac{2}{3}t\right).$$