HW # 11

1) \[ \frac{d^2 y(t)}{dt^2} - 9 y(t) = \frac{d^2 x(t)}{dt^2} - \frac{d x(t)}{dt} - 6 x(t) \]

a) \[ H(j\omega) = \frac{x(j\omega)}{y(j\omega)} = \frac{(j\omega)^2 - (j\omega) - 6}{(j\omega)^2 - 9} = \frac{j\omega + 2}{j\omega + 3} \]

b) \[ 1 - \frac{1}{j\omega + 3} = \delta(t) - e^{-3t} u(t) \]

c) \[ G(j\omega) = \frac{1}{H(j\omega)} = \frac{j\omega + 3}{j\omega + 2} \]

d) \[ 1 + \frac{1}{j\omega + 2} = \delta(t) + e^{-2t} u(t) \]

2) \[ \frac{d^2 y(t)}{dt^2} + 7 \frac{dy(t)}{dt} + 10 y(t) = \frac{d^2 x(t)}{dt^2} + 8 \frac{dx(t)}{dt} + 15 x(t) \]

a) \[ H(j\omega) = \frac{(j\omega)^2 + 8j\omega + 15}{(j\omega)^2 + 7j\omega + 10} = \frac{(j\omega + 5)(j\omega + 3)}{(j\omega + 5)(j\omega + 2)} = \frac{j\omega + 3}{j\omega + 2} \]

b) \[ 1 - \frac{1}{j\omega + 2} = \delta(t) - e^{-2t} u(t) \]

c) \[ G(j\omega) = \frac{1}{H(j\omega)} = \frac{j\omega + 2}{j\omega + 3} \]

d) \[ 1 - \frac{1}{j\omega + 3} = \delta(t) + e^{-3t} u(t) \]

3) a) \[ H(j\omega) = \frac{(j\omega)^2 + 7(j\omega) + 10}{(j\omega)^2 + 8(j\omega) + 15} = \frac{(j\omega + 5)(j\omega + 2)}{(j\omega + 5)(j\omega + 3)} = \frac{j\omega + 3}{j\omega + 2} \]

b) \[ 1 - \frac{1}{j\omega + 3} = \delta(t) - e^{-3t} u(t) \]

c) \[ G(j\omega) = \frac{j\omega + 2}{j\omega + 2} \]

d) \[ 1 + \frac{1}{j\omega + 2} = \delta(t) + e^{-2t} u(t) \]
4) a) \( H(j\omega) = \frac{(j\omega)^2 + 4(j\omega) + 3}{(j\omega)^2 + 5(j\omega) + 6} = \frac{(j\omega + 3)(j\omega + 1)}{(j\omega + 2)(j\omega + 3)} = \frac{j\omega + 1}{j\omega + 2} \)

b) \( 1 - \frac{1}{j\omega + 2} = \delta(t) - e^{-2t}u(t) \)

c) \( G(j\omega) = \frac{j\omega + 2}{j\omega + 1} \)

d) \( 1 + \frac{1}{j\omega + 1} = \delta(t) + e^{-t}u(t) \)

5) a) \( H(j\omega) = \frac{(j\omega)^2 + 8(j\omega) + 1}{(j\omega)^2 + 4j\omega + 4} = \frac{(j\omega + 1)^2}{(j\omega + 2)^2} \)

b) Solve using differential eqns

\( H(j\omega) = 1 - \frac{2}{j\omega + 2} + \frac{1}{(j\omega + 2)^2} = \delta(t) + \frac{2}{j\omega + 2} \)

\( = \delta(t) - 2e^{-2t}u(t) + t e^{-2t}u(t) \)

c) \( G(j\omega) = \frac{(j\omega + 2)^2}{(j\omega + 1)^2} \)

d) \( 1 + \frac{\omega}{j\omega + 1} + \frac{1}{(j\omega + 1)^2} = \delta(t) + \left[ e^{-t} + t e^{-t} \right] u(t) \)

6) a) \( H(j\omega) = \frac{(j\omega)^2 + 8j\omega + 15}{(j\omega)^2 + 10j\omega + 21} = \frac{(j\omega + 3)(j\omega + 5)}{(j\omega + 3)(j\omega + 7)} = \frac{j\omega + 5}{j\omega + 7} \)

b) \( 1 - \frac{2}{j\omega + 7} = \delta(t) - 2e^{-7t}u(t) \)

c) \( G(j\omega) = \frac{j\omega + 7}{j\omega + 5} \)

d) \( 1 + \frac{2}{j\omega + 5} = 1 + 2e^{-5t}u(t) \)