Exercise 1. Representing all the relevant intermediate steps, find a minimum weight spanning tree on the graph shown in Fig. 1 using, respectively,

A) the Prim-Dijkstra algorithm choosing node 5 as the root vertex [pt. 10],

B) the Kruskal algorithm [pt. 10].

Exercise 2. Consider the graph shown in Fig. 2. Note that every arc in the figure represents two distinct directed arcs with opposite directions and same weight (shown next to the arc). Using a graphical or matrix based representation of each intermediate iteration, find the shortest path from every node to node 5 as indicated below.

A) Run the first two iterations of the Dijkstra algorithm and show the path costs at the end of the second iteration [pt. 15].
B) Identify the paths found at the end of the second iteration [pt. 10].

C) Continue to run iterations until a stop condition is reached, reporting the paths found and their respective costs [pt. 15].

Exercise 3. Consider the open network of three queues Q1, Q2, and Q3. Each queue has a single server. The service times at the queues are independent and exponentially distributed with mean 1/μ1, 1/μ2, and 1/μ3, respectively. Customers (or jobs) entering the network form a Poisson arrival process with rate λ. Upon entering the network of queues, a job chooses to enter Q1 with probability p, or Q3 with probability 1 − p. Jobs leaving Q3 will depart from the network for good. Jobs leaving Q1 will enter Q2. Jobs leaving Q2 will depart from the network for good.

A) Find the stability conditions of the network of queues [pt. 10].

B) Find N, defined as the average number of jobs in the entire network of queues at steady state [pt. 15].

C) Find T, defined as the average time spent in the network of queues by the generic job [pt. 15].

D) Assume μ1 = μ2 = μ3 = μ. Find pT, defined as the value of p which guarantees a fair system, i.e., the average time spent by a job going through both Q1 and Q2 (T1 + T2) is equal to the average time spent by a job going through Q3 (T3) [pt. 15].

E) Find the stability conditions of the network of queues when p = pT and μ1 = μ2 = μ3 = μ (answer must contain only λ and μ) [pt. 15].