Problem #2

\( \mu_1 = \mu_2 = \mu \)

B) \[
\begin{align*}
\hat{p}_1 &= \frac{dP}{d\mu_1} \\
\hat{p}_2 &= \frac{d(1-P)}{d\mu_2}
\end{align*}
\]

\[
P(\text{Server 1 is busy, Server 2 is busy}) = \frac{d^2}{\mu^2} P(1-P)
\]

\[
\max \{ P(1-P)^2 \} \rightarrow P_{\text{max}} = 0.5
\]

C) Given two packets in router, we have 3 cases

\[
P(n_1 = 1, n_2 = 1) = (1-P_1)(1-P_2) \cdot \hat{p}_1 \cdot \hat{p}_2
\]

\[
P(n_1 = 2, n_2 = 0) = (1-P_1)(1-P_2) \cdot \hat{p}_2^2
\]

\[
P(n_1 = 0, n_2 = 2) = (1-P_1)(1-P_2) \cdot \hat{p}_2^2
\]

\[
P(\text{of interest}) = \frac{\hat{p}_1 \cdot \hat{p}_2}{\hat{p}_1^2 + \hat{p}_1 \hat{p}_2 + \hat{p}_2^2}
\]

\[
\left| \frac{1}{P - P_{\text{max}}} \right| = \frac{1}{3}
\]
Problem # 3

\[ d = 4 \text{ jobs/ min} \]
\[ P = 0.25 \]

\[ \frac{1}{\mu_1} = 8 \text{ min}^{-1} \quad \frac{1}{\mu_2} = 2 \text{ min}^{-1} \quad \frac{1}{\mu_3} = 4 \text{ min}^{-1} \]

A) \[ P_1 = \frac{d}{\mu_1} = \frac{4}{8} = 0.5 \quad \text{not stable} \]

\[ P_2 = \frac{d \cdot P}{\mu_2} = \frac{4 \cdot 0.25}{\frac{1}{2}} = 2 < 1 \quad \text{not stable} \]

\[ P_3 = \frac{d (1-P)}{\mu_3} = \frac{4 \cdot 0.75}{\frac{1}{3}} = 3 \cdot 4 = 12 \quad \text{not stable} \]

B) Product form

Assume now that \( P_1 < 1, P_2 < 1, P_3 < 1 \)

\[ P(n_1, n_2, n_3) = (1-P_1)^n_1 \cdot (1-P_2)^n_2 \cdot (1-P_3)^n_3 \]
c) \[ N_1 = \frac{p_1}{1 - p_1} \]
\[ N_2 = \frac{p_2}{1 - p_2} \]
\[ N_3 = \frac{p_3}{1 - p_3} \]

d) Total time in network of Queue by average job?
\[ T = \frac{N}{d} = \frac{N_1 + N_2 + N_3}{d} \]