Stochastic dynamical wake modeling using partially available field measurements

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Track 4: Wind turbines and wind plants

Introductory Summary

Low-fidelity analytical models of turbine wakes have traditionally been used for wind farm planning, performance evaluation, and demonstrating the utility of advanced control algorithms in increasing the annual energy production. In practice, however, it remains challenging to correctly estimate the flow and achieve significant performance gains using conventional low-fidelity models. This is due to the over-simplified static nature of wake predictions from models that are agnostic to the complex aerodynamic interactions among turbines. To improve the predictive capability of low-fidelity models while remaining amenable to control design, we offer a stochastic dynamical modeling framework for capturing the effect of atmospheric turbulence on the thrust force and power generation as determined by the actuator disk concept. In this approach, we use stochastically forced linear models of the turbulent velocity field to augment the analytically computed wake velocity and achieve consistency with higher-fidelity models in capturing power and thrust force measurements. The power-spectral densities of our stochastic models are identified via convex optimization to ensure consistency with partially available velocity statistics or power and thrust force measurements. Our results provide insight into the significance of sparse field measurements in recovering the statistical signature of the flow using stochastic linear models.

Keywords: *Convex optimization, stochastically forced Navier-Stokes equations, wake modeling*

Introduction

A major challenge in wind farm control arises from the importance of nonlinear aerodynamic interactions that can result in a reduction in power production and an elevation in dynamic loads while inducing wake recovery. An accurate representation of wake turbulence can thus notably affect the performance of control strategies in improving energy production and structural durability. While high-fidelity models, e.g., large-eddy simulations (LES) [1–3], capture such complex wake interactions, they are computationally expensive and are thus not suitable for the development of online model-based control strategies that can adapt to time-varying atmospheric conditions informed by supervisory control and data acquisition (SCADA) measurements. On the other hand, low-fidelity analytical models such as the Frandsen [4] or the Jensen-Park [5, 6] models predict the wind velocity in the wake of turbines under steady atmospheric conditions. Such models neglect the time-varying near-field turbulence behind the wind turbine and are often combined with linear wake superposition laws providing an over-simplified static prediction of the wind velocity. In the absence of a turbulence model that can induce mixing in the turbine wakes, velocity deficits are typically over-predicted by such models leading to inaccurate predictions of quantities of interest for wind farm control, i.e., the load and power corresponding to each turbine.

In this work, we seek *linear dynamical models* of the velocity field in a wind farm that compensate the shortcomings of low-fidelity models in a *data-driven manner*, i.e., by accounting for second-order velocity correlations that are pertinent in the prediction of turbulence intensities as well as thrust force and power based on the actuator disk concept. To this end, we adopt the stochastic dynamical modeling framework of [7–9] to model the effect of atmospheric turbulence as an input stochastic forcing for the linearized Navier-Stokes (NS) equations around a static velocity profile provided by a low-fidelity wake-expansion model. The power spectral density of the stochastic input is prescribed by inverse problems that are formulated to match statistical data informed by SCADA measurements or LES. Our approach offers a data-driven dynamical enhancement to low-fidelity models improving their predictive capability without adding to their dimensional complexity giving rise to linear time-invariant (LTI) models that are well-suited for estimation and control.

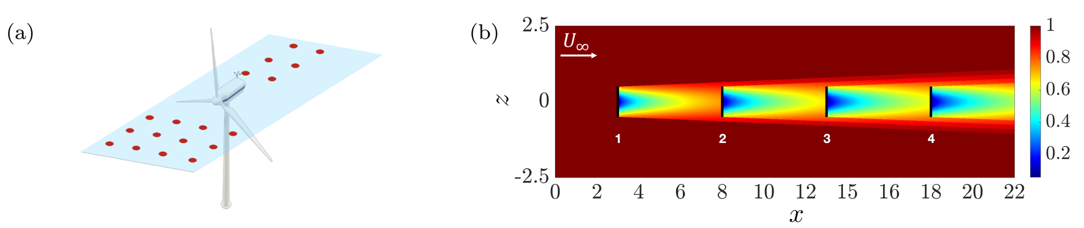

Figure 1: (a) Schematic of hub-height computational plane with data points highlighted in red; (b) Hub-height streamwise velocity $\bar{\mathbf{u}}(x, z)$ generated using the analytical wake-expansion model of [10].

Methods

The wind velocity field $\mathbf{u}$ can be decomposed into the sum of a time-averaged mean $\bar{\mathbf{u}}$ and zero-mean fluctuations $\mathbf{v}$ as

$$\mathbf{u} \;=\; \bar{\mathbf{u}} + \mathbf{v}, \quad \bar{\mathbf{u}} \;=\; \mathbf{E}\left[\mathbf{u}\right], \quad \mathbf{E}\left[\mathbf{v}\right] \;=\; 0 \tag{1}$$

where $\mathbf{E}\left[\cdot\right]$ denotes the time-average operator. The velocity fluctuations $\mathbf{v}$, which we will use to capture the effect of atmospheric turbulence on the wake model, are assumed to be stochastic Gaussian processes.

Substituting the decomposition for $\mathbf{u}$ into the power and thrust force relations offered by the actuator disk concept [11, Chapter 3] yields

$$F = \frac{1}{2}\rho A C_T (\bar{\mathbf{u}}^2 + \overline{\mathbf{v}^2}), \quad P = \frac{1}{2}\rho A C_P (\bar{\mathbf{u}}^3 + 3\,\bar{\mathbf{u}}\,\overline{\mathbf{v}^2}) \tag{2}$$

where the overline denotes an average over the surface of the rotor disk $\mathcal{S}$ and the properties of the fluctuation field $\mathbf{v}$, namely its zero mean (cf. Equation (1)) and skewness (due to its Gaussian distribution), have been used to eliminate certain terms. Based on Equation (2), the scalar quantities that we obtain for the thrust force and power of each turbine will not only depend on the mean disk-averaged velocity $\bar{\mathbf{u}}$ at the turbine, but also on the disk-averaged second-order statistics of the fluctuation field $\mathbf{v}$.

While analytical models provide a static prediction of the effective velocity in the wind farm (similar to $\bar{\mathbf{u}}$), the fluctuation field $\mathbf{v}$ provides an additional degree-of-freedom whose second-order statistics can be modeled to improve predictions of the thrust force and power. Thus, given a set of available thrust force $\{\bar{F}_i\}$ and power $\{\bar{P}_i\}$ measurements for the turbines across a wind farm, the dynamics of $\mathbf{v}$ can be sought to augment the predictions of static analytical models by providing the necessary surface integrals $\overline{\mathbf{v}^2}$ to better predict the available data (cf. Equation (2)). Together with the *prior* low-fidelity model that predicts $\bar{\mathbf{u}}$, the dynamical model identified for $\mathbf{v}$ can provide a class of low-complexity models that are more accurate in predicting quantities that depend on turbulent flow statistics. Following [8, 9], we assume the dynamics of velocity fluctuations that complement the static predictions of $\bar{\mathbf{u}}$ to be governed by the stochastically forced linearized NS equations

$$\begin{aligned} \mathbf{v}_t &= -(\nabla\cdot\mathbf{v})\,\bar{\mathbf{u}} - (\nabla\cdot\bar{\mathbf{u}})\,\mathbf{v} - \nabla p + \frac{1}{Re}\Delta\mathbf{v} - K^{-1}\mathbf{v} + \mathbf{d} \\ 0 &= \nabla\cdot\mathbf{v} \end{aligned} \tag{3}$$

where p is the vector of pressure fluctuations, ∇ is the gradient operator, $\Delta = \nabla\cdot\nabla$ is the Laplacian, and the Reynolds number $Re = U_\infty d_0/\nu$ is defined in terms of the rotor diameter d_0, free-stream velocity U_∞, and kinematic viscosity ν. Moreover, the volume penalization term $K^{-1}\mathbf{v}$ is used to capture the effect of turbine rotors and nacelles on the velocity field, i.e., $K \to 0$ within solid structures and $K \to \infty$ within the fluid; see [12] for details. After eliminating p, Equations (3) can be brought into the evolution form

$$\begin{aligned} \dot{\boldsymbol{\psi}}(t) &= A\,\boldsymbol{\psi}(t) + B\,\mathbf{d}(t) \\ \mathbf{v}(t) &= C\,\boldsymbol{\psi}(t). \end{aligned} \tag{4}$$

Here, $\boldsymbol{\psi}$ is the state, $\mathbf{d}$ is a stationary zero-mean stochastic process, A is the linearized dynamic generator, B is an input matrix, and C is an output matrix that relates $\boldsymbol{\psi}$ to $\mathbf{v}$; see [8] for an example of matrices.

The necessary surface integrals $\overline{\mathbf{v}^2}$ that allow us to match the set of partially available power and thrust forces measurements across the wind farm constitute entries of the output covariance matrix $\Phi = \lim_{t\to\infty} \mathbf{E}\,[\mathbf{v}(t)\,\mathbf{v}^*(t)]$, which are linearly related to the state covariance matrix $X = \lim_{t\to\infty} \mathbf{E}\,[\boldsymbol{\psi}(t)\,\boldsymbol{\psi}^*(t)]$ via $\Phi = CXC^*$. The inverse modeling framework of [7–9] provides the means to identify the statistics of forcing $\mathbf{d}$ and input matrix B in Equations (4) in order to match a partially available set of second-order statistics $\overline{\mathbf{v}^2}$ at given spatial locations. In addition to matching power and thrust force, this approach also provides the means to match a partially available set of turbulence intensities $\overline{\mathbf{v}^2}$ (from LES or LIDAR measurements) at predetermined locations across the farm and predict it over the remainder of the spatial domain in a way that is consistent with the linearized NS equations. For brevity, we refrain from expanding on the formulation and solution of structured covariance completion problems; see [7–9] for details. We next apply our approach to the problem of matching LES-informed power and thrust force measurements in a 4×1 array of turbines. We also show how access to turbulence intensities at various diameters away from the turbines can change the ability of the linearized NS equations to complete the statistical signature of the flow fields across the entire farm.

Results and Conclusions

We first consider a 2D computational domain of size $L_x \times L_z = 22 \times 5$; $x \in [\,0,\ 22\,]$ and $z \in [\,-2.5,\ 2.5\,]$ with turbines of unit diameter located at $x = \{3, 8, 13, 18\}$ and $z = 0$ in a turbulent flow with $Re = 10^8$. The analytical wake model proposed by [10] with a wake growth rate of $k^\star = 0.03$ and $\beta = (1+\sqrt{1-C_T}/(2\sqrt{(1-C_T)}))$ with $C_P = 0.4858$ and $C_T = 0.7871$ results in the velocity profile shown in Figure 1(b). Choices of C_P and C_T correspond to the maximum power generated by a 5MW NREL turbine [13] using an LES code that leverages blade momentum element theory [2, 3].

As shown in Figure 2, the velocity field $\bar{\mathbf{u}}$ predicted by the low-fidelity analytical model yields quantitatively poor predictions of the thrust force and power generation at the turbines. Furthermore, the monotonically decreasing $\bar{\mathbf{u}}$ fails to capture the increase in the thrust force and power after the second turbine. We thus seek the statistics of velocity fluctuations $\mathbf{v} = [\,u\ w\,]^T$ around $\bar{\mathbf{u}}$ to improve predictions of the low-fidelity model. We use a 2nd-order central difference scheme with $\Delta x = 1$ and $\Delta z = 0.5$ to discretize the differential operators in the linearized equations (3). Homogenous Dirichlet and Neumann boundary conditions are applied at the top and bottom boundaries and extrapolated boundary conditions are implemented at the inlet and outlet of the domain; see [14] for details. As shown in Figure 2, our modeling approach provides the necessary dynamical perturbation to the linearized NS equations for matching the LES data (cf. Figure 2). While matching both $\{\bar{F}_i\}$ and $\{\bar{P}_i\}$ is not possible because of the limited degrees of freedom in Equations (2), a least-squares balance in matching F and P can provide a reasonable simultaneous estimation for both.

We next focus on the problem of predicting the streamwise turbulence intensity uu at the hub height of single wind turbine; see Figure 1(a). We consider a 2D computational domain of size $L_x \times L_z = 5 \times 5$ where

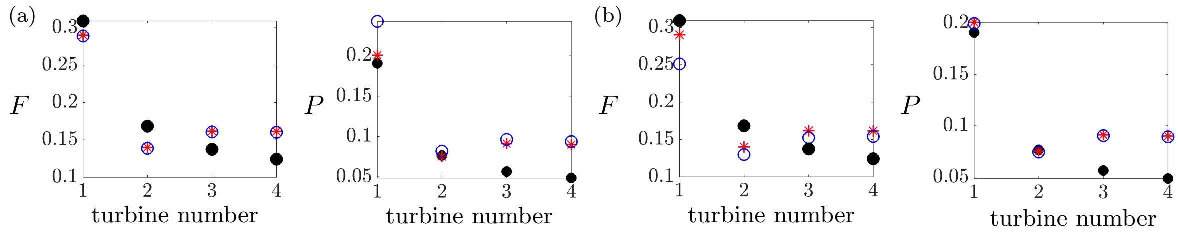

Figure 2: (a) Results for matching F and predicting P; (b) Results for matching P and predicting F. LES data ($*$); results of analytical model [10] (●); and from our data-enhanced stochastic dynamical model (○).

$x \in [0,\ 5]$ and $z \in [-2.5,\ 2.5]$. The turbine of unit diameter is located at $x = 3$ and $z = 0$. While the discretization scheme and boundary conditions are the same as the first case study, we use a more refined grid with $\Delta x = 0.5$ and $\Delta z = 0.5$ to better assess the effect of statistics at various locations downstream of the turbine. We use LES generated turbulent intensities at various locations within the computational domain to train our stochastic dynamical models. For consistency, we use all data points before the turbine to match the inflow turbulence conditions with that of LES. We consider three cases in which the available training dataset contains the streamwise turbulent intensity at various distances away from the turbine (see Figure 1(a) for an illustration): (i) at the turbine location $x = 2$ and points at 1 diameter away (Figure 3(b)), (ii) at $x = 2$ and points at 1 and 2 diameters away (Figure 3(c)), and (iii)at $x = 2$ and points at 1, 2, and 3 diameters away (Figure 3(d)). The preliminary results shown in Figure 3 demonstrate the ability of the stochastically forced linearized NS equations in capturing the dominant trends of uu in the wake of the turbine. It is also evident that for the considered turbine and atmospheric conditions, access to flow statistics at 3 diameters away form the turbine can significantly improve the completion of the statistical signature of the flow at hub height. This quality of completion demonstrates the ability of our linear stochastic dynamical models in predicting the dominant features of the flow physics in addition to matching quantities of interest in farm planning and control. Our ongoing efforts focus on extensions of the framework to the 3D domain to better capture vortex shedding effects from yawed turbines.

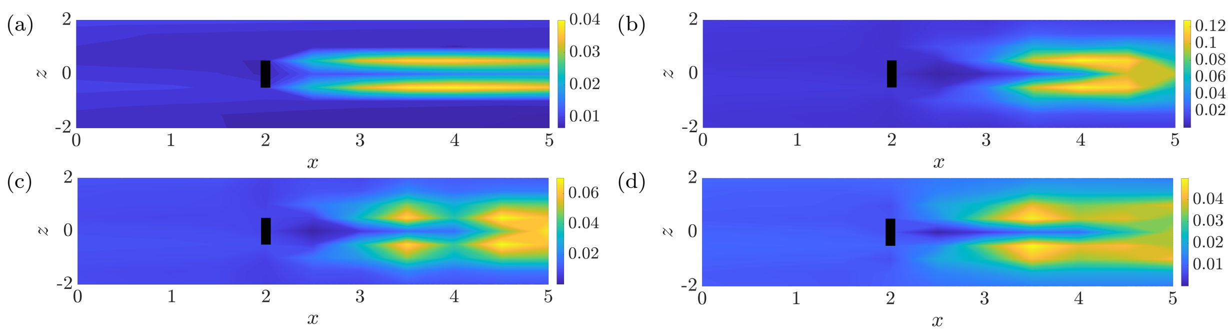

Figure 3: Preliminary results for the predictions of streamwise turbulent intensity using the data-enhanced model (4). (a) LES results. Results obtained using inflow statistics in addition to statistics at a 1 diameter downstream (b), 1 and 2 diameters downstream (c), and 1, 2, and 3 diameters downstream (d).

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