Stochastic dynamical wake modeling for wind farms

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Research Problem

Low-fidelity wake modeling

The oversimplified static nature of low-fidelity wake models that do not account for the complex aerodynamic interactions among turbines limits their utility for closed-loop model-based wind farm control.

Question

Can we use data to introduce dynamical augmentations to low-fidelity static wake models and accurately capture power and thrust force measurements in accordance with large-eddy simulations (LES) and SCADA data?

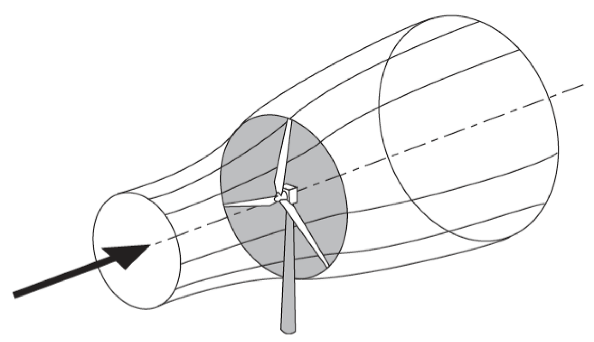
Low-Fidelity Wake Modeling

The actuator disk concept for energy extraction:

The turbine rotor is modeled as a disk and its behaviour is analysed within a control volume.

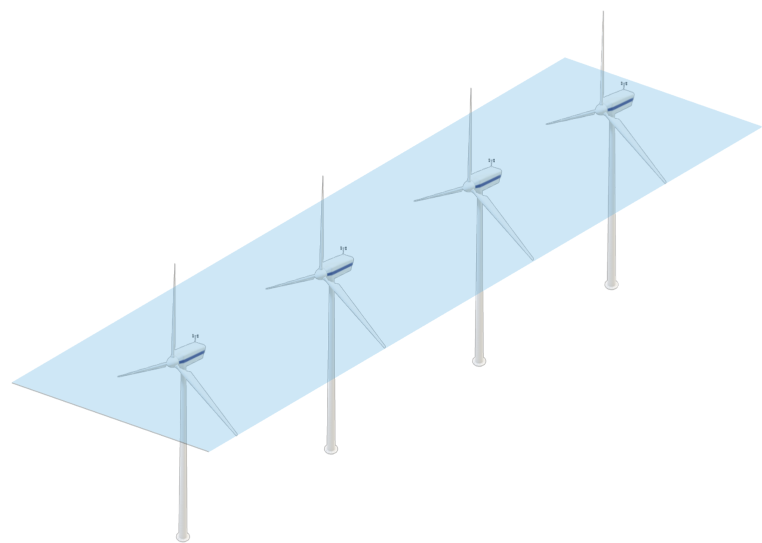
Thrust force: $F \;=\; \frac{1}{2}\,\rho\,A\,C_T\,\mathbf{u}^2$

Extracted power: $P \;=\; \frac{1}{2}\,\rho\,A\,C_P\,\mathbf{u}^3$



Analytical wake models:

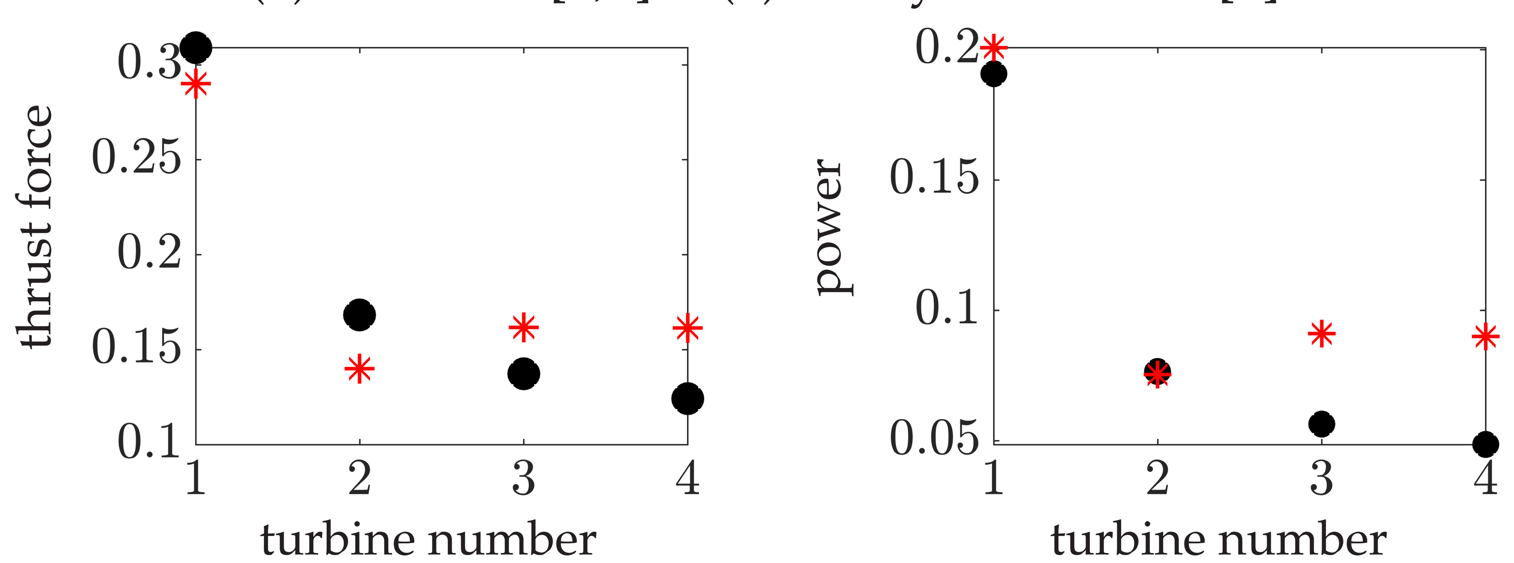
- Jensen model
- Park model
- Frandsen model
- Gaussian deficit model



⋆ **Example:**

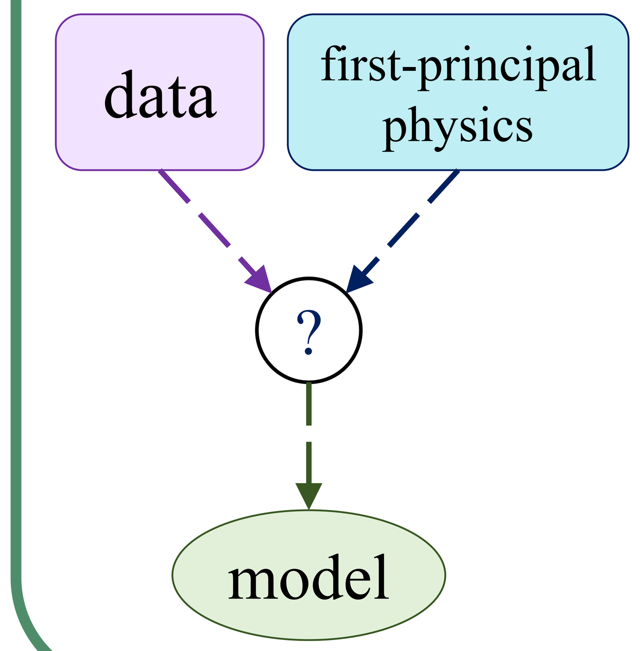
4×1 wind farm

(∗) LES data [1,2] (•) Analytical model [6]



⋆ **Model predictions fail to capture quantities and trends.**

Data-driven vs Physics-based Modeling



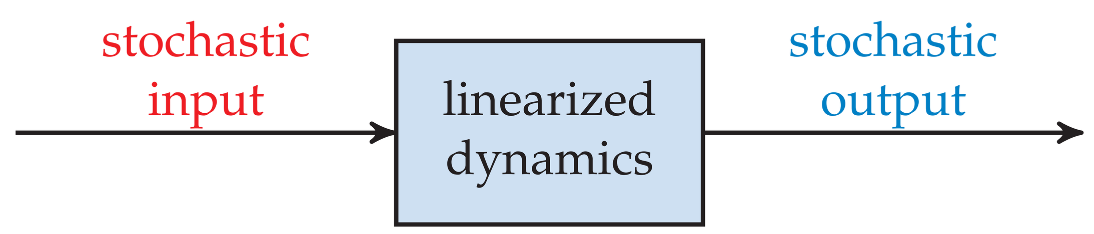
Data-driven: flexible but not robust; small-data issues

Physics-based: good robustness; trade-off (complexity vs accuracy)

⋆ How can data be used to refine the predictive capability of physics-based models?

Approach: Stochastic Dynamical Modeling [3,4]

Stochastically forced linearized equations



- view second-order statistics as data for **inverse problems**
- identify input statistics to account for available velocity statistics in output

Stochastic Wake Models [5]

- **Model dynamics of fluctuations ($\mathbf{v}$) around base flow ($\bar{\mathbf{u}}$):**

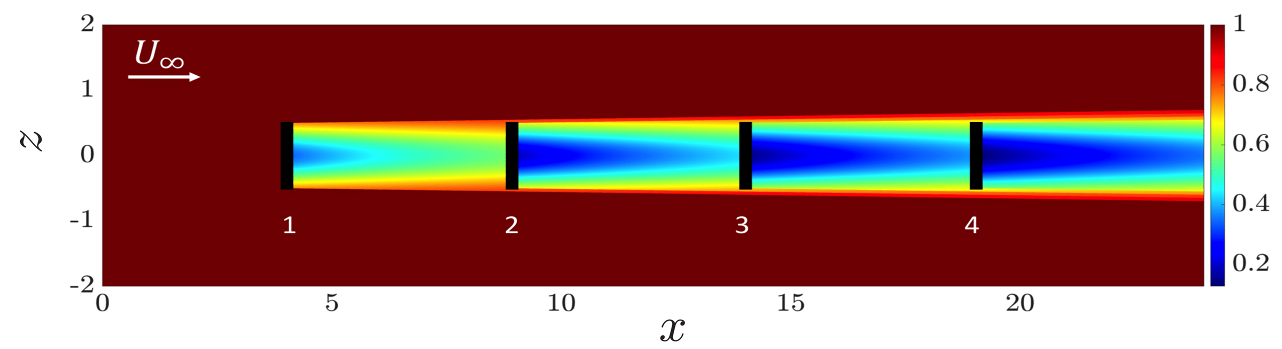
$$\mathbf{u} \;=\; \bar{\mathbf{u}} + \mathbf{v} \qquad \bar{\mathbf{u}} \;=\; [\mathbf{u}] \qquad [\mathbf{v}] \;=\; 0$$

$$F \;=\; \frac{1}{2}\,\rho\,A\,C_T\left(\bar{\mathbf{u}}^2 + \overline{\mathbf{v}^2}\right) \qquad P \;=\; \frac{1}{2}\,\rho\,A\,C_P\left(\bar{\mathbf{u}}^3 + 3\,\bar{\mathbf{u}}\,\overline{\mathbf{v}^2}\right)$$

$$\overline{\mathbf{v}^2} \;:=\; \int_{\mathcal{S}} \left[\mathbf{v}(\mathbf{x},t)^2\right] \mathrm{d}\mathbf{x}$$

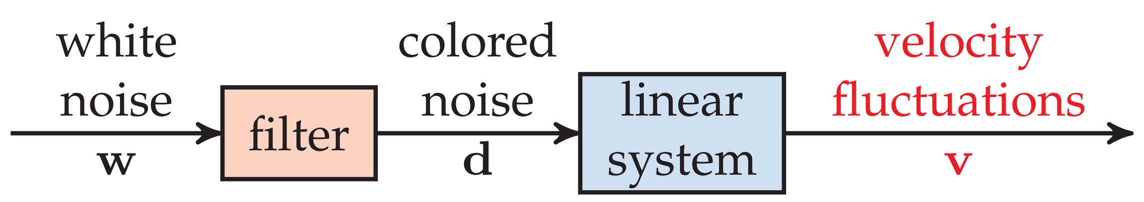
⋆ **Analytical base flow $\bar{\mathbf{u}}$: Gaussian deficit model [6]**

$$\bar{\mathbf{u}}(x,z) \;=\; U_\infty - U_\infty\left(1 - \sqrt{1 - \frac{C_T}{8\,(k^\star x/d_0 + 0.2\sqrt{\beta})^2}}\right) \mathrm{e}^{\left(-\frac{1}{2\,(k^\star x/d_0 + 0.2\sqrt{\beta})^2}\left(\frac{z}{d_0}\right)^2\right)}$$



⋆ **Model fluctuation dynamics using linearized Navier-Stokes and shape forcing statistics to match LES data [5]**

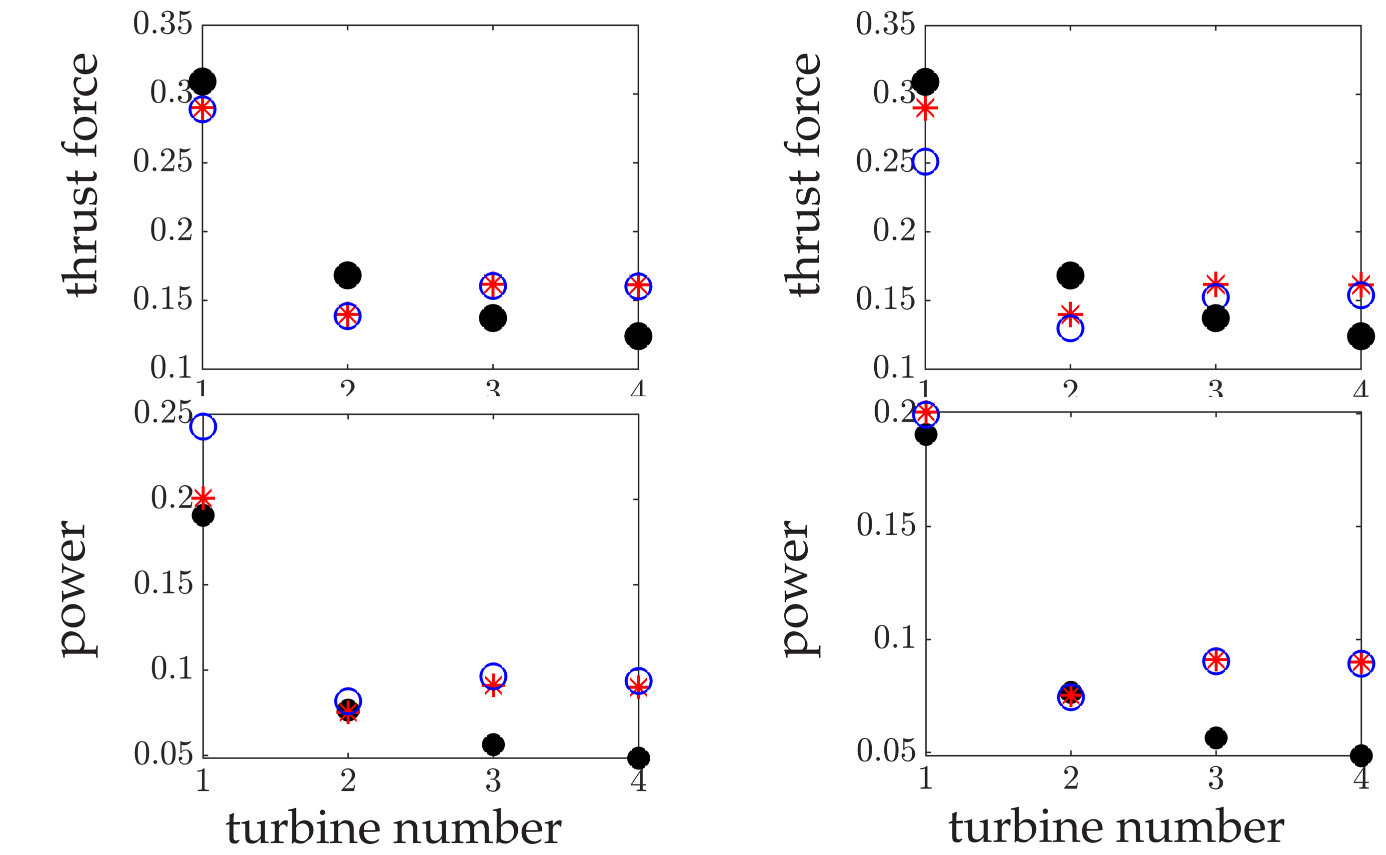
$$\dot{\boldsymbol{\psi}} \;=\; A\,\boldsymbol{\psi} + B\,\mathbf{d}$$
$$\mathbf{v} \;=\; C\,\boldsymbol{\psi}$$



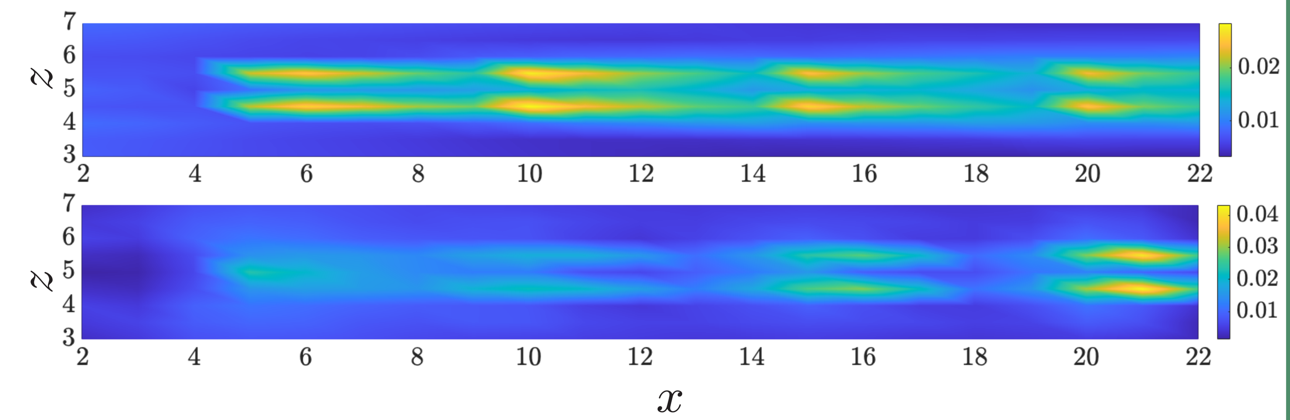
Results

- **Predictions of power and thrust force**

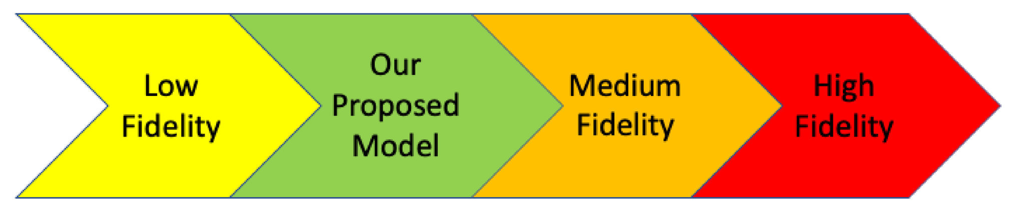
(∗) LES data [1,2] (•) Analytical model [6] (○) Data-enhanced model [5]



- **Prediction of velocity intensities**



Conclusion



Stochastic dynamical models improve predictive capability of low-fidelity wake models in capturing power and thrust force in wind farms, in addition to turbulent intensities.

Outlook:

- Extension to 3D flows to capture vortex shedding effects and wake curl
- Model-based feedback control

References

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