Homework 6

Problem 1, 10 marks

We can ice skate because the pressure on the ice lowers its freezing temperature creating a thin film of water under the skate blades. Calculate the freezing temperature of ice for a 70 kg person standing on ice skate blades of dimension 0.25 x 12.0 inches.

Data: 1 inch = 2.54 cm; $\rho(H_2O(\ell)) = 0.997$ g/cm^3; $\rho(H_2O(s)) = 0.917$ g/cm^3; $\Delta H_{fus}(H_2O) = 6.01$ kJ/mol; $g = 9.8$ m/s^2

don't forget to convert!!

Use Clapeyron Equation:

integrate

assume independent of T

$$\frac{dP}{dT} = \frac{\Delta \bar{H}}{T\Delta \bar{V}} \Rightarrow dP = \frac{\Delta \bar{H}}{\Delta \bar{V}}\frac{dT}{T} \Rightarrow \Delta P = \frac{\Delta \bar{H}}{\Delta \bar{V}} \ln\left(\frac{T_2}{T_1}\right)$$

use:

$$P_2 = \frac{\overset{\text{Force}}{F}}{\underset{\text{Area}}{A}} \Rightarrow \frac{m \cdot g}{A} = \text{answer}$$

$T_1 = 273.15\,K$

and

$$\Delta \bar{V} = V_\ell - V_s \Rightarrow (m/\rho_\ell) - (m/\rho_s) = \text{answer}$$

Then, plug back into $\Delta P = \frac{\Delta \bar{H}}{\Delta \bar{V}} \ln\left(\frac{T_2}{T_1}\right)$ and solve for T_2

* use Kelvin

to check yourself, make sure all units cancel out except for Kelvin.

Looking for 4 things when grading (highlited)

How to derive Clapeyron Equation

https://chem.libretexts.org/Bookshelves/Physical_and_Theoretical_Chemistry_Textbook_Maps/Physical_Chemistry_(LibreTexts)/23%3A_Phase_Equilibria/23.03%3A_The_Chemical_Potentials_of_a_Pure_Substance_in_Two_Phases_in_Equilibrium

Problem 2, 10 marks

Freeze drying (lyophilization) is a process where water contained in a sample is evaporated into the gas phase at low pressures and temperatures below the freezing temperature of water. What is the maximum pressure at which a sample can be freeze dried if the temperature is not to exceed -10.5 °C? → Kelvin

P_1

Data: $\Delta H^0_{fus} = 6.01$ kJ/mol; $\Delta H^0_{vap} = 40.65$ kJ/mol; the vapor pressure of ice is 611 Pa at 273.15 K.

Enthalpy is a state function so,:

$\Delta H_{sub} = \Delta H_{fus} + \Delta H_{vap}$ = answer

Use Clapeyron Equation:

$$\frac{dP}{dT} = \frac{\Delta \bar{H}}{T\Delta \bar{V}} \Rightarrow \text{what is } \Delta\bar{V}?$$

The Clapeyron equation can be developed further for phase equilibria involving the gas phase as one of the phases. This is the case for either sublimation (solid → gas) or vaporization (liquid → gas). In the case of vaporization, the change in molar volume can be expressed

$$\Delta V = V_{gas} - V_{liquid}$$

Since substances undergo a very large increase in molar volume upon vaporization, the molar volume of the condensed phase (liquid in this case) is negligibly small compared to the molar volume of the gas (i.e., $V_{gas} \gg V_{liquid}$). So,

$$\Delta V \approx V_{gas}$$

explanation

$$\frac{dT}{T^2} = \frac{1}{T^2}dT = \int T^{-2}dT = \frac{T^{-1}}{-1}\Big|_{T_1}^{T_2} = \frac{T_2^{-1}}{-1} - \left(\frac{T_1^{-1}}{-1}\right) = T_2^{-1} + T_1^{-1} = \frac{1}{T_2} + \frac{1}{T_1}$$

$$\frac{dT}{T^2} = -d\left(\frac{1}{T}\right)$$

So, use ideal gas law:

$V = \frac{RT}{P}$ and plug into clapeyron eq:

$$\frac{dP}{dT} = \frac{\Delta \bar{H}}{T\Delta \bar{V}} \Rightarrow \frac{dP}{dT} = \frac{P\Delta H}{RT^2} \Rightarrow dP = \frac{P\Delta H}{R}\frac{dT}{T^2} \Rightarrow \int_{P_1}^{P_2}\frac{dP}{P} = -\frac{\Delta H}{R}\int_{T_1}^{T_2} d\left(\frac{1}{T}\right)$$

How to derive this

https://chem.libretexts.org/Bookshelves/Physical_and_Theoretical_Chemistry_Textbook_Maps/Physical_Chemistry_(Fleming)/08%3A_Phase_Equilibrium/8.05%3A_The_Clausius-Clapeyron_Equation

$$\Rightarrow \ln\left(\frac{P_2}{P_1}\right) = -\frac{\Delta H}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

this is the Clausius-Clapeyron eq.

Plug in #'s and solve for P_2

Looking for these 4 things

Problem 3, 10 marks

convert

The vapor pressure of water at 20 °C is 17.54 torr. Using this data and $\Delta H^0_{vap} = 40.65$ kJ/mol for water calculate ΔG^0_{298} for the change $H_2O(\ell) \rightarrow H_2O(g)$? (Hint: the form of the van't Hoff equation is identical to the Clausius-Clapeyron equation)

Comparison is $K_p(T) = P(T)$

$$\ln\left(\frac{P_2}{P_1}\right) = -\frac{\Delta H}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

so, $P_2 = K_{P(298)}$

Looking for these 2 things

$$\Delta G^0_{298} = -RT\ln(K_P)$$