

REVIEW FOR MATH 102 FIRST MIDTERM (SPRING 2008)

1) Find the following limits:

a)  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 - 3x + 2}$

b)  $\lim_{x \rightarrow 1} \frac{|x^2 - 1|}{x - 1}$

c)  $\lim_{x \rightarrow \infty} \frac{3x^2 + 2x + \sqrt{x}}{5x^2 + 1}$

d)  $\lim_{x \rightarrow -\infty} \frac{3x + 2}{\sqrt{x^2 + 5}}$

e)  $\lim_{x \rightarrow \infty} \frac{3x^3 + 2x + \sin x}{x^3 + 2x^2 + 1}$

f)  $\lim_{x \rightarrow 0} \frac{\sin^2(2x^2)}{x^4}$

g)  $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x^2}$

2) Find  $y'$ .

a)  $y = x^3 \cdot (\sin \sqrt{x} + 1)$

b)  $y = \sin^2(2x + x^{1/3})$

c)  $y = \frac{x^3 + \cos x}{x^2 + 3}$

d)  $y = \tan\left(\frac{x^2}{\sqrt{x+1}}\right)$

e)  $y^2 + xy - x^2 = 1$

3) Find all the points on the curve  $y = x^3 - 6x^2 + 9x + 4$  where the tangent line is parallel to the  $x$ -axis.

4) If  $y = \frac{\sin x + 4}{x + 1}$ , then find the equation of the tangent line at  $x = 0$ .

5) For what values of  $a$  and  $b$  will the function

$$g(x) = \begin{cases} ax + b & x \leq -1 \\ ax^3 + x + 2b, & x > -1 \end{cases}$$

be differentiable for all values of  $x$ ?

6) a) Find two numbers whose sum is 20, and their product is maximum.

b) Find two numbers whose difference is 20, and their product is minimum.

7) Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola  $y = 8 - x^2$ .

8) Find the dimensions of the right circular cylinder of maximum volume inscribed in a sphere of radius 10cm.

9) Find all the critical points of the following functions on the given interval. Determine the absolute minimum and absolute maximum values, if exists.

a)  $f(x) = x^3 - 6x^2 + 9x + 1$  on the interval  $[-1, 4]$ .

b)  $f(x) = |x^2 - 2x - 3|$  on the interval  $[-2, 4]$ .

10) Sketch the graph of the following functions (Find all the critical points, increasing & decreasing intervals, inflection points, concavity, the asymptotes):

a)  $f(x) = 2x^3 + 3x^2 - 12x + 5$

b)  $f(x) = \frac{x}{x^2 - 4}$

c)  $f(x) = 8x^2 - x^4$

d)  $f(x) = x - 3\sqrt[3]{x}$

e)  $f(x) = \sqrt{4 - x^2}$

# ANSWERS OF REVIEW QUESTIONS FOR MATH 102 MIDTERM 1 (SPRING 2008)

1) a) 4, b) No Limit, c)  $\frac{3}{5}$ , d) -3, e) 3, f) 4, g)  $-\frac{1}{2}$

2) a)  $y' = 3x^2 \cdot (\sin \sqrt{x} + 1) + x^3 \cdot \frac{\cos \sqrt{x}}{2\sqrt{x}}$

b)  $y' = 2 \sin 2x + \sqrt[3]{x} \cdot \cos 2x + \sqrt[3]{x} \cdot (2 + \frac{1}{2\sqrt[3]{x^2}})$

c)  $y' = \frac{x^4 - 2x^2 \sin x - 2x \cos x + 3 \sin x}{(x^2 + 3)^2}$

d)  $y' = \sec^2(\frac{x^2}{\sqrt{x+1}}) \cdot \frac{3x^2 + 4x}{(x+1)^{\frac{3}{2}}}$

e)  $y' = \frac{2x-y}{2y+x}$

3) 1,3

4)  $y = -3x$

5)  $a = \frac{-1}{2}$ ,  $b = 1$

6) a) 10, 10

b) 10, -10

7)  $\sqrt{\frac{8}{3}}$

8)  $r = 5\sqrt{2}$ ,  $h = 10\sqrt{2}$

9) a)  $x = -1$  absolute min,  $x = 1$  and  $x = 4$  absolute max.

b)  $x = -2$  and  $x = 4$  absolute max,  $x = -1$  and  $x = 3$  absolute min.