REVIEW FOR MATH 102 SECOND MIDTERM (SPRING 2008)

1) Find $g'(x)$

a) $g(x) = \int_{x^2}^{\cos x} \ln t dt$

b) $g(x) = \int_0^1 \frac{\sin t}{t} dt$

2) Find the area between the following curves.

a) $y = x^2 - 3x - 5$ and $y = 2x + 1$

b) $y = x^2 - 5x - 2$ and $y = -x^2 + x + 6$

c) $y = \sqrt[3]{x}$ and $y = 2x$

d) $y = x^3 - x$ and $y = 3x$

e) $x = y^2 - 4y$ and $x = 2y - y^2$

3) Compute the following integrals (you can use the table of integrals).

a) $\int_2^3 \frac{x^2}{\sqrt{x^3-2}}\ dx$

b) $\int_0^\pi \sin^3 x . \cos^3 x\ dx$

c) $\int_{-\pi}^{\pi} \sin x^3\ dx$

d) $\int_0^1 \frac{e^{\sqrt{x}}\ dx}{\sqrt{x}}$

e) $\int_0^1 x^3 . \sqrt{x^2+1}\ dx$

f) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cot x\ dx$

g) $\int_0^1 \frac{e^x}{e^{2x}+1}\ dx$

h) $\int_e^{e^2} \frac{1}{x . \ln x}\ dx$

4) Find y'.

a) $y = x^x$

b) $y = \frac{(x+2)^7 (x^2-8)^{1/4}}{(x^3+7)^{14}}$

5) Find the following limits.

a) $\lim_{x\to\infty} x^{\frac{1}{x}}$

b) $\lim_{x\to 0^+} x^x$

6) Find the volume of the solids obtained by rotating the defined regions about the given axis.

a) Region: $y = \frac{1}{x}$, $y = 0$, $x = 1$, $x = 3$ rotate about the x-axis.

b) Region: $y = x^2$, $y = 2x$ rotate about the x-axis.

c) Region: $y = \sqrt{x}$, $y = x$ rotate about y-axis.

d) Region: $y = 4 - x^2$, $y = 0$ rotate about the axis $y = -2$

7) Find the following integrals (you can use the table of integrals).

a) $\int e^x.x \; dx$

b) $\int \frac{1}{4+9x^2} \; dx$

c) $\int \frac{1}{\sqrt{2x-x^2}} \; dx$

d) $\int \frac{1}{x.\sqrt{x^2-4}} \; dx$

e) $\int \frac{1}{1-4x^2} \; dx$

f) $\int x^2.\sqrt{9+x^2} \; dx$

g) $\int \sin^4 x \; dx$

h) $\int x.\sin x \; dx$

8) Determine whether the following improper integrals are convergent or divergent. Find its value if it is convergent.

a) $\int_0^1 \frac{1}{x} \; dx$

b) $\int_1^\infty \frac{1}{x^2} \; dx$

c) $\int_{-\infty}^0 e^x \; dx$

ANSWERS OF REVIEW PROBLEMS FOR MATH 102 - 2. MIDTERM (SPRING 2008)

1) a) $-\sin x.\ln(\cos x) - 2x.\ln(x^2)$ b) 0

2) a) $\frac{353}{6}$ b) $\frac{125}{3}$ c) $\frac{1}{8}$ d) 8 e) 9

3) a) $\frac{2}{3}(5-\sqrt{6})$ b) 0 c) 0 d) $2e-2$ e) $\frac{2\sqrt{2}}{15}$ f) $\frac{1}{2}\ln 3$ g) $\arctan e - \frac{\pi}{4}$ h) $\ln 2$

4) a) $y' = x^x.(\ln x + 1)$ b) $y' = (\frac{(x+2)^7(x^2-8)^{1/4}}{(x^3+7)^{14}}).(\frac{7}{x+2} + \frac{x}{2(x^2-8)} - \frac{42x^2}{x^3+7})$

5) a) 1 b) 1

6) a) $\frac{2\pi}{3}$ b) $\frac{64\pi}{15}$ c) $\frac{2\pi}{15}$ d) $\frac{256\pi}{5}$

7) a) $e^x.(x-1)+C$ b) $\frac{1}{6}\arctan(\frac{3x}{2})+C$ c) $\arcsin(x-1)+C$ d) $\frac{1}{2}\arccos(\frac{2}{x})+C$

e) $\frac{1}{4}\ln(\frac{x+1/2}{x-1/2})+C$ f) $\frac{x}{8}(9+2x^2).\sqrt{9+x^2} - \frac{81}{8}\ln(x+\sqrt{9+x^2})+C$

g) $-\frac{\sin^3 x.\cos x}{4} + \frac{3}{8}x - \frac{3\sin^2 x}{16} + C$ h) $\sin x - x.\cos x + C$

8) a) divergent. b) convergent, 1 c) convergent, 1