(1.18) $x^3 - 6x = 4$

Find 3 real-valued possibilities for $\sqrt[3]{2+2i} + \sqrt[3]{2-2i}$.

$2+2i = 2\sqrt{2}\, e^{i\pi/4} = r^3 e^{3i\phi} \Rightarrow r = 8^{1/6} = \sqrt{2}\,,\ 3\phi = \frac{\pi}{4} + 2n\pi$

$\phi = \frac{\pi}{12}\,,\ \frac{9\pi}{12}\,,\ \frac{17\pi}{12}$

$2-2i = 2\sqrt{2}\, e^{i\frac{7\pi}{4}} = r^3 e^{3i\phi} \Rightarrow r = \sqrt{2}\,,\ 3\phi = \frac{7\pi}{4} + 2n\pi$

$\phi = \frac{7\pi}{12}\,,\ \frac{15\pi}{12}\,,\ \frac{23\pi}{12}$

$\sqrt[3]{2+2i} + \sqrt[3]{2-2i}$ is in $\mathbb{R}$ when imaginary parts vanish:

$\sin\left(\frac{\pi}{12}\right) + \sin\left(\frac{23\pi}{12}\right) = 0$

$\sin\left(\frac{9\pi}{12}\right) + \sin\left(\frac{15\pi}{12}\right) = 0$

$\sin\left(\frac{17\pi}{12}\right) + \sin\left(\frac{7\pi}{12}\right) = 0$

$\Rightarrow$ $\mathbb{R}$-valued solutions:

$x_1 = \sqrt{2}\left(\cos\frac{\pi}{12} + \cos\frac{23\pi}{12}\right) = 2\sqrt{2}\cos\frac{\pi}{12} = \boxed{1+\sqrt{3}} = $

$x_2 = \sqrt{2}\left(\cos\frac{9\pi}{12} + \cos\frac{15\pi}{12}\right) = -2\sqrt{2}\underbrace{\cos\frac{\pi}{4}}_{\frac{\sqrt{2}}{2}} = \boxed{-2} = x_2$

$x_3 = \sqrt{2}\left(\cos\frac{17\pi}{12} + \cos\frac{7\pi}{12}\right) = -2\sqrt{2}\cos\frac{5\pi}{12} = \boxed{1-\sqrt{3}} = x_3$

Note that $\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \frac{\sqrt{2}}{2}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)$