(10) (2) a. $\int_0^i e^z\,dz = e^z\Big|_0^i = \boxed{e^i - 1}$ as $e^z = (e^z)'$ and e^z is entire

(5)

(3) b. $\int_{\pi/2}^{\pi/2+i} \cos(2z)\,dz = \frac{1}{2}\sin(2z)\Big|_{\pi/2}^{\pi/2+i} = \frac{1}{2}\sin(\pi+2i)$

$$= \frac{1}{2}\,\frac{e^{i(\pi+2i)} - e^{-i(\pi+2i)}}{2i}$$

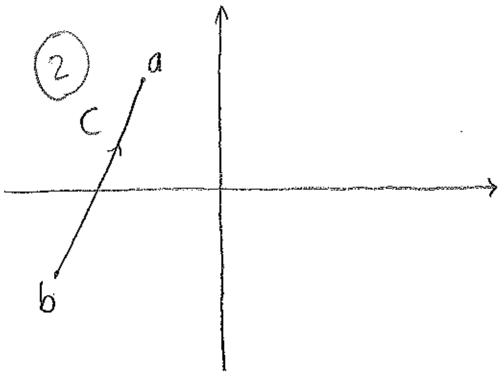
$$= \frac{1}{4i}\, e^{i\pi-2} - e^{-i\pi+2}$$

$$= \frac{1}{4i}\left[e^{-2}(\underbrace{\cos\pi}_{-1} + i\cancel{\sin\pi}) - e^{2}(\underbrace{\cos\pi}_{-1} + i\cancel{\sin\pi})\right]$$

$$= \boxed{\frac{1}{4i}\left(e^2 - \frac{1}{e^2}\right)}$$

(12) Let $a, b \in$ left half-plane, i.e. $\mathrm{Re}\,a, \mathrm{Re}\,b < 0$.

(10) Prove that $|e^a - e^b| < |a-b|$.



C is a smooth curve of length $|a-b|$

Let $f(z) = e^z$

Note that $|e^z| = e^x \le e^{\mathrm{Re}\,a} < 1$

$\forall$ point on C.

↑ uniform bound (2)

$$\Rightarrow \left|\int_C f(z)\,dz\right| = |e^a - e^b| \le \underbrace{e^{\mathrm{Re}\,a}}_{<1}\,|a-b| < |a-b|$$

as $e^z = (e^z)'$ (1) and e^z is entire (1)

M-L formula (2)

(1) for teo.