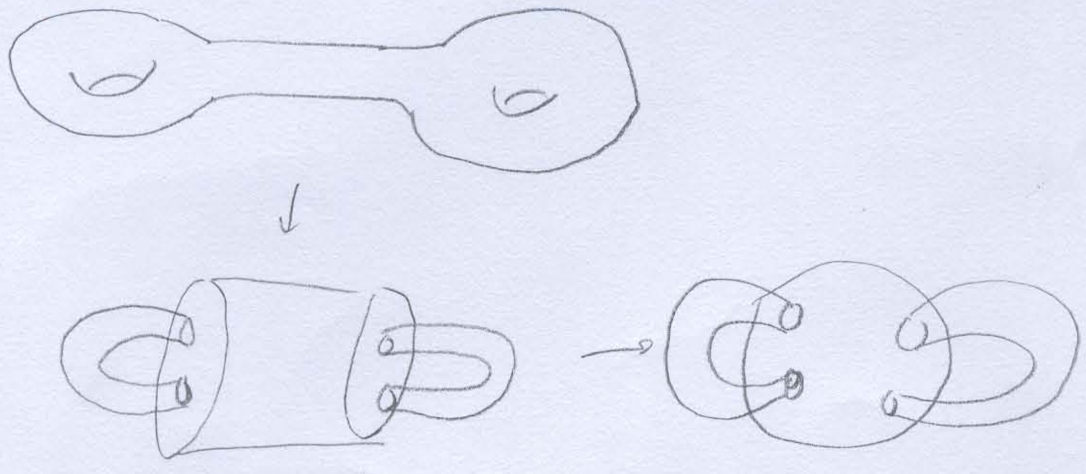
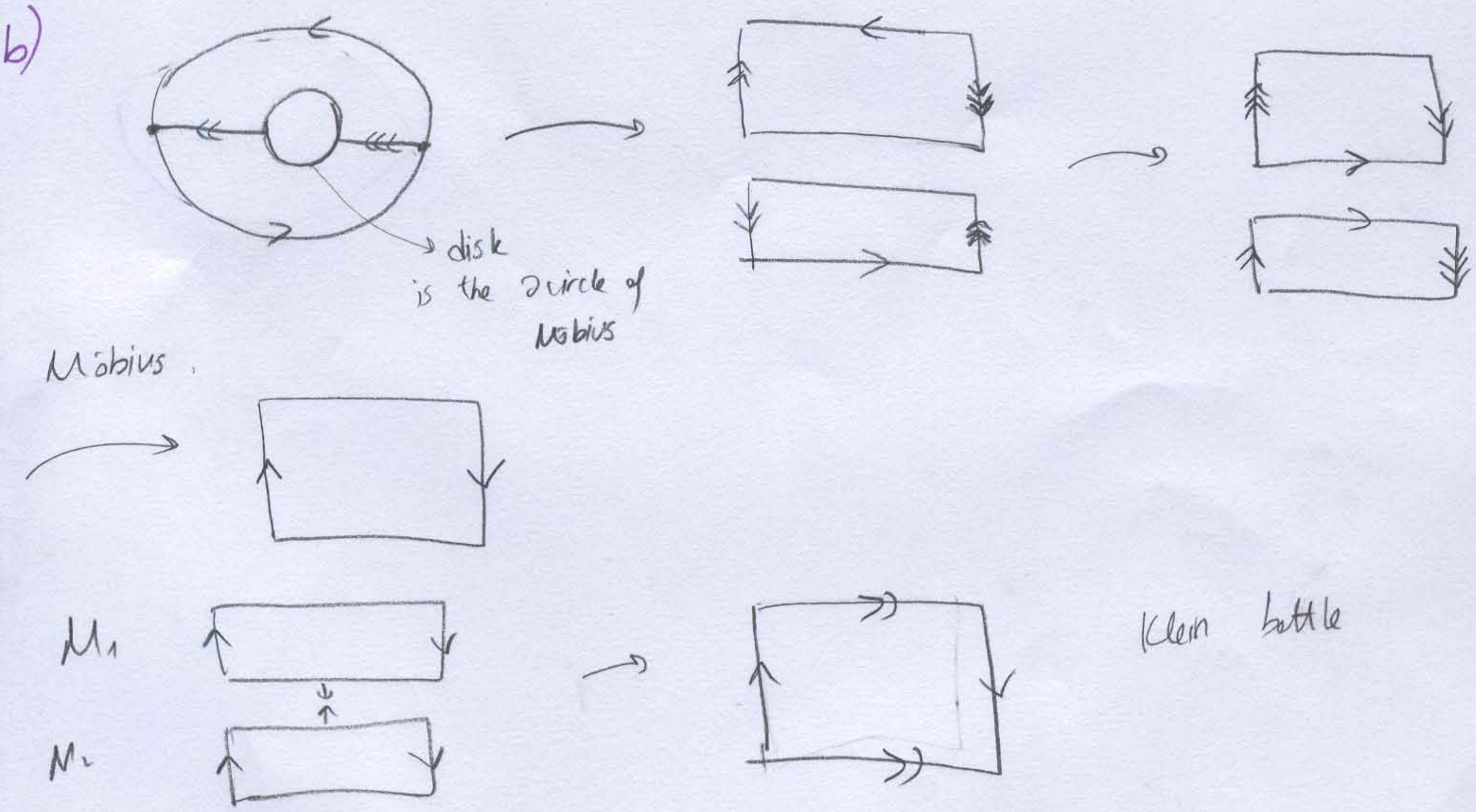
HW # 11 – SOLUTIONS

3 a)



b)



22 They are not homeomorphic, first one is genus 6 second one is genus 5.

32 Let S be a compact connected surface with genus g. Then, after capping off each boundary circle with a disc we will get a closed surface $\hat{S}$ with genus g by definition of the genus. If S is orientable, then $\hat{S}$ is orientable. Then by classification of closed surfaces $\hat{S}$ is homeomorphic to a sphere with g handles. $\hat{S} \simeq \Sigma_g$

$\Rightarrow S$ is homeomorphic to $\Sigma_g - n$ discs removed.

If S is not orientable $\Rightarrow \hat{S}$ is not orientable

$\Rightarrow \hat{S}$ is homeomorphic to a surface with q disks replaced by Möbius strips. $\Rightarrow S$ is homeomorphic to a surface which is obtained by removing n discs from the surface with q discs replaced by Möbius strips.

(83) The first surface S is orientable and have two boundaries.

S is homotopic to  S^1. $\chi(S^1) = v - e + f = 2 - 4 = -2$

$\chi(S) = \chi(S^1) = -2$

Let S^* be the closed surface obtained by capping off the 2 boundaries of S with 2 discs $\Rightarrow \chi(S^*) = \chi(S) + 2$

$\Rightarrow \chi(S^*) = 0 = 2 - 2g \Rightarrow g = 1 \Rightarrow S^* \simeq T^2$

since it is orientable

so S is homeomorphic to torus $-$ 2 discs.

The second surface S_1 is also orientable. It has one boundary. Let S_1^* be the surface obtained by capping of the boundary of S_1 with a disk. Then $\chi(S_1^*) = \chi(S_1) + 1$

S_1 is also homotopic to  S_1'. $\chi(S_1') = 2 - 5 = -3$

$\Rightarrow \chi(S_1^*) = \chi(S_1) + 1 = \chi(S_1') + 1 = -2 = 2 - 2g \Rightarrow g = 2$

$\Rightarrow S' \simeq \Sigma_2 \Rightarrow S \simeq \Sigma_2 - 2$ discs..