HW #6 — SOLUTIONS

(26) For $n \in \mathbb{Z}$ define

$$\varphi_n = (x,y) \mapsto \left(x+n, \left(\tfrac{1}{2}\right) - \left(\tfrac{1}{2}\right)(-1)^n + y(-1)^n\right)$$

when n is odd $\varphi_n = (x,y) \mapsto (x+n, 1-y)$

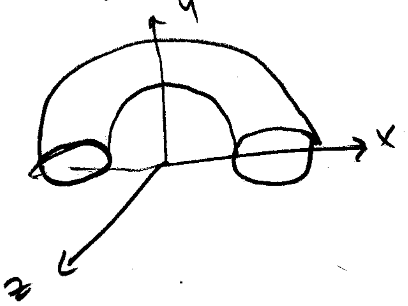
" " " even $\varphi_n = (x,y) \mapsto (x+n, y)$

$\mathbb{E}^1 \times [0,1] / \mathbb{Z}$ has the Möbius strip as orbit space.

(27) $g(x,y,z) = (x,-y,z)$

g is generator of $\mathbb{Z}_2$ $g^2 = id.$

$T^2/\mathbb{Z}_2 \cong S^1 \times I$



(1) $F(x,t) = \dfrac{f(x)t + x(1-t)}{|f(x)t + x(1-t)|}$ when $f(x) \neq -x \ \forall x \in C$

is a homotopy between f and Identity.

because claim: f is cts

Assume $f(x)t + x(1-t) = 0 \Rightarrow f(x) = \dfrac{(-x)(1-t)}{t}$

$\Rightarrow t = 1/2 \quad f(x) = -x$ $\times$.

$\left\| \dfrac{f(x)}{-x} \right\| = \dfrac{1-t}{t} = 1$

so F is a continuous function which is homotopy between f and id contradicting the hypothesis.

(3) Define the homotopy

$F = D \times I \to D$ $\qquad F(r,\theta,0) = (r,\theta) = $ identity

$((r,\theta),t) \mapsto (r, \theta + 2\pi r t)$ $\quad F(r,\theta,1) = (r, \theta + 2\pi r) = h$

clearly continous so h is homotopic to id.

consider $F_t = D \times \{t\} \to D$ $\quad (r,\theta) \mapsto (r, \theta + 2\pi r t)$

it is continuous map from a compact space to a Hausdorff space.

claim: F is onto

let $(r,\theta) \in D$ $\quad \Rightarrow \quad F(r, \theta - 2\pi r t) = (r,\theta)$

$(r, \theta - 2\pi r t) \in D \times \{t\}$

so F_t is a homeomorphism.

(5) f is not onto $\Rightarrow \exists p \notin f(x)$ $\quad p \in S^n$.

Now consider the stereographic projection

$h = S^n - \{p\} \to E^n$

$h \circ f = X \to E^n$ Now let c be a constant. We can write the straight line homotopy $F : X \times I \to E^n$

$(h \circ f)(x)\, t + c(1-t) = F(x,t)$

so $h \circ f$ is homotopic to c.

Consider $h^{-1}(F(x,t))$ it is continuous from $X \times I$ to S^n

$h^{-1}(F(x,0)) = h^{-1}(c)$

$h^{-1}(F(x,1)) = h^{-1}(h(f(x))) = f(x)$

so f is homotopic to $h^{-1}(c)$ which is a constant.