Math 405/538 Differential Geometry
Midterm Exam 2

December 24, 2008

1) (5 pts each) For each of (a)-(d) below: If the proposition is true, write TRUE. If the proposition is false, write FALSE. No explanations are required for this problem.

a. If S is a regular surface in $\mathbf{R}^3$, then there is a family of parametrizations for S so that the coordinate curves for the parametrizations are lines of curvature of S.

b. Let α_1 and α_2 be two curves in a surface S with $\alpha_1(0) = \alpha_2(0) = p$ in S and $\alpha_1'(0) = \alpha_2'(0) = V$ in T_pS. Then the geodesic curvatures of α_1 and α_2 at p are same.

c. Let S be a sphere with Gaussian curvature $K = 2$ everywhere. Let α be a curve in S. Then there is no point on α such that the curvature of α (as a space curve) is 1.

d. Let S_1 and S_2 be two isometric surfaces in $\mathbf{R}^3$. Then there is a rigid motion which maps S_1 onto S_2.

2) (15 pts) Construct a surface S which contains a point p such that the principal curvatures at p are 3 and -2. Compute the first and second fundamental form at p for S.

3) (25 pts) Let S_φ and S_ψ be the surfaces parametrized as

$$\varphi(u,v) = (u\cos v, u\sin v, v)$$
$$\psi(u,v) = (u\cos v, u\sin v, \ln u)$$

a. Show that the Gaussian curvatures are same for each (u,v).

b. Decide whether $\psi \circ \varphi^{-1}$ is a local isometry or not.

4) (15 pts) A diffeomorphism $\psi : S_1 \to S_2$ is said to be *area preserving* if the area of any region $R \subset S_1$ is equal to the area of $\psi(R)$. Show that if ψ is area preserving and conformal, then ψ is an isometry.

5) (25 pts) Let S be a surface of revolution by rotating the curve $\alpha(u) = (f(u), 0, g(u))$ around z-axis. i.e. $S = \{(f(u)\cos\theta, f(u)\sin\theta, g(u))\}$

a. Show that the meridian curves $(\theta = c)$ are geodesic.
b. Which parallel curves $(u = c)$ are geodesic? Explain.