Math 405/538 Differential Geometry
Midterm Exam 1

November 3, 2008

1) a. (15 pts) Consider the helix $\alpha(s) = (a\cos(\frac{s}{c}), a\sin(\frac{s}{c}), \frac{bs}{c})$ where $c = \sqrt{a^2+b^2}$. Compute $\kappa(s), \tau(s), T(s), N(s), B(s)$.

b. (10 pts) Find a regular differentiable curve $\beta(s)$ parametrized by arclength such that $\kappa(s) = 1$, $\tau(s) = 1$ for all $s \in \mathbf{R}$. Is such a curve unique? Explain.

2) (25 pts) Show that there is no regular differentiable curve $\alpha(s)$ parametrized by arclength so that binormal lines all pass through a common point p in $\mathbf{R}^3$ (The binormal line is the line through the point $\alpha(s)$ in the direction of the binormal vector $B(s)$).

3) a. (20 pts) Let $S = \{(x, y, z) \in \mathbf{R}^3 \mid x^2 + y^2 = 1\}$ be a surface which is a cylinder in $\mathbf{R}^3$. Define a parametrization $\varphi : U \to S$ and calculate its first fundamental form for S on $\varphi(U)$.

b. (5 pts) By using the first fundamental form in part (a), calculate the area of the following cylinder $S_0 = \{(x, y, z) \in \mathbf{R}^3 \mid x^2 + y^2 = 1\ ,\ |z| \leq 1\}$

4) (25 pts) Show that if a surface S meets a plane P in a single point p, then this plane coincides with the tangent plane of S at p, i.e. $T_pS = P$.