A MINICOURSE by Klaus Niederkrüger

For the lectures, the main reference is

“ Niederkrüger, Wendl, *Weak Symplectic Fillings and Holomorphic Curves*, arXiv:1003.3923 ”

Monday and Tuesday Lectures

Giroux Torsion as a fillability obstruction

In three dimensional contact topology an important feature is the amount of twisting of a contact structure. The most common way to measure it is by studying certain embeddings of thickened tori I x T^2, in which the contact structure rotates an integer number of times. The supremum of this rotation number is called the Giroux torsion of a contact manifold.

It has been known for a long time that the twisting of a contact structure is an obstruction for finding a symplectic manifold whose boundary is the given contact manifold. For example, it is known that every contact structure on a Seifert fibered manifold that is transverse to the fibers and invariant under translation by the S^1 direction is the boundary of a symplectic manifold.

By contrast, overtwisted contact structures are not even fillable in a weak sense. In the first two lectures, I want to explain a simple proof of the well-known fact (proved in different generality by Eliashberg; Gay; Ghiggini-Honda) that positive Giroux torsion always represents an obstruction for the strong fillability of the manifold, and depending on homological conditions may also be an obstruction to the weak fillability. The proof is based on holomorphic curves with boundary, and is very similar to the non-fillability proof for overtwisted manifolds given by Eliashberg and Gromov in the 80's. It gives also a very graphical explanation for the role of the cohomology with respect to this type of fillability questions.

Wednesday Lectures

Toroidal handles

The aim of these talks will be to explain a handle construction that extends to _weak_ fillings. This allows us to construct many examples of weakly fillable manifolds (that often for other reasons are known not to be strongly fillable).

The manifolds constructed will mostly be products of a surface with a circle as studied for the first time by Lutz with a contact structure that should be invariant under the circle action. In many cases this allows us to directly read off from the dividing sets on the surface whether the manifold is weakly fillable.

Friday Lecture

Different notions of fillability in higher dimension

The aim of this last lecture will be to give a quick overview of fillability in higher dimensions. The main emphasis will be on explaining a notion of weak fillability, again sketch that the main difference between a strong filling and a weak filling is a cohomological one, and state several results about non-fillability for manifolds of dimensions five and higher.