

# Holomorphic Curves, Planar Open Books and Symplectic Fillings

*A MINICOURSE by Chris Wendl*

The overarching theme of this minicourse will be the properties of pseudoholomorphic curves and their use in proving global results about symplectic or contact manifolds based on more "localized" information. We will aim primarily to understand the proofs of two recent results:

- (1) Every weak symplectic filling of a planar contact 3-manifold is a blowup of a Stein filling (joint with K. Niederkrueger).
- (2) There exist infinite sequences of tight but non-fillable contact 3-manifolds such that each cannot be obtained from the previous one by contact surgery (joint with J. Latschev and M. Hutchings).

In both cases, the proofs take some inspiration from McDuff's 1990 paper on rational and ruled symplectic 4-manifolds, so as preparation, we will begin by discussing McDuff's results and some relevant background material on closed holomorphic curves and Lefschetz pencils. Then we will generalize the discussion to punctured holomorphic curves in symplectic cobordisms, and explain the relationship with open books and Lefschetz fibrations in order to sketch proofs of the two theorems mentioned above.

Here is a tentative plan of the lectures. Numbers in brackets refer to relevant references in the literature for each topic (see reference list below), though I will not always follow the same approach as in the references.

**DAILY PROGRAM is in the next page.**

### **Lecture 1 (Monday):**

Introduction / Properties of closed holomorphic curves 1 Summary of main goals, local properties of holomorphic curves (regularity, similarity principle, positivity of intersections) [12]

### **Lecture 2 (Monday):**

Properties of closed holomorphic curves 2 Gromov compactness [5], Fredholm theory and (generic) transversality [12], automatic transversality [5,8]

### **Lecture 3 (Wednesday):**

McDuff's results on symplectic 4-manifolds Adjunction formula [5], Lefschetz pencils and symplectic structures [2], McDuff's theorem on rational and ruled symplectic 4-manifolds [4,5]

### **Lecture 4 (Wednesday):**

Symplectizations and holomorphic open books Stable Hamiltonian structures, symplectizations of contact manifolds, compactness [1] and Fredholm theory [8] for punctured holomorphic curves, existence of holomorphic open books [10]

### **Lecture 5 (Friday):**

Holomorphic curves in symplectic fillings and cobordisms Intersection theory of punctured holomorphic curves [7], uniqueness in holomorphic open books [11], classifying weak fillings of planar contact manifolds [6,9], sketch of obstructions to exact cobordisms [3]

## REFERENCES:

- [1] F. Bourgeois, Y. Eliashberg, H. Hofer, K. Wysocki and E. Zehnder, "Compactness results in symplectic field theory", *Geom. Topol.* 7 (2003), 799--888
- [2] R. Gompf and A. Stipsicz, "4-manifolds and Kirby calculus", American Mathematical Society, 1999
- [3] J. Latschev and C. Wendl, "Algebraic torsion in contact manifolds", Preprint <http://arxiv.org/abs/1009.3262> (with an appendix by M. Hutchings)
- [4] D. McDuff, "The structure of rational and ruled symplectic 4-manifolds", *J. Amer. Math. Soc.* 3 (1990), no. 3, 679--712
- [5] D. McDuff and D. Salamon, "J-holomorphic curves and symplectic topology", American Mathematical Society, 2004
- [6] K. Niederkrüger and C. Wendl, "Weak symplectic fillings and holomorphic curves", Preprint <http://arxiv.org/abs/1003.3923>
- [7] R. Siefring, "Intersection theory of punctured pseudoholomorphic curves", Preprint <http://arxiv.org/abs/0907.0470>
- [8] C. Wendl, "Automatic transversality and orbifolds of punctured holomorphic curves in dimension four", *Comment. Math. Helv.* 85 (2010), no. 2, 347--407
- [9] C. Wendl, "Strongly fillable contact manifolds and J-holomorphic foliations", *Duke Math. J.* 151 (2010), no. 3, 337-384
- [10] C. Wendl, "Open book decompositions and stable Hamiltonian structures", *Expo. Math.* 28 (2010), no. 2, 187-199
- [11] C. Wendl, "A hierarchy of local symplectic filling obstructions for contact 3-manifolds", Preprint <http://arxiv.org/abs/1009.2746>
- [12] C. Wendl, "Lectures on Holomorphic Curves in Symplectic and Contact Geometry", Preprint <http://arxiv.org/abs/1011.1690>