Bidirected Flows & b-matching

Consider the problem:

$$x \geqq 0, \text{ integer}$$

$Ax = b$; A integer matrix ($\cancel{\ne} 0, \pm 1$ matrix)

b integer vector

Min(Max) cx. , c arbitrary

Where $a_{i,j}$ (entries in A) satisfy the condition

$$\sum_i |a_{ij}| \leq 2 \quad \forall j.$$

(Each column of A contains either 2 ~~&~~ or less ± 1, ~~and~~ or One ± 2), ~~or none of these~~)

Such a problem is called <u>bidirected flows</u>

If each column has either one non zero which is a ± 1, or one $+1$ and one -1, we get our (directed) min-cost flow problem

If it has two $+1$ or two -1, we get b-matching. one ± 2 allows SP in Undir. graphs

Algorithms for matching can be extended to polynomial algorithms for these as well (and also can be converted to ~~SP algo~~ strongly polynomial algorithms)

Cliques in Special graphs:

Given an undirected graph $G = [V; E]$ with edge weights $w(e)$, $e \in E$.

Def <u>Line graph</u> $L(G)$ of G is defined as follows.

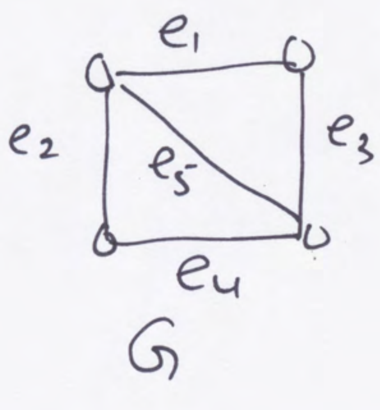
these are vertices in $L(G)$

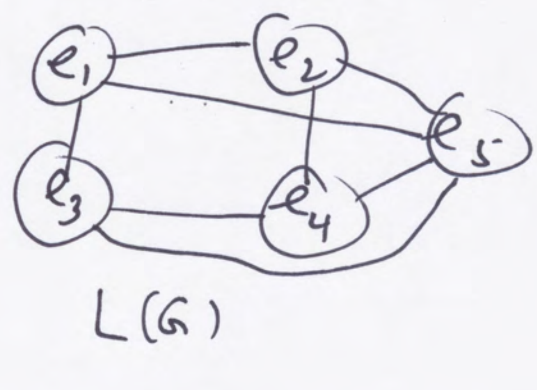
$$L(G): [E ; F]$$

$$F: \{(e_1, e_2): e_1 \in E, e_2 \in E, (e_1, e_2) \text{ share a vertex in common}\}$$

i.e incident at some vertex

Example

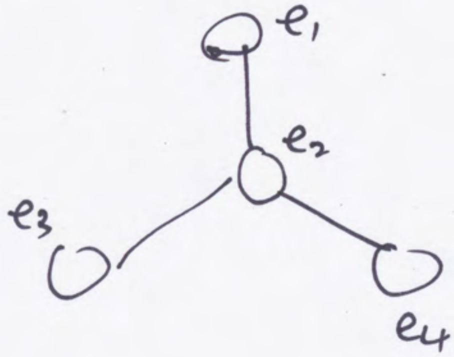


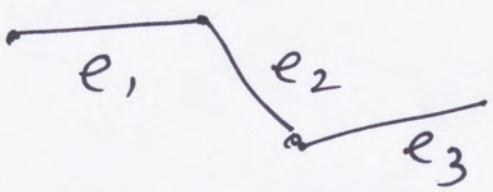


A matching M in G is an anti-clique (independent set) in $L(G)$ and conversely.

Find max cardinality matching in G is the same as finding largest independent set in $L(G)$ and similar results hold for max weighted matching and max wtd ind. set.

Notice that $L(G)$ <u>can not have</u> as a <u>node-induced</u> subgraph, a <u>claw</u>





Where is e_4?

Such graphs are called Claw-free graphs.
Not all Claw-free graphs are line graphs of some graph. So finding largest (wtd) ind. set in claw free graphs extends matchi

There are several other applications of matching
(See book by L. Lovasz & M. Plummer)

One of these applies to multi-commodity
flows and goes under the name
T- joins and T- cuts
Another deals with decomposing a simple polygon
into convex polygons (See Chazelle and Dobkin)

There is also an application in physics
on spin-models

End of Matching

We move next to multi-commodity
flows. Download A. Schrijver's notes.