CS 6381 - Lecture 22-B 10·28·2020 (1)

Blocking Systems & Bottleneck optimization Problems

References

1. D.R. Fulkerson, O. Gross : "A Production Line Assignment Problem". RM 1102 (1953) RAND

2. O. Gross : "The Bottleneck assignment Problem" P. 1630 (1959) RAND

3. H. Pollack : "The Maximum Capacity Route through a Network" O.R. (1960) pp. 722-736

4. T.C. Hu : "The Maximum Capacity Route Problem" O.R. (1961) pp. 898-900

5. J. Edmonds, D.R. Fulkerson : "Bottleneck Extrema" RM 5375 PR RAND (1968)

Maximum Capacity Route = FAT-Path Problem

Given a graph $G = [V; E]$ with edge capacities $C(e) \geq 0$, $e \in E$; Want $\max\limits_{P \in \mathcal{P}} \min\limits_{e \in P} C(e)$

where $\mathcal{P}$: set of s-t paths.

Bottleneck Assignment Problem (O. Gross)

Consider an assembly line with n processing stations (in an ordered manner). Assume we have n processors and if j^{th} processor is assigned to i^{th} station, rate of productivity is r_{ij} (if j can not do i^{th} process, $r_{ij} = 0$)

We want to assign to each service station, a processor; one processor can only do one job. We want to maximize the production rate of the assembly line.

Let E be the set of cells in an $n \times n$ matrix

(i, j) cell has a weight r_{ij} $\quad i = 1 \cdots n$, $\; j = 1 \cdots n$

Let $\mathcal{P} = \{ P \subseteq$ ~~E~~ cells of E : there is exactly one cell in ~~ith row & j on~~ each row and each column of $E \}$

Corresponds to an assignment.

Want $\quad \displaystyle \max_{P \in \mathcal{P}} \min_{\text{cell} \in P} r_{ij}$

3. Let $E = \{1, 2, \ldots, 2k+1\}$: Set of elected officials who pass laws.

$\mathcal{P}$: Subset of E that can pass a law

i.e $\{P \subseteq E : |P| = k+1\}$

$\mathcal{P}$ is a clutter on E : no nested sets.

$\mathcal{K}$: Subsets of E that can block passage of a bill.

$= \{K \subseteq E : |K| = k+1\}$

Occurs in multiperson game theory in what are called <u>Majority Games</u>

4. There are other bottleneck problems all of which lead to Blocking Systems.

We study these now.