

Where does this lead us? . It leads us to what are called Set-Packing Problems and Set-Covering problems.

Set packing problems

Let E be a finite set; Let \mathcal{P} be a family of subsets of E . Let A be an incidence matrix (0-1 entries) where ~~columns~~ ^{rows} represent $P \in \mathcal{P}$, ~~rows~~ ^{columns} represent $e \in E$. Let $w(e), e \in E$ be non-negative (possibly integer) weights (Capacity of $e \in E$) Consider, the integer Linear Program.

$$\text{Max } \sum_{P \in \mathcal{P}} y(P) = \sum_{i=1}^m y_i$$

$$\text{s.t. } A^t y \leq w$$

$$y \geq 0, \text{ integer.}$$

This is called a set packing problem.

In the same context, the integer LP

$$\text{Min } \sum y_i$$

$$A^t y \geq w$$

$$y \geq 0, \text{ integer}$$

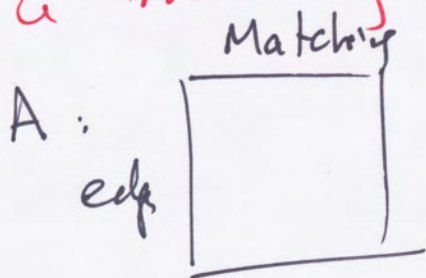
is called a Set-covering problem.

Examples:

Coloring problems

Edge-coloring : Given an undirected graph $G = [V; E]$, a valid edge-coloring gives colors to edges so that two edges incident at a vertex receive different colors. We want min # of colors required

Note Edges that receive the same color form a matching (if coloring is valid)

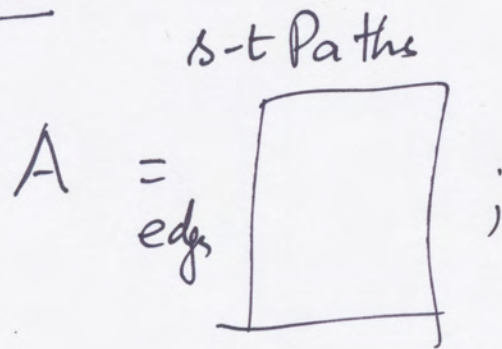


$$\left. \begin{array}{l} y \geq 0, \text{ integer} \\ A y \geq \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \\ \text{Min } \sum_j y_j \end{array} \right\}$$

Set-covering

For node coloring, we must use independent ⁽³⁾
Sets instead of Matching \rightarrow Again a Set
Covering problem

Max flow



$$y \geq 0$$

$$Ay \leq c$$

$$\text{Max } \sum_j y_j$$

Set ~~path~~ packing;

Path-Packing Problem

Shortest Path: Cut-Packing Problem

This leads via Linear Programming duality to
Blocking and Anti-Blocking Polyhedra

Which we take up next.

Blocking Polyhedra:

(4)

Set Packing

$$\begin{aligned} \text{Max } \sum y_j &= e^t y \\ x \geq 0: \quad & A^t y \leq w \\ & y \geq 0, (\text{int}) \end{aligned}$$

(P) viewed as LP.

$$\text{Min } w^t x$$

$$Ax \geq \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} = e \quad \text{(D)}$$

$$x \geq 0$$

Set of feasible solutions to dual is an unbounded convex polyhedron and is denoted by B. B can be expressed as a sum of convex-hull of its extreme points and non-neg. orthant R_+^n .

Def: i th row is inessential if $\exists \lambda$ such that

$$\lambda \geq 0, \quad A_i \cdot \geq \sum_{j \neq i} \lambda_j A_j, \quad \sum_{j \neq i} \lambda_j = 1$$

(i th constraint is redundant in (D))

Def: A is proper if \exists inessential rows.
 (equivalent to non-nestedness of sets)

Def The blocker \hat{B} of B is defined by:

$$B = \{y: y \geq 0, y^t x \geq 1, \forall x \in B\}$$

(Equivalent to $P \cap K \neq \emptyset \forall P \in \mathcal{P}, \forall K \in \mathcal{K}$)

Let extreme points of B be b^1, b^2, \dots, b^r

Let B be a $r \times n$ matrix whose rows are b^1, b^2, \dots, b^r . Let $\mathcal{A} = \{y: y \geq 0, By \geq (1)\}$

Then (i) $\hat{\mathcal{A}} = B$.

(ii) B is proper assuming A is proper.

ext pts of A are rows of A.

(iii) $\hat{A} = B$.

(i.e. Blocker of blocker is the original system)

$$\hat{A} = \{x: x \geq 0, x^t y \geq 1, \forall y \in \mathcal{A}\}$$

We can extend all of the results in Blocking (6)
Systems to Blocking Polyhedra.