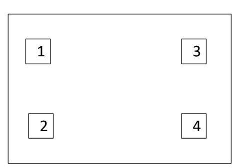
CS 6381 : **Assignment #4& Exam**

Due Date:

1. Let Q be a finite set and let $(S_1, S_2, ..., S_k)$ be a family of subsets of Q. A **system of distinct reperesentatives** (SDR) is a set $(q_1, q_2, ..., q_k)$ of distinct elements of Q, such that $q_i \in S_i; i = 1, 2, .., k$. Show how to test if an SDR exists and how to find one if it does using maximum flow.

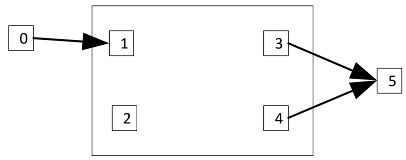
2. Elimination of Sports Teams: Consider a scenario with a set T of teams in some league. At some point in the season, some games hav been palyed so far and their outcomes are known. More games are scheduled and we know the upcoming schedule. We want to know if some team is eliminated. A team is eliminated if regardless of the outcomes of the future games, this team has no chance of winning. Each game is won by one team and the other loses the game. [There are no ties]. Let w_i be the number of wins for team i so far [this is given]. Let $r_{i,j} = r_{j,i}$ be the number of games between teams i and j scheduled to be played in future. [this is also given]. We wish to determine whether or not a team s is eliminated. Show how to use max-flow to answer this question. [Hint: Check the book by Cunningham et al.]

3. In the following graph, we do not know the internal connections and capacities. However, we know the values of $F^*_{1,3}, F^*_{1,4}, F^*_{2,3}$, and $F^*_{2,4}$ where $F^*_{i,j}$ represents maximum flows with i as origin and j as destination.

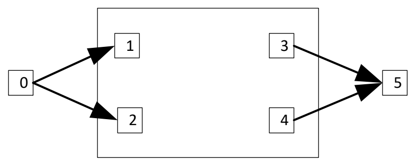


We wish to get either exact values or tightest bounds on maximum flow $F^*_{0,5}$ for each of the following problems [you may use the exact solutions of the previous problems (call them F^*_a, F^*_b etc) in arriving at bounds for the later ones]:

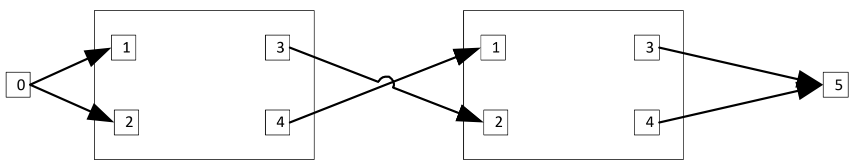
(a)



(b)



(c)



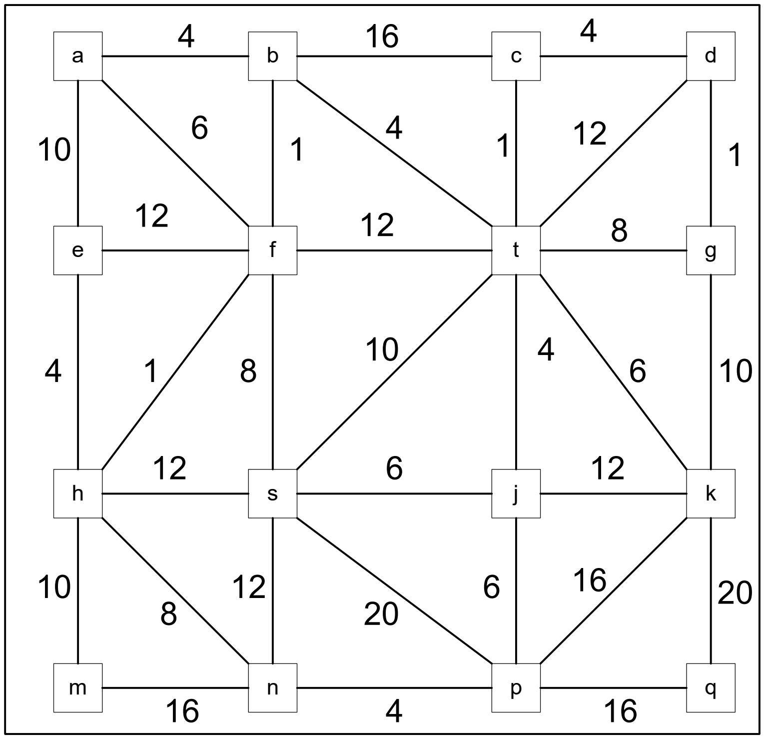
4. Determine maximum weight perfect matching in the following graph: Start with the following dual solution:

   [a=4,b=0,c=16,d=0,e=12,f=2,t=12,g=1,h=0,s=12]

   [j=0,k=12,m=16,n=8,p=16,q=8] for both parts.

   (a) First remove all "diagonal edges" to get a bipartite graph and solve this problem.

(b) Do this for the general graph.



5. **Let $G = (V, E)$ be an undirected graph. Let $\{u_{i,j}\}; (i, j) \in E$ be non-negative capacities. Suppose $u_{x,y} > \sum_{j \neq y} u_{x,j}$. Show that (x, y) is an edge in Gomory Hu cut-tree.