Assignment #1:

1. Consider the following problem: $m =$ the number of machines $= 1$; $n =$ the number of jobs $= n$; $r_j =$ "release" date – time at which processing of job j can begin; $p_j =$ total processing time for job j; $w_j =$ weight of job j. [Assume all these are input data of the problem and are positive numbers]. We want a schedule S that minimizes $\sum_{j=1}^{n} w_j F_j(S)$ where $F_j(S)$ is the "flow" time of job j in S and is given by

$$F_j(S) = C_j(S) - r_j$$

where $C_j(S)$ is the completion time of job j in S.

   (a) First consider the case where $w_j = 1 \forall j$. If preemptions are allowed, show that SRPT rule is optimal. SRPT rule assigns for processing at all times the job with the *shortest remaining processing time*. [Make sure to argue that inserted idle time is not present in optimal schedules]

   (b) Now consider the case with general values for w_j. Devise an algorithms to solve this problem again allowing preemptions. [Make sure to argue that inserted idle time is not present in optimal schedules]

   (c) Now consider the case where preemption is not allowed. In this part, let $w_j = 1 \forall j$. Show that we can no longer assume that optimal solutions have no inserted idle time.

2. Consider the following problem: $m = 1; r_j = 0 \forall j; p_j, w_j$ are as in the previous problem. We want a schedule S that minimizes $\sum_{j=1}^{n} w_j [1 - e^{-\alpha C_j(S)}]$ where $0 < \alpha < 1$. Show that WDSPT [weighted discounted shortest processing time) rule produces an optimal schedule . This is a schedule that processes jobs in decreasing order of $\frac{w_j e^{-\alpha p_j}}{1 - e^{-\alpha p_j}}$. [Make sure you consider inserted idle time and preemption.]