Geometric optics of Bloch waves in a chiral and dissipative medium

Chuanwei Zhang[1] and Qian Niu[2]
[1]*Department of Physics and Astronomy, Washington State University, Pullman, Washington 99164, USA*
[2]*Department of Physics, The University of Texas, Austin, Texas 78712, USA*
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We present a geometric optics theory for the transport of quantum particles (or classical waves) in a chiral and dissipative periodic crystal subject to slowly varying perturbations in space and time. Taking account of some properties of particles and media neglected in previous theory, we find important additional terms in the equations of motion of particles. The (energy) current density field, which traces the geometric optics rays, is not only governed by the Bloch band energy dispersion but also involves there additional fields. These are the angular momentum of the particle, the dissipation dipole density, and various geometric gauge fields in the extended phase space spanned by space time and its reciprocal, momentum, and frequency. For simplicity, the theory is presented using light propagation in photonic crystals.

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I. INTRODUCTION

The transport property of quantum particles (or classical waves) in media is one of the central problems in physics. A textbook example is the dynamics of Bloch electrons in solid-state crystals [1,2]. For a long time, the semiclassical equations of motion (EOM) of electrons have been known to be solely determined by the energy-band dispersion [1]. Recently it was found that geometric gauge fields due to Berry phase [3] effects may modify the EOM of particles [2,4] in a chiral medium, leading to important physical phenomena and applications [5–12]. Here "chiral" means that the time reversal or spatial inversion symmetry in the medium is broken.

However, the EOM in current literature neglect several important properties of particles (or waves) and media [e.g., self-rotation of particles, frequency (or energy) dependence, and dissipation of the media]. In this article, we derive a set of EOM of particles where the effects of these properties are included. For simplicity of the presentation, we consider a concrete physical system: light propagation in photonic crystals [13], although the results are applicable to many other systems [11]. Instead of wave-packet dynamics commonly used in previous literature, we adopt a geometric optics approach that has been proved to be extremely valuable for describing light propagation in usual optical materials. However, the traditional approach of geometric optics is inadequate because the light wavelength is typically the same order as the unit cell size of the photonic crystals within which the dielectric constant varies.

The geometric optics theory is presented for light propagation in a frequency band of a chiral and dissipative photonic crystal under perturbations which vary slowly in space and time compared to the lattice constant and the band gaps. We focus on the Poynting vector field, averaged over the crystal unit cell, which traces the geometric optics rays (i.e., the EOM). It involves the photon wave-packet center velocity and also contains two additional terms due to the photon orbital angular momentum (POAM) and the dissipation of the medium that are absent in previous wave-packet theory of photons [8,9]. We show that, in a concrete example, the term from POAM can dramatically modify the transport properties of light in photonic crystals. The propagation of light is also affected by various geometric gauge fields due to Berry phase effects. Both POAM and Berry phase originate from the chirality of the medium or the particle. Two such gauge fields mimic closely the electric and magnetic fields for electrons [12], and two other fields arise from Berry curvatures in the momentum-frequency space, previously called reciprocal electromagnetic fields in the context of electron semiclassical dynamics [4,14]. The physical origin of the reciprocal electric field is illustrated. There are also fields from Berry curvatures in other facets of the extended phase space. All these fields can be calculated based on the band-structure wave functions of the photonic crystal.

The article is organized as follows: Section II introduces the wave equations and the wave-packet dynamics for light propagation in photonic crystals. In Sec. III, a geometric optics theory for light propagation in photonic crystals is developed. Section IV gives an example of the application of the theory. Finally, we summarize and conclude in Sec. V.

II. WAVE EQUATIONS AND WAVE PACKET DYNAMICS

Consider light propagating in an inhomogeneous photonic crystal characterized by the dielectric permittivity tensor $\varepsilon(\mathbf{r})$ and magnetic permeability tensor $\mu(\mathbf{r})$, where ε and μ are spatially fast-varying periodic functions with slowly spatial modulation. ε and μ may also have smooth temporal modulation, which, together with dissipation of the medium, will be discussed in Sec. III D for simplicity of the presentation. The Maxwell equations without external sources (i.e., currents) can be written as a Schrödinger-equation like form

$$[\mathcal{H}(-i\partial_{\mathbf{r}},\mathbf{r}) - \Xi(\omega,\mathbf{r})]\Phi(\mathbf{r}) = 0, \tag{1}$$

where $\mathcal{H} = \begin{pmatrix} 0 & i\partial_{\mathbf{r}}\times \\ -i\partial_{\mathbf{r}}\times & 0 \end{pmatrix}$, $\Phi = (\mathbf{E},\mathbf{H})^T/\sqrt{2}$ is the electromagnetic wave function, $\Xi(\omega) = \omega\begin{pmatrix} \varepsilon(\omega) & 0 \\ 0 & \mu(\omega) \end{pmatrix}$ and the frequency ω corresponds to $i\partial_t$ in the time-dependent Maxwell equation. In multiferroic materials with strong magnetoelectric coupling, $\Xi(\omega)$ may also have nondiagonal terms [15]. The form of the wave equation (1) is very general. For instance, it can be taken as the Schrödinger equation for $\Xi = \hbar\omega$ and $\mathcal{H} = -\hbar^2\partial_{\mathbf{r}}^2/2m + V(\mathbf{r})$.

Without considering dissipation and frequency dependence of the medium, the wave equation (1) can be treated using the

wave-packet dynamics. The central idea [4] is to construct a wave packet

$$|\Phi\rangle = \sqrt{\rho}\int d^3q a(\mathbf{q},t)|\psi(\mathbf{q},\mathbf{r}_c)\rangle \tag{2}$$

with a mean wave vector

$$\mathbf{q}_c = \int d^3q |a(\mathbf{q},t)|^2\mathbf{q} \tag{3}$$

and a preassigned wave-packet center position

$$\mathbf{r}_c = [\langle\Phi|\hat{\mathbf{r}}|\Theta_0\Phi\rangle + \langle\Theta_0\Phi|\hat{\mathbf{r}}|\Phi\rangle]/2\rho. \tag{4}$$

Here $\psi(\mathbf{q},\mathbf{r}_c) = (\mathbf{E},\mathbf{H})^T/\sqrt{2\rho} = e^{i\mathbf{q}\cdot\hat{\mathbf{r}}}\phi(\mathbf{q},\mathbf{r}_c)$ is the Bloch eigenstate of the local wave equation

$$[\mathcal{H}_0 - \Xi_0(\omega_{c0},\mathbf{r}_c)]\psi(\mathbf{q},\mathbf{r}_c) = 0 \tag{5}$$

with the eigenenergy $\omega_{c0} = \omega_{c0}(\mathbf{q},\mathbf{r}_c)$, $\mathcal{H}_0 - \Xi_0$ is the local operator with the required periodicity of the unperturbed crystal, $\Theta_0 = \partial\Xi_0/\partial\omega_{c0}$, $\phi(\mathbf{q},\mathbf{r}_c)$ is the periodic part of the Bloch wave and satisfies $\langle\phi|\Theta_0|\phi\rangle = 1$, and $\rho = \langle\Phi|\Theta_0|\Phi\rangle$ is the normalization factor. In a medium without frequency dependence, we have $\rho = (\mathbf{E}\cdot\mathbf{D} + \mathbf{H}\cdot\mathbf{B})/2$, the energy density of photons. While in the Schrödinger equation, ρ is the wave density. The amplitude $a(\mathbf{q},t)$ satisfy the normalization condition $\int d^3q|a(\mathbf{q},t)|^2 = 1$.

Using a time-dependent variational principle with the Lagrangian

$$L = \langle\Phi|[\Xi(i\partial_t,\mathbf{r}) - \mathcal{H}(-i\partial_{\mathbf{r}},\mathbf{r})]|\Phi\rangle, \tag{6}$$

we obtain the semiclassical EOM for photons

$$\dot{\mathbf{r}}_c = \frac{\partial\omega_c}{\partial\mathbf{q}_c} - \Omega_{\mathbf{qq}}\dot{\mathbf{q}}_c - \Omega_{\mathbf{qr}}\dot{\mathbf{r}}_c, \tag{7}$$

$$\dot{\mathbf{q}}_c = -\frac{\partial\omega_c}{\partial\mathbf{r}_c} + \Omega_{\mathbf{rq}}\dot{\mathbf{q}}_c + \Omega_{\mathbf{rr}}\dot{\mathbf{r}}_c, \tag{8}$$

by following a similar derivation as that in Ref. [4] for electrons, where

$$\omega_c = \omega_{c0} + \mathrm{Im}\left\langle\frac{\partial\phi}{\partial\mathbf{r}_c}\middle|(\mathcal{H}_0 - \Xi_0)\middle|\frac{\partial\phi}{\partial\mathbf{q}_c}\right\rangle \tag{9}$$

is the total energy,

$$\Omega^{\alpha\beta}_{\mathbf{qq}} = i\left\langle\frac{\partial\phi}{\partial\mathbf{q}_\alpha}\middle|\Theta_0\middle|\frac{\partial\phi}{\partial\mathbf{q}_\beta}\right\rangle + \text{c.c.} \tag{10}$$

is the Berry curvature in the momentum space, $\Omega_{\mathbf{qr}}$ and $\Omega_{\mathbf{rr}}$ have similar definitions.

III. GEOMETRIC OPTICS APPROACH

The geometric optics approximation we adopt does not involve the concept of wave packet. Substituting the WKB trial wave function $\Phi(\mathbf{r},t) = \exp[iS(\mathbf{r},t)]u(\mathbf{r})$ into the time-dependent Maxwell equation, we obtain

$$[\mathcal{H}(-i\partial_{\mathbf{r}} + \mathbf{p}) - \Xi(\omega,\mathbf{r})]u = 0, \tag{11}$$

where $\mathbf{p} = \partial_{\mathbf{r}}S$ is the wave vector, the frequency $\omega = -\partial_t S$ is the same as that defined in Eq. (1). $\mathbf{p}$ plays a similar role as $\mathbf{q}_c$ in the wave-packet dynamics. In the geometric optics theory for an inhomogeneous medium, $\mathbf{p}$'s direction is perpendicular to the wave front that has a constant phase S.

Because the spatial modulation of the medium is slowly varying, u can be taken as a periodic function of $\mathbf{r}$ over a few lattice sites. The spatial modulation of u can be accounted by taking u as a function of a slowly varying variable $\mathbf{R}$, where $\mathbf{R}$ describes variations on a length scale much larger than the lattice spacing. $\mathbf{R}$ plays the same role as $\mathbf{r}_c$. For a fixed $\mathbf{R}$, u is a periodic function of the fast-varying variable $\bar{\mathbf{r}}$. With the slowly varying variable $\mathbf{R}$ and the fast varying variable $\bar{\mathbf{r}}$, Eq. (11) can be rewritten as

$$[\mathcal{H}(-i\partial_{\bar{\mathbf{r}}} - i\partial_{\mathbf{R}} + \mathbf{p}) - \Xi(\omega,\bar{\mathbf{r}},\mathbf{R})]u = 0. \tag{12}$$

By choosing a suitable phase S, we can take $\mathbf{p}$ and ω as functions of $\mathbf{R}$.

Because u varies slowly with respect to $\mathbf{R}$, we may solve Eq. (12) perturbatively. Up to the first-order correction $\partial_{\mathbf{R}}u$, we can take the gradient expansion $\mathcal{H}(-i\partial_{\bar{\mathbf{r}}} - i\partial_{\mathbf{R}} + \mathbf{p}) \approx \mathcal{H}_0(-i\partial_{\bar{\mathbf{r}}} + \mathbf{p}) + \Delta\mathcal{H}$ with

$$\Delta\mathcal{H} = -\frac{i}{2}\left(\partial_{\mathbf{R}}\cdot\frac{\partial\mathcal{H}_0}{\partial\mathbf{p}} + \frac{\partial\mathcal{H}_0}{\partial\mathbf{p}}\cdot\partial_{\mathbf{R}}\right), \tag{13}$$

and $\Xi(\omega,\bar{\mathbf{r}},\mathbf{R}) = \Xi_0(\omega_0) + \Theta_0\omega_1$. Here we denote

$$\Theta_0 = \frac{\partial\Xi_0(\omega_0)}{\partial\omega_0} \tag{14}$$

and expand the eigenenergy and wave function to the first order: $\omega = \omega_0 + \omega_1$, $u = u_0 + u_1$. With these expansions, the zero order of Eq. (12) gives

$$[\mathcal{H}_0(-i\partial_{\bar{\mathbf{r}}} + \mathbf{p}) - \Xi_0]u_0 = 0, \tag{15}$$

which is similar as the local wave equation (5) in the wave-packet dynamics. For a fixed $\mathbf{R}$, Eq. (15) yields a Bloch wave function $u_0 = \sqrt{\rho}f$, where we choose the normalization condition $\langle f|\Theta_0|f\rangle = \int d^3\bar{\mathbf{r}}f^*(\bar{\mathbf{r}})\Theta_0 f(\bar{\mathbf{r}}) = 1$. Here and later in this article, an inner product corresponds to an integration over a unit lattice cell. f plays the same role as ϕ in the wave-packet dynamics. We emphasize that the wave function f is a periodic function of the fast-varying variable $\bar{\mathbf{r}}$ that originates from the periodicity of the photonic crystals. f describes the periodic wave property of light in photonic crystals that does not exist in traditional geometric optical approach for an inhomogeneous medium. The periodic wave function can lead to Eq. (15) and together with suitable boundary conditions yields the energy dispersion relation $\omega_0 = \omega_0(\mathbf{p},\mathbf{R})$. In general f may be a function of $\bar{\mathbf{r}},\mathbf{R},\mathbf{p}$, and ω_0, depending on the way to record the wave function. In this article, we will take f as a function of $\bar{\mathbf{r}},\mathbf{p},\mathbf{R}$ only by replacing ω_0 in u_0 as $\omega_0(\mathbf{p},\mathbf{R})$ using the dispersion relation.

The first order of the gradient expansion of Eq. (12) is

$$\mathcal{L}_0u_1 = \mathcal{G}u_0, \tag{16}$$

where $\mathcal{L}_0 = \mathcal{H}_0 - \Xi_0$, $\mathcal{G} = -\Delta\mathcal{H} + \Theta_0\omega_1$. Note that the semiclassical geometric optics approach differs from the well-known Luttinger-Kohn treatment [16], which is a full quantum mechanical perturbative approach to perturbed periodic systems. In Luttinger-Kohn treatment, the wave function is expanded around the band minimum. While in the geometric optics approach, the gradient expansion is around the local Bloch wave function.

A. Energy correction and Berry phase

Multiplying each side of Eq. (16) with $\langle u_0|$, we obtain the first-order energy correction $\omega_1 = \langle f|\Delta\mathcal{H}|f\rangle$. A straightforward but tedious evaluation of ω_1 yields

$$\omega_1 = \mathrm{Im}\left\langle \frac{\partial f}{\partial \mathbf{R}}\right| \cdot \mathcal{L}_0 \left|\frac{\partial f}{\partial \mathbf{p}}\right\rangle - \mathrm{Im}\langle f|\Theta_0\left|\frac{df}{dt}\right\rangle. \tag{17}$$

The first term $\omega_{1a} = \mathrm{Im}\langle \frac{\partial f}{\partial \mathbf{R}}| \cdot \mathcal{L}_0|\frac{\partial f}{\partial \mathbf{p}}\rangle$ is the same as that derived from the wave-packet dynamics. In the second term $\omega_{1b} = -\mathrm{Im}\langle f|\Theta_0|\frac{df}{dt}\rangle$, $\frac{df}{dt} = \dot{\mathbf{R}}\cdot\frac{\partial f}{\partial \mathbf{R}} + \dot{\mathbf{p}}\cdot\frac{\partial f}{\partial \mathbf{p}}$ since we assume no explicit time dependence of the medium. Therefore along a path $\mathcal{C}$, a Berry phase $\gamma = \int_{\mathcal{C}} \mathbf{A_R}\cdot d\mathbf{R} + \mathbf{A_p}\cdot d\mathbf{p}$ is accumulated. Here $\mathbf{A_R}$ and $\mathbf{A_p}$ are Berry connections in position and momentum spaces, with $\mathbf{A_R} = -\mathrm{Im}\langle f|\Theta_0|\frac{\partial f}{\partial \mathbf{R}}\rangle$. $\mathbf{A_p}$ has a similar definition.

The procedure to obtain the energy correction (17) also yields the continuity equation

$$\frac{\partial \rho}{\partial t} + \partial_{\mathbf{R}}\cdot\mathbf{J} = 0, \tag{18}$$

where

$$\mathbf{J} = \langle u|\frac{\partial \mathcal{H}}{\partial \mathbf{p}}|u\rangle = \mathbf{E}\times\mathbf{H} \tag{19}$$

is the Poynting vector (the energy current density) of light. Note that ρ does not vary with time without considering the dissipation of the medium, i.e., $\frac{\partial \rho}{\partial t} = 0$. At the zero order, $\mathbf{J}_0 = \rho\mathbf{v}_0$ with $\mathbf{v}_0 = \partial\omega_0/\partial\mathbf{p}$ as the zero order of the local velocity. In the Schrödinger equation, $\mathbf{J}$ is the current density of particles.

B. Poynting vector field

Substituting $u = u_0 + u_1$ into (19), we find the Poynting vector can be rewritten as

$$\mathbf{J} = \rho\left(\frac{\partial\bar{\omega}}{\partial\mathbf{p}} - \Omega_{\mathbf{pp}}\dot{\mathbf{p}} - \Omega_{\mathbf{pR}}\mathbf{v}_0\right) + \partial_{\mathbf{R}}\times\rho\mathbf{m}. \tag{20}$$

Here $\bar{\omega} = \omega_0 + \omega_{1a}$, $\dot{\mathbf{p}} = \mathbf{v}_0\cdot\frac{\partial\mathbf{p}}{\partial\mathbf{R}}$, $\partial_{\mathbf{R}} = \frac{\partial}{\partial\mathbf{R}} + \frac{\partial\mathbf{p}}{\partial\mathbf{R}}\cdot\frac{\partial}{\partial\mathbf{p}}$, and $\mathbf{m}$ is the POAM defined in Eq. (21). The Berry curvatures $\Omega_{\mathbf{pp}}$ and $\Omega_{\mathbf{pR}}$ have the same definitions as that in the wave-packet dynamics [Eq. (10)] with the replacement of the notations $\{\mathbf{r}_c,\mathbf{q}_c,\omega_{c0},\phi\}\rightarrow\{\mathbf{R},\mathbf{p},\omega_0,f\}$. The first three terms in Eq. (20) describe the energy flow due to the translational motion of photons with a velocity same as Eq. (7) for the wave-packet dynamics.

It is clear from Eq. (20) that the energy current density $\mathbf{J}$ cannot be simply determined by the translational motion of photons. $\mathbf{J}$ contains an additional term $\partial_{\mathbf{R}}\times\rho\mathbf{m}$, which is absent in Eq. (7) for the wave-packet dynamics. Note that

$$\mathbf{m} = -\frac{i}{2}\left\langle\frac{\partial f}{\partial\mathbf{p}}\right|\times(\Xi_0 - \mathcal{H}_0)\left|\frac{\partial f}{\partial\mathbf{p}}\right\rangle \tag{21}$$

is the POAM. For noninteracting electrons in crystals (no frequency dependence), $\mathbf{m}$ is just the orbital magnetic moment of Bloch electrons [4]. In the wave-packet treatment of electron dynamics, $\mathbf{m}$ originates from the finite spread of the wave packet. The wave packet generally rotates about its center position, giving rise to an orbital magnetic moment $\mathbf{m} = -e/2\langle W|(\mathbf{r}-\mathbf{r}_c)\times\hat{\mathbf{v}}|W\rangle$, where $|W\rangle$ is the wave packet and $\hat{\mathbf{v}}$ is the velocity operator [17]. Therefore the term $\partial_{\mathbf{R}}\times\rho\mathbf{m}$ corresponds to a local current density field arising from the self-rotation of photons (i.e., POAM) in photonic crystals.

Note that both $\mathbf{p} = \partial_{\mathbf{r}}S$, and $\omega = -\partial_t S$ are gauge-dependent quantities, i.e., they depend on the choice of S. In the static case, a natural choice of the gauge is to ensure that energy is the same for different positions, i.e., $\partial_{\mathbf{R}}\omega = 0$. Under this gauge, we find $\dot{\mathbf{p}} = -\frac{\partial\omega_0}{\partial\mathbf{R}}$, which is the same as Eq. (8) in the wave-packet dynamics up to the first order of the gradient expansion. It is easy to check all terms in Eq. (20) are gauge independent. Therefore the Poynting vector $\mathbf{J}$ does not depend the choice of S. The equations for $\mathbf{J} = \rho\dot{\mathbf{R}}$ and $\dot{\mathbf{p}}$ determine the geometric ray self-consistently. With these two equations, we can obtain the position and momentum parameters $\mathbf{R}$,$\mathbf{p}$ at any time.

The gauge-invariant momentum and energy defined in the wave-packet dynamics can be related with $\mathbf{p}$, ω through $\mathbf{q}_c = \mathbf{p} - \mathbf{A_R}$, $\omega_c = \omega_0 + \omega_1 + \mathrm{Im}\langle f|\Theta_0|\frac{df}{dt}\rangle$. Under this transformation, we can show that the equations for $\dot{\mathbf{R}}$ and $\dot{\mathbf{p}}$ transfer to Eqs. (7) and (8) derived from the wave-packet dynamics (except the term $\partial_{\mathbf{R}}\times\rho\mathbf{m}$). Note that the term $\Omega_{\mathbf{RR}}\dot{\mathbf{R}}$ in Eq. (8) (with $\mathbf{R}$ replacing $\mathbf{r}_c$) can be rewritten as $\dot{\mathbf{R}}\times\mathcal{B}$, where $\mathcal{B} = \frac{1}{2}\epsilon_{\alpha\beta\gamma}\Omega^{\beta\gamma}_{\mathbf{RR}}$ is an *effective magnetic field* for photons.

C. Reciprocal electric field

It is well known that the term $-\Omega_{\mathbf{pp}}\dot{\mathbf{p}}$ in the equation of motion for $\dot{\mathbf{R}}$ can be rewritten as $-\dot{\mathbf{p}}\times\Omega$ with $\Omega = \frac{1}{2}\epsilon_{\alpha\beta\gamma}\Omega^{\beta\gamma}_{\mathbf{pp}}$ as the *reciprocal magnetic field* in the momentum space [4]. Interestingly, a *reciprocal electric field*may also exist in a frequency-dependent medium. In a frequency-dependent medium [18], if the wave function f is chosen in a way that has explicit ω_0 dependence, the derivative $\frac{\partial}{\partial\chi}$ (χ can be $\mathbf{R}$, $\mathbf{p}$) becomes $\frac{\partial}{\partial\chi} + \frac{\partial\omega_0}{\partial\chi}\frac{\partial}{\partial\omega_0}$ and many new terms depending on the Berry curvature in the frequency space may appear in the Poynting vector (20). For instance, the second term in Eq. (20) can be rewritten as

$$-\rho\dot{\mathbf{p}}\times[\Omega + \mathbf{v}_0\times\Upsilon], \tag{22}$$

where

$$\Upsilon = i\left\langle\frac{\partial f}{\partial\omega_0}\right|\Theta_0\left|\frac{\partial f}{\partial\mathbf{p}}\right\rangle + \mathrm{c.c.} \tag{23}$$

is the *reciprocal electric field* in the momentum-frequency space. It is interesting that the form of Υ obtained from the simple substitution here agrees with that obtained in Ref. [14] for quasiparticle dynamics in an interacting Fermi liquid based on the complicated Keldysh formalism, where the self-energy of the quasiparticles has an energy dependence.

D. Slow time modulation and dissipation of the medium

In the presence of slow time modulation of ε and μ, characterized by a slow variable T (similar as $\mathbf{R}$), additional terms $\Omega_{T\mathbf{q}}$ and $-\Omega_{T\mathbf{R}}$ appear in the equations of motion (7) [also in (20) and (8), respectively]. The term $-\Omega_{T\mathbf{R}} = \frac{\partial\mathbf{A_R}}{\partial T} - \frac{\partial A_T}{\partial\mathbf{R}}$ (A_T is the Berry connection in the T space) can be taken as an *effective electric field* for photons with $(-\mathbf{A_R}, A_T)$ serving as the gauge potential of the effective magnetic and electric fields. Note that the effective magnetic field $\mathcal{B}$ and

electric field $-\Omega_{T\mathbf{R}}$ differ from the real magnetic and electric fields ($\mathbf{E},\mathbf{H}$) which are contained in the wave functions of photons.

In previous discussion, we have neglected the dissipation of the medium for simplicity of the presentation. We note that the dissipation of the medium can be incorporated by adding an anti-Hermitian part $i\Gamma = (\Xi - \Xi^\dagger)/2$ to the operator $\Xi = \bar{\Xi} + i\Gamma$, here Γ is assumed to be a small perturbation to the Hermitian part $\bar{\Xi} = (\Xi + \Xi^\dagger)/2$. Θ_0 in previous equations is now replaced with $\Theta_0 = \partial\bar{\Xi}_0/\partial\omega$. The continuity equation (18) becomes

$$\partial_T \rho + \nabla_{\mathbf{R}} \cdot \mathbf{J} = -\rho/\tau, \tag{24}$$

where $\tau^{-1} = 2\langle f|\Gamma|f\rangle$ describes the dissipation of the medium. The right-hand side of the continuity equation (24) represents the change of the energy density and Poynting vector field due to the dissipation of the medium. For an isotropic medium (ε and μ are scalar) with $\mathbf{H} = 0$, we have

$$\tau^{-1} = 2\omega_0 \mathrm{Im}\varepsilon(\omega_0)/\ \mathrm{Re}\varepsilon(\omega_0). \tag{25}$$

We see $\tau^{-1}\mathrm{Re}n/2$ corresponds to the attenuation constant or absorption coefficient of a plane-wave propagating in an absorptive medium [19], where n is the index of refraction of the medium.

The dissipation of the medium yields an additional term to the Poynting vector field (20)

$$\rho\mathcal{F} = \rho\mathrm{Im}\left[\langle f|\tau^{-1}\Theta_0\left|\frac{\partial f}{\partial \mathbf{p}}\right\rangle - 2\langle f|\Gamma\left|\frac{\partial f}{\partial \mathbf{p}}\right\rangle\right], \tag{26}$$

which is also absent in Eq. (7) for the wave-packet dynamics. Clearly this term is gauge invariant since $\tau^{-1} = 2\langle f|\Gamma|f\rangle$. For an isotropic medium without frequency dependence, this term is zero. However, it can become nonzero in an anisotropic medium (e.g., $\varepsilon_{xx} \neq \varepsilon_{yy} \neq \varepsilon_{zz}$). $\rho\mathcal{F}$ is similar as the spin torque dipole density in spin transport theory [20], where the existence of the spin torque reflects the fact that spin is not conserved microscopically in systems with spin-orbit coupling. The term $\rho\mathcal{F}$ originates from the dissipation of the medium, which induces the inconservation of the density ρ (i.e., $\dot{\rho} \neq 0$). We thus name $\rho\mathcal{F}$ as the *dissipation dipole density*.

E. Degenerate bands

In the above discussion, we consider only a single nondegenerate energy band. The generalization of the theory to degenerate bands (e.g., left and right circular polarization states of photons) is straightforward [21]. All above formulas are still valid by replacing the local wave function f with $\sum_i \chi_\alpha f_\alpha$, where f_α is the Bloch wave function at band α and χ_α is the superposition coefficient that can be determined through

$$i\dot{\chi}_\alpha = \left[\langle f_\alpha|\mathcal{H}|f_\beta\rangle - i\langle f_\alpha|\Theta_0\left|\frac{df_\beta}{dt}\right\rangle\right]\chi_\beta. \tag{27}$$

In a uniform medium without frequency dependence, Eq. (27) can be simplified as $\dot{\chi}_\alpha = -i\dot{\mathbf{p}}\cdot\mathbf{A}_{\mathbf{p}}\chi_\beta$ on a helical basis, which is the same as that derived in Ref. [8] using wave-packet dynamics. Equations (20), (26), and (27) and the equation for $\dot{\mathbf{p}}$ are the new set of the EOM for light propagation in photonic crystals.

IV. TRANSVERSE ENERGY CURRENT FROM POAM

Finally, we show that the additional energy current $\partial_{\mathbf{R}} \times \rho\mathbf{m}$ in $\mathbf{J}$ can be nonzero and may significantly modify the photon transport in photonic crystals using a concrete example. Consider a 2D hexagonal photonic crystal with a uniform isotropic permeability tensor $\mu_0\delta_{ij}$, and an isotropic but spatially varying permittivity tensor $\varepsilon(\mathbf{r})\varepsilon_0$, with [10]

$$\begin{aligned}\varepsilon_{ii}(\mathbf{r}) &= \varepsilon[1 + \varsigma V_{\mathbf{G}}(\mathbf{r})],\\ \varepsilon_{xy}(\mathbf{r}) &= -\varepsilon_{yx}(\mathbf{r}) = -i\varepsilon[\eta_0 + \eta_1 V_{\mathbf{G}}(\mathbf{r})].\end{aligned} \tag{28}$$

Here $V_{\mathbf{G}}(\mathbf{r}) = 2\sum_{n=1}^{3}\cos(\mathbf{G}_n\cdot\mathbf{r})$, $\mathbf{G}_n$ are the three equal-length reciprocal vectors in the xy plane, rotated 120° relative to each other. The $\varepsilon_{xy}(\mathbf{r})$ are the Faraday terms which explicitly break the time-reversal symmetry and lead to nonzero Berry phase contribution to the photon transport.

We focus on the decoupled TE set $\{E_x, E_y, H_z\}$ of the electromagnetic fields. Around the corner K of the first Brillouin zone (BZ) (see the inset of Fig. 1), the photon dynamics are governed by an effective Hamiltonian [10]

$$H = \omega_0 I + c(p_x\sigma_x + p_y\sigma_y + \Delta\sigma_z)/2, \tag{29}$$

as shown in Ref. [10], where $\omega_0 = c|\mathbf{G}|(1-\varsigma/4)/\sqrt{3}$, $\Delta = |\mathbf{G}|(\frac{3}{2}\eta_1 - 3\varsigma\eta_0)/\sqrt{3}$, $\mathbf{p}$ is the deviation of the 2D Bloch vector from the BZ corner K, and c is the speed of light. From the Hamiltonian, we find

$$\Omega_z = \pm\tfrac{1}{2}\Delta(p^2+\Delta^2)^{-3/2}, \tag{30}$$

$$\mathbf{m}_z = \pm\frac{c}{4}\Delta(p^2+\Delta^2)^{-1}, \tag{31}$$

where $\pm$ correspond to two photonic bands.

We assume the parameter $\eta_1(x)$ is slowly varying along the x direction. Consider a photon frequency at the lower band, a nonzero Ω_z leads to a transverse energy current

$$\mathbf{J}_\perp = -\Omega_{\mathbf{pp}}\dot{\mathbf{p}} = -\dot{\mathbf{p}}\times\Omega = -\frac{\sqrt{3}c|G|\Delta^2}{8(p^2+\Delta^2)^2}\frac{d\eta_1}{dx}\mathbf{e}_y \tag{32}$$

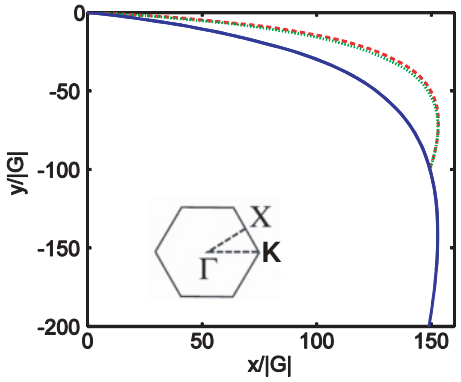

FIG. 1. (Color online) Plot of the trajectories of a photon in a 2D (xy) hexagonal photonic crystal obtained from the geometric optics approach. The length unit is $|G|^{-1} = a/2\pi$, where a is the lattice constant. $\lambda = 0.5$, $\eta_0 = 0.008$, $d\eta_1/dx = 4\times10^{-6}$ with $\eta_1(t=0) = 0$. The inital wave vector of the photon has $\mathbf{p} = (\Delta/3,0)$ from the BZ corner $\mathbf{K}$. (Solid line) Real trajectory of the photon. (Dashed line) Trajectory without $\partial_{\mathbf{R}}\times\rho\mathbf{m}$. (Dotted line) Trajectory without both $\partial_{\mathbf{R}}\times\rho\mathbf{m}$ and $\Omega_{\mathbf{pR}}\dot{\mathbf{R}}$.

along the y direction. However, the nonzero $\mathbf{m}$ also yields another transverse energy current

$$\mathbf{J}'_{\perp} = \partial_{\mathbf{R}} \times \mathbf{m} = \frac{\sqrt{3}c|G|(p^2 - \Delta^2)}{8(p^2 + \Delta^2)^2} \frac{d\eta_1}{dx} \mathbf{e}_y, \tag{33}$$

which is absent in previous literature. Note that these two currents have the same directions around $\mathbf{p} = 0$ and are comparable in magnitudes. In Fig. 1, we plot the trajectories of a photon with and without taking account of the contribution (33) from the POAM. We see the clear distinction between these two trajectories, which indicates that the transverse energy current from the POAM cannot be neglected in the transport of photons in photonic crystals. Finally we emphasize that the photon trajectories in Fig. 1 are obtained from our first-order geometric optics treatment and it would be interesting to compare them with the fully correct photon trajectories obtained from *ab initio* calculation in the future.

V. CONCLUSION

In summary, we derive a set of semiclassical EOM of particles in a chiral and dissipative periodic medium subject to slowly varying perturbations in space and time. The effects of several important properties of particles (or waves) and media that are neglected in current literature [e.g., self rotation of particles, frequency (or energy) dependence and dissipation of the media] have been included in the EOM. The EOM are derived based on a geometric optics approach for light propagation in periodic photonic crystals. We show that Berry curvatures in position, time, momentum, and frequency spaces; the dissipation dipole density of the medium; and the photon angular momentum may affect the Poynting vector field of photons that traces the geometric optics rays. Reciprocal as well as effective electromagnetic fields for photons may arise from the Berry phase effects in photonic crystals. We emphasize that although the geometric optics theory and the EOM are presented for the light propagation in photonic crystals, they can also be applied to many other systems, to name a few [11], electron dynamics in Bloch bands, x-ray propagation in natural solid crystals that can be taken as photonic crystals for x rays, light propagation in multiferroic materials, quasiparticle dynamics in a superfluid, and transport of acoustic waves.

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