

Spin-orbit coupling and perpendicular Zeeman field for fermionic cold atoms: Observation of the intrinsic anomalous Hall effect

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(Received 4 May 2010; published 27 August 2010)

We propose a scheme for generating Rashba spin-orbit coupling and a perpendicular Zeeman field *simultaneously* for cold fermionic atoms in a harmonic trap through the coupling between atoms and laser fields. The realization of Rashba spin-orbit coupling and a perpendicular Zeeman field provides opportunities for exploring many topological phenomena using cold fermionic atoms. We focus on the intrinsic anomalous Hall effect and show that it may be observed through the response of atomic density to a rotation of the harmonic trap.

DOI: [10.1103/PhysRevA.82.021607](https://doi.org/10.1103/PhysRevA.82.021607)

PACS number(s): 67.85.-d, 03.65.Vf, 03.75.Ss, 71.70.Ej

Two important ingredients for manipulating electron spin dynamics and designing spin devices in spintronics [1] are spin-orbit coupling and a Zeeman field. For instance, Rashba spin-orbit coupling (RSOC) (assumed in the x - y plane), together with a *perpendicular* Zeeman field (PZF) (along \mathbf{e}_z), yield a transverse (along \mathbf{e}_y) topological Hall current with an applied electric field (along \mathbf{e}_x). Such an intrinsic current was proposed to be one feasible explanation of the experimentally observed anomalous Hall effect (AHE) and spin Hall effect (SHE) in ferromagnetic semiconductors [2,3]. However, the scattering of electrons from impurities and defects in the solid, leading to extrinsic AHE and SHE, makes the experimental observation of intrinsic AHE and SHE very difficult [4].

Ultracold-atom gases experience an environment essentially free from impurities and defects and therefore provide an ideal platform for emulating many condensed-matter models or even observing new phenomena. One important recent effort along this line is the study of atomtronics, which, in analogy to electronics and spintronics, aims to realize devices and circuits using cold atoms [5]. One natural and important question in atomtronics deals with how to generate effective spin-orbit coupling and Zeeman fields. Great progress has been made recently on the generation of RSOC by considering the coupling between cold atoms and laser fields [6–9], which leads to a series of important applications [10–13].

However, the direction of the generated Zeeman field in these previous schemes [6,9–13] is in the spin-orbit coupling plane. Such *in-plane* Zeeman fields cannot open a band gap between different energy branches in the energy spectrum. The band gap, together with RSOC, is the physical origin of many topological phenomena. For instance, the band gap is necessary for the observation of the intrinsic AHE [2]. It is also the key ingredient of the recently broadly discussed schemes on the creation of a chiral $p_x + ip_y$ -wave superfluid or superconductor from an s -wave superfluid or superconductor [14,15] for the observation of non-Abelian statistics and topological quantum computation [16]. In contrast, a band gap can be opened in the presence of RSOC and PZF, but a scheme for generating them simultaneously for cold atoms is still absent.

In this article, we propose a scheme for creating RSOC and PZF *simultaneously* for a fermionic atomic gas in a harmonic trap. The realization of RSOC and PZF may open opportunities for the observation of many topological phenomena in cold

atoms because of the nonzero Berry phase induced by RSOC and PZF. Here we focus on one of them: observation of the intrinsic AHE. In solid-state systems, the AHE has been observed in transport experiments for electrons (i.e., measuring charge currents or voltages). Such transport experiments are not suitable for cold atoms in a harmonic trap. We find that the time of flight of cold atoms in the presence of RSOC and PZF can only yield a small asymmetry of the atomic density; therefore, it may not be suitable for observing the intrinsic AHE. Instead, we consider the response of atom density to an external rotation of the trap, which corresponds to an effective magnetic field for atoms. Because the intrinsic AHE is not a quantized effect, the Strěda formula [17] that was proposed for studying quantum Hall effects in cold atoms [18] does not apply. We find that the atomic density response to the rotation contains not only contributions from the anomalous Hall conductivity, but also a new term from the orbit magnetic moments of atoms that is absent in previous literature [19]. Both contributions originate from the topological properties of RSOC and PZF.

Consider ultracold Fermi atoms with a tripod electronic level scheme [Fig. 1(a)]. States $|1\rangle$, $|2\rangle$, and $|3\rangle$ are three hyperfine ground states, and state $|4\rangle$ is an excited state. The Fermi gas is confined in a quasi-two-dimensional (x - y plane) harmonic trap. Along the z direction, the atomic dynamics is “frozen” by a deep optical lattice, leading to a multilayered system. The ground states $|1\rangle$, $|2\rangle$, and $|3\rangle$ are coupled with state $|4\rangle$ using three lasers with corresponding Rabi frequencies Ω_{a1} , Ω_{a2} , and Ω_{a3} . In the interaction representation, the single-particle Hamiltonian is

$$H = p^2/2m + V_{\text{ext}} + H_I - \mu, \quad (1)$$

where $H_I = \hbar\Delta|4\rangle\langle 4| - \hbar(\Omega_{a1}|4\rangle\langle 1| + \Omega_{a2}|4\rangle\langle 2| + \Omega_{a3}|4\rangle\langle 3| + \text{H.c.})$ describes the laser-atom interaction, where Δ is the detuning to state $|4\rangle$. V_{ext} is the external potential that includes the harmonic trap as well as potentials created by other laser fields and μ is the chemical potential.

The Rabi frequencies Ω_{ai} can be parameterized as $\Omega_{a1} = \Omega_a \sin\theta \cos\varphi e^{iS_1}$, $\Omega_{a2} = \Omega_a \sin\theta \sin\varphi e^{iS_2}$, $\Omega_{a3} = \Omega_a \cos\theta e^{iS_3}$, and $\Omega_a = \sqrt{|\Omega_{a1}|^2 + |\Omega_{a2}|^2 + |\Omega_{a3}|^2}$. The diagonalization of H_I yields two degenerate dark states: $|D_1\rangle = \sin\varphi e^{iS_{31}}|1\rangle - \cos\varphi e^{iS_{32}}|2\rangle$ and $|D_2\rangle = \cos\theta \cos\varphi e^{iS_{31}}|1\rangle + \cos\theta \sin\varphi e^{iS_{32}}|2\rangle - \sin\theta|3\rangle$, with $S_{ij} = S_i - S_j$. We choose

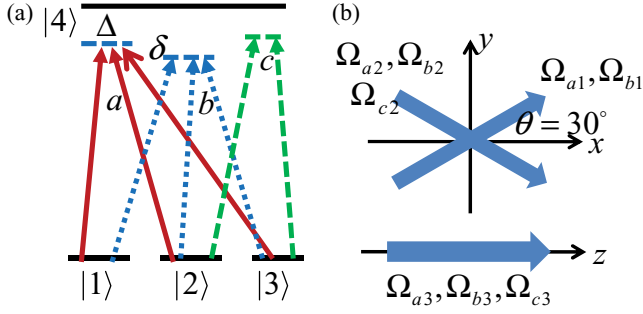


FIG. 1. (Color online) Schematic representation of the light-atom interaction for the generation of the effective Hamiltonian (4). a , b , and c are three sets of lasers. Solid line, Ω_{ai} ; dotted line, Ω_{bi} ; dashed line, Ω_{ci} . Δ is the laser detuning of laser set a ; δ is a small shift of the detuning for laser sets b and c from Δ . (b) The configuration of the laser beams. All lasers are uniform plane waves.

a laser configuration illustrated in Fig. 1(b) with the parameters $\varphi = \pi/4$, $\theta = \arctan \sqrt{2}$, $S_i = \mathbf{q}_i \cdot \mathbf{R}$, $\mathbf{q}_1 = q_0(\cos \frac{\pi}{6} \mathbf{e}_x + \sin \frac{\pi}{6} \mathbf{e}_y)$, $\mathbf{q}_2 = q_0(\cos \frac{\pi}{6} \mathbf{e}_x - \sin \frac{\pi}{6} \mathbf{e}_y)$, and $\mathbf{q}_3 = q_1 \mathbf{e}_z$, $\mathbf{R} = (x, y, z)$. The lasers Ω_{a1} and Ω_{a2} are in the x - y plane, Ω_{a3} is along the z direction, and $|\Omega_{a1}| = |\Omega_{a2}| = |\Omega_{a3}|$. Note that these three lasers are uniform plane waves, which are different from the optical lattices used in a previous scheme [13] to generate RSOC. The effective low-energy Hamiltonian is obtained by projecting the Hamiltonian (1) onto the subspace of the degenerate dark states spanned by $|D_i\rangle$,

$$H_e = \gamma k^2 - \mu - \alpha(k_x \sigma_z + k_y \sigma_x) + V, \quad (2)$$

where $\gamma = \hbar^2/2m$ and $\alpha = \gamma q_0/\sqrt{3}$, $V_{ij} = \langle D_i | V_{\text{ext}} | D_j \rangle$ is the effective external potential. The external harmonic trap $V_{\text{trap}} = V(\mathbf{r}) \sum_{i=1}^3 |i\rangle \langle i|$ is chosen to be spin-independent to avoid heating of atoms, where $V(\mathbf{r}) = m\omega_i^2 r^2/2$, $\mathbf{r} = (x, y)$ is the coordinate in the x - y plane and ω_i is the trapping frequency.

Two additional laser sets b and c with Rabi frequencies Ω_{bi} and Ω_{ci} in Fig. 1 induce two Raman transitions between different hyperfine ground states, yielding an effective coupling interaction,

$$H_Z = -\frac{\hbar}{\Delta} \begin{pmatrix} |\Omega_{b1}^2| & \Omega_{b1}^* \Omega_{b2} & \Omega_{b1}^* \Omega_{b3} \\ \Omega_{b2}^* \Omega_{b1} & F_1 & F_3 \\ \Omega_{b3}^* \Omega_{b1} & F_3^* & F_2 \end{pmatrix}, \quad (3)$$

for atoms at the hyperfine ground states $|1\rangle$, $|2\rangle$, and $|3\rangle$, where $F_1 = |\Omega_{b2}^2| + |\Omega_{c2}^2|$, $F_2 = |\Omega_{b3}^2| + |\Omega_{c3}^2|$, and $F_3 = \Omega_{b2}^* \Omega_{b3} + \Omega_{c2}^* \Omega_{c3}$. We choose the detunings Δ_b and Δ_c for the two sets (b and c) of lasers as $\Delta_b = \Delta + \delta$ and $\Delta_c = \Delta - \delta$, with $\delta \sim 2\pi \times 80 \text{ MHz} \ll \Delta$. The small shifts of the detunings do not change the effective Rabi coupling between different hyperfine ground states, but remove the interference among different sets of lasers. The optical potentials generated by the laser sets b and c are taken as external potentials. With suitably chosen Rabi frequencies ($\Omega_{b1} = i\sqrt{3}\Omega_0 e^{iS_1}$, $\Omega_{b2} = -\Omega_0 e^{iS_2}$, $\Omega_{b3} = \Omega_0 e^{iS_3}$, $\Omega_{c2} = \sqrt{2}e^{i\pi/3}\Omega_0 e^{iS_2}$, and $\Omega_{c3} = \sqrt{2}\Omega_0 e^{iS_3}$), the Hamiltonian (3) reduces to $H_Z = i h_0(|D_2\rangle \langle D_1| - |D_1\rangle \langle D_2|)$, with $h_0 = 3\hbar\Omega_0^2/\Delta$. Here Ω_0 is the magnitude of the Rabi frequency of the laser $b2$. Within the

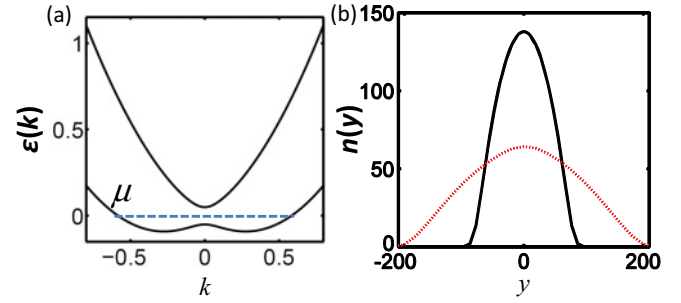


FIG. 2. (Color online) (a) The spin-orbit band structure. $h_0 = 0.05 E_R$; $\alpha^2/\gamma = 1/3 E_R$. We use the wavelength $\lambda = 767 \text{ nm}$ of the ^{40}K D_2 transition line, the wave vector $k_R = 2\pi/\lambda$, the recoil frequency $\omega_R = \hbar k_R^2/2m = 2\pi \times 8.5 \text{ KHz}$, the recoil energy $E_R = \hbar\omega_R$, as the units of length, wave vector, frequency, and energy, respectively. E_R corresponds to a temperature $T = 2 \mu\text{K}$. (b) The time-of-flight image of atoms in the presence of PSOC, PZF, and gravitational force. The unit of the density is $k_R^2 = 6.7 \times 10^9 \text{ cm}^{-2}$. $\mu = 0$, $T = 20 \text{ nK}$, $\omega_t = 2\pi \times 10 \text{ Hz}$. Solid line, $t = 0$; dotted line, $t = 19 \text{ ms}$.

dark-state basis $|D_i\rangle$, $H_Z = h_0 \sigma_y$. Under a new dark-state basis $|\chi_1\rangle = (|D_1\rangle - i|D_2\rangle)/\sqrt{2}$, $|\chi_2\rangle = (-i|D_1\rangle + |D_2\rangle)/\sqrt{2}$, which corresponds to a unitary rotation of $|D_i\rangle$, the Hamiltonian (2) becomes

$$H_e = \gamma k^2 - \mu + \alpha(k_x \sigma_y - k_y \sigma_x) + h_0 \sigma_z + V(\mathbf{r}), \quad (4)$$

where the third term is the RSOC and the fourth term is the PZF. All eight lasers used for the generation of RSOC and PZF are uniform plane waves; therefore, they do not lead to spatial periodic modulation of the atomic density. In addition, these lasers propagate only along three different directions (the same as other tripod schemes [6,10,11,13]); therefore, our scheme should be feasible in experiments.

Under the local density approximation with the local chemical potential $\mu(\mathbf{r}) = \mu - m\omega_i^2 r^2/2$, the Hamiltonian (4) has two eigenenergies $\epsilon_{k\pm} = \gamma k^2 - \mu(\mathbf{r}) \pm E_0$ with $E_0 = \sqrt{h_0^2 + \alpha^2 k^2}$. There is an energy gap $E_g = 2h_0$ opening between two spin-orbit bands at $k = 0$ [Fig. 2(a)]. The intrinsic AHE is nonzero only when the chemical potential $\mu(\mathbf{r})$ lies inside the gap.

The dynamics of cold Fermi atoms are described by the semiclassical equations of motion [20],

$$\dot{\mathbf{r}} = \partial \epsilon_{\mathbf{k}} / \partial \mathbf{k} - \dot{\mathbf{k}} \times \boldsymbol{\Gamma}_z, \quad \dot{\mathbf{k}} = \mathbf{F}/\hbar, \quad (5)$$

where $\boldsymbol{\Gamma}(\mathbf{k}) = \alpha^2 h_0 / 2(\alpha^2 k^2 + h_0^2)^{3/2} \mathbf{e}_z$ is the Berry curvature in the momentum space and the physical origin of many topological phenomena. When there is a nonzero external force \mathbf{F} along the x direction, an anomalous velocity $\mathbf{v}_{\text{AH}} = -\dot{\mathbf{k}} \times \boldsymbol{\Gamma}$ of atoms is induced along the y direction. \mathbf{v}_{AH} is the physical origin of the intrinsic AHE in electronic systems. However, it is difficult to perform a similar transport measurement commonly used for electronic systems for cold atoms in a harmonic trap. In the schemes for the observation of SHE in cold atoms [7–9], the time-of-flight image has been proposed. Because of the anomalous velocity \mathbf{v}_{AH} , we expect that the time-of-flight expansion of cold atoms with RSOC, PZF, and

the gravitational force $\mathbf{F} = m\mathbf{g}$ (along the x direction) has a transverse shift along the y direction.

We assume that the harmonic trap in the x - y plane is suddenly turned off at $t = 0$, and atoms start to expand, following the equations of motion (5). The column density along the y direction after the expansion depends on the initial atom distribution in both momentum and real spaces,

$$n(y_f, T, t) = \int d^2\mathbf{r}_i \int \frac{d^2\mathbf{k}}{(2\pi)^2} \delta[y_f - y_i - \Delta y(t)] f, \quad (6)$$

where T is the temperature and t is the time of flight. \mathbf{r}_i (y_i) is the initial position of atoms, and $f(\mathbf{k}, \mathbf{r}_i, T) = 1/(\exp\{[\varepsilon_{\mathbf{k}} - \mu(\mathbf{r}_i)]/k_B T\} + 1)$ is the Fermi-Dirac distribution of atoms. y_f is the position of atoms at time t . $\Delta y(t) = \int_{k_x}^{k_x + Ft/\hbar} dk'_x \left(\frac{1}{F} \frac{\partial \varepsilon_{\mathbf{k}}}{\partial k_y} + \Gamma_z(k'_x, k_y) \right)$ is the distance of flight. $\delta(y)$ is the δ function.

In Fig. 2(b), we plot the column density $n(y_f, T, t)$ at two different times. At $t = 0$, the initial column density is symmetric along the y axis. After a time t , $n(y_f, T, t)$ becomes asymmetric because of the anomalous velocity of atoms. However, the expansion dynamics of atoms is dominated by the first term in $\Delta y(t)$, and the differences of the atomic densities at $\pm y_f$ correspond to only a small percentage (below 3%) of the total density. Therefore, it may be hard to observe them in experiments. The difference between electrons and cold atoms comes from the fact that $\partial \varepsilon_{\mathbf{k}}/\partial k_y$ in an electronic system does not contribute to the overall transverse motion (the observed Hall current is purely from the anomalous velocity), while the initial atom velocities $\partial \varepsilon_{\mathbf{k}}/\partial k_y$ dominate the expansion process and the asymmetry of the column density should be small in an atomic system. Therefore, we need develop other techniques for the observation of the intrinsic AHE.

Recently, the response of atomic density to a rotation of the trap has been proposed for measuring quantum Hall conductivity for cold atoms based on the well-known Strěda formula [17] $\sigma = \partial n/\partial B$. Here the rotation for atoms is equivalent to the magnetic field for electrons. However, the Strěda formula does not apply to the AHE because it is not quantized. Nevertheless, the density response to the rotation still contains rich information about the intrinsic AHE. Consider a rotation of the harmonic trap along the z axis [21]; the Hamiltonian can be written as

$$H = \hbar^2 q^2/2m + \alpha(q_x \sigma_y - q_y \sigma_x) + h_0 \sigma_z + m(\omega_r^2 - \varpi^2)r^2/2 - \mu \quad (7)$$

in the rotation frame, where $\hbar\mathbf{q} = \hbar\mathbf{k} - m\varpi\hat{\mathbf{z}} \times \mathbf{r}$ is the mechanical momentum of atoms [20] and ϖ is the rotation frequency of the trap. The density of atoms is

$$n(\mathbf{r}) = (2\pi)^{-2} \int d^2\mathbf{q} [1 + m\varpi \Gamma_z/\hbar] f(\mathbf{q}, \mathbf{r}, \varpi), \quad (8)$$

where $m\varpi \Gamma_z/\hbar$ is a correction to the well-known constant density of states $1/(2\pi)^2$ in the presence of nonzero Berry curvature fields and the rotation [19]. $f(\mathbf{q}, \mathbf{r}, \varpi) = 1/(\exp\{[\varepsilon_{\mathbf{m}} - \mu(\mathbf{r})]/k_B T\} + 1)$ is the Fermi-Dirac distribution of atoms, where the energy of atoms $\varepsilon_{\mathbf{m}} = \varepsilon_{\mathbf{q}} - M_z \varpi$ contains a correction $-M_z \varpi = -m\omega \alpha^2 h_0/4\hbar(\alpha^2 q^2 + h_0^2)$, known as the magnetization energy for electrons in the solid state [20].

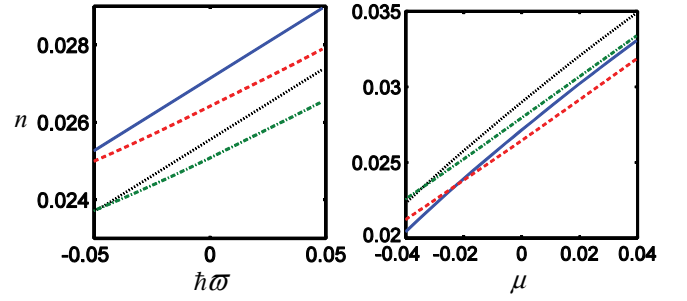


FIG. 3. (Color online) Plot of the atom density $n(\varpi, \mu, T)$ with respect to $\hbar\varpi$ and μ . The units are the same as that in Fig. 2. $h_0 = 0.05E_R$; $\alpha^2/\gamma = 1/3E_R$. Solid and dotted lines, $T = 2$ nK; dashed and dash-dotted lines, $T = 200$ nK. (a) Solid and dashed lines, $\mu = 0$; dotted and dash-dotted lines, $\mu = -0.01E_R$. (b) Solid and dashed lines, $\varpi = 0$; dotted and dash-dotted lines, $\varpi = 0.05\omega_R$.

We assume that the rotation frequency ϖ of the trap is slowly increased to keep the same temperature of the system. The local chemical potential can be fixed by increasing ω_r with ϖ . The response of the atom density to ϖ is

$$\frac{\partial n}{\partial \varpi} = \int \frac{d^2\mathbf{q}}{(2\pi)^2} \left[\frac{m}{\hbar} \Gamma_z f + \left(1 + \frac{m\varpi \Gamma_z}{\hbar} \right) \frac{\partial f}{\partial \mu} M_z \right] \quad (9)$$

for fixed μ and T . At $T = 0$ and $\varpi = 0$, Eq. (9) reduces to

$$\frac{\partial n}{\partial \varpi} = \frac{m}{\hbar} \int^\mu \frac{d^2\mathbf{q}}{(2\pi)^2} \Gamma_z + \frac{M_z}{4\pi} \frac{1}{d\varepsilon_{\mathbf{q}}/dq^2|_{\varepsilon_{\mathbf{q}}=\mu}}. \quad (10)$$

Here the first term is the anomalous Hall conductivity σ_{xy} for cold atoms. In the parameter region $|\mu(r)| < h_0$ (i.e., the chemical potential lies in the band gap), it yields $\sigma_{xy} = m[1 - h_0/\sqrt{\alpha^2 q_F^2(r) + h_0^2}]/4\pi\hbar$, where the Fermi wave vector $q_F(r)$ is obtained from $\varepsilon_{\mathbf{q}_F} = \mu(r)$. In the parameter region $\alpha^2/\gamma \gg h_0$, $q_F^2 \approx \alpha^2/\gamma^2$ and $\sigma_{xy} \approx m/4\pi\hbar$. The second term $\bar{\sigma}_{xy}$, originating from the nonzero orbit magnetic moment M_z , is an additional contribution to $\partial n/\partial \varpi$ that was missing in the previous literature [19] for electron systems. Equation (10) is a generalization of the Strěda formula for the AHEs. By varying parameters and measuring the density response, we can extract information not only about the anomalous Hall conductivity, but also the magnetic moment that is generally hard to measure in solid-state systems. In the parameter region $\alpha^2/\gamma \gg h_0$,

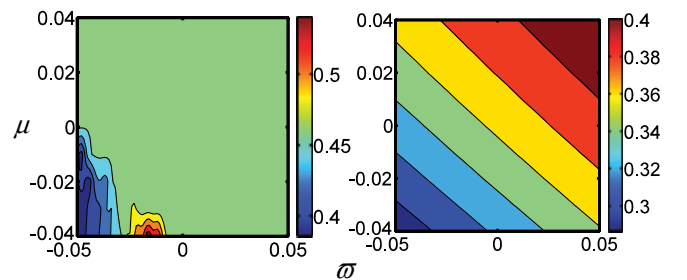


FIG. 4. (Color online) Plot of $\partial n(\varpi, \mu, T)/\partial \varpi$ with respect to ϖ and μ . The unit of $\partial n(\varpi, \mu, T)/\partial \varpi$ is taken to be $m/2\pi\hbar$. The other units are the same as that in Fig. 2. $h_0 = 0.05E_R$; $\alpha^2/\gamma = E_R/3$. (a) $T = 2$ nK; (b) $T = 200$ nK.

$\bar{\sigma}_{xy} \approx \gamma m h_0 / 8\pi \hbar \alpha^2 \ll m / 4\pi \hbar \approx \sigma_{xy}$. Therefore, σ_{xy} dominates in Eq. (9) in this region, and the density response $\partial n / \partial \varpi$ yields a rough measurement for the anomalous Hall conductivity.

We numerically calculate the density n and density response $\partial n / \partial \varpi$ as functions of the parameters (μ, ϖ, T) , and plot them in Figs. 3 and 4 for $T = 200$ and 2 nK. The presence of the harmonic trap changes the chemical potential at r by $-m(\omega_i^2 - \varpi^2)r^2/2$. In a realistic experiment, the effective trapping frequency $\sqrt{\omega_i^2 - \varpi^2}$ in the presence of rotation may be slightly different from the initial trapping frequency ω_{i0} without rotation to keep the same temperature of the system [22]. This can be overcome by comparing the densities at different spatial points r, r^* such that $\mu - \frac{1}{2}m(\omega_i^2 - \varpi^2)r^{*2} = \mu - \frac{1}{2}m\omega_{i0}^2 r^2$ to keep the same local chemical potential [22]. With this method, we can measure the density response to the rotation with the fixed temperature and chemical potential. In addition, the rotation of the system requires an asymmetric harmonic trap [21], which does not affect our results because it only changes the spatial positions for the measurement of the density change at a fixed chemical potential through a different local chemical potential dependence $\mu(\mathbf{r}) = \mu - m[(\omega_{ix}^2 - \varpi^2)x^2 + (\omega_{iy}^2 - \varpi^2)y^2]/2$. We adopt a set of parameters: $\alpha^2/\gamma = E_R/3 = 2\pi\hbar \times 2.8$ KHz, $h_0 =$

$2\pi\hbar \times 425$ Hz, $\omega_{i0} = 2\pi \times 50$ Hz, $\varpi_{\max} = 0.05\omega_R = 2\pi \times 425$ Hz, $\omega_i \approx \sqrt{\omega_{i0}^2 + \varpi^2}$, and $\omega_i - \varpi = \omega_{i0}^2/(\omega_i + \varpi) \approx 2\pi \times 3$ Hz. At $r = 0.1\lambda\omega_R/\pi\omega_{i0} = 4.2$ μm , the chemical potential μ changes by $0.01E_R$. From Figs. 3 and 4, we see that a maximum density change of the order of 3×10^7 cm^{-2} can be observed with a rotation frequency ϖ_{\max} , which corresponds to about 10% of the total density and can be observed in a realistic experiment. The density variation at a medium temperature $T = 200$ nK is of the same order as that at a low temperature $T = 2$ nK. Note that the multilayer structure induced by the optical lattice confinement along the z direction can further enhance the signal.

In summary, we propose a scheme for creating RSOC and PZF simultaneously for cold-atom gases. We show that, by measuring the atomic density response to a rotation of the trap, the intrinsic AHE can be observed for cold fermionic atoms in a harmonic trap. We emphasize that the creation of RSOC and PZF brings new opportunities for studying many topological phenomena, such as chiral p -wave superfluids, anomalous and spin Hall insulators, etc.

We thank Di Xiao and Qian Niu for helpful discussion. This work is supported by the ARO (W911NF-09-1-0248) and DARPA-YFA (N66001-10-1-4025).

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