

Berry Phase Effects on the Dynamics of Quasiparticles in a Superfluid with a Vortex

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We study quasiparticle dynamics in a Bose-Einstein condensate with a vortex by following the center of mass motion of a Bogoliubov wave packet, and find important Berry-phase effects due to the background flow. We show that the Berry phase invalidates the usual canonical relation between the mechanical momentum and position variables, leading to important modifications of quasiparticle statistics and thermodynamic properties of the condensates. Applying these results to a vortex in an infinite uniform superfluid, we find that the total transverse force acting on the vortex is proportional to the superfluid density. We propose an experimental setup to directly observe Berry phase effects through measuring local thermal atoms' momentum distribution around a vortex.

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Quasiparticles in a superfluid play an essential role in its thermodynamic as well as dynamic properties [1]. Owing to the realization of Bose-Einstein condensates (BEC) with ultracold atoms, much detailed investigation of the quasiparticles becomes possible [2,3]. In the past several years, extensive experimental and theoretical work has been devoted to the study of quasiparticle dynamics in static BECs [3–6], while quasiparticles in a flowing condensate have not received much attention [7].

In this Letter, we study the quasiparticle dynamics in a BEC superfluid with a vortex, a typical flowing condensate. We adopt a semiclassical approach by following the center of mass motion of a Bogoliubov wave packet, finding important Berry phase effects [8] due to the background flow. Unlike the case with a static BEC, the quasiparticle position and momentum are no longer canonical variables, and there are also important modifications to the quasiparticle statistics and thermodynamic properties of the condensates. We use these results to calculate the momentum circulation around a vortex in an infinite superfluid and find the total transverse force acting on the vortex is proportional to the superfluid density [9], which rules out the existence of the Iordanskii force that is supposed to originate in the asymmetric scattering of quasiparticles by the vortex [10–12]. Finally, we propose an experimental setup to directly observe Berry phase effects through measuring local thermal atoms' momentum distribution around a vortex.

Wave packet dynamics.—For simplicity, we present the theory here for a single vortex, although it is applicable to a general flowing condensate such as vortex lattices. The dynamics of the condensate are described by the Gross-Pitaevskii equation [13]

$$i \frac{\partial \psi_0}{\partial t} = \left(\frac{\mathbf{p}^2}{2} + g |\psi_0|^2 + V(\mathbf{r}) - \boldsymbol{\Omega} \cdot (\mathbf{r} \times \mathbf{p}) \right) \psi_0, \quad (1)$$

where, for simpler notation, we have used units such that \hbar and the atomic mass m are both unity, g is the interatom

coupling constant, and $V(\mathbf{r})$ is the trapping potential. We have also included an angular momentum term to take into account the possibility of using a rotating reference frame, with $\boldsymbol{\Omega}$ being its angular frequency. The condensate wave function may be written as $\psi_0 = \sqrt{n(\mathbf{r})} e^{i\alpha(\mathbf{r})} e^{-i\mu t}$, where μ is the chemical potential, $n(\mathbf{r})$ and $\alpha(\mathbf{r})$ are the vortex density and phase, respectively. The gradient of phase yields local velocity of the vortex $\mathbf{v}_s = \nabla \alpha(\mathbf{r})$.

A small deviation $\delta\psi e^{-i\mu t}$ (elementary excitations) from the condensate wave function ψ_0 is described by the Bogoliubov equation [13]

$$i\sigma_z \frac{\partial}{\partial t} |\Phi\rangle = \mathcal{Q} |\Phi\rangle, \quad \mathcal{Q} = \begin{pmatrix} H_+ & H_2 e^{2i\alpha(\mathbf{r})} \\ H_2 e^{-2i\alpha(\mathbf{r})} & H_- \end{pmatrix}. \quad (2)$$

The wave function Φ has two components, u and v , which are related to $\delta\psi$ through $\delta\psi = u e^{-i\omega t} + v^* e^{i\omega t}$, where ω is the quasiparticle energy. The entries of the matrix operator are given by $H_{\pm} = \mathbf{p}^2/2 + 2gn(\mathbf{r}) + V(\mathbf{r}) - \mu \mp \boldsymbol{\Omega} \cdot (\mathbf{r} \times \mathbf{p})$, and $H_2(\mathbf{r}) = gn(\mathbf{r})$. Because of the Pauli matrix σ_z on the left-hand side, the Bogoliubov equation is non-Hermitian.

We now consider a quasiparticle wave packet centered at \mathbf{r}_c with its spread small compared to the length scale of the slowly varying potentials [including trap potential $V(\mathbf{r})$, condensate wave function ψ_0 , and terms related to $\boldsymbol{\Omega}$]. The dynamics of the wave packet is approximately governed by the local Bogoliubov operator $\mathcal{Q}_c \equiv \mathcal{Q}(\mathbf{p}, \mathbf{r} = \mathbf{r}_c)$ plus its linear gradient correction. The local Bogoliubov operator has plane wave eigenstates $\mathcal{Q}_c e^{i\mathbf{q} \cdot \mathbf{r}} |\phi(\mathbf{q}, \mathbf{r}_c)\rangle = \omega_c \sigma_z e^{i\mathbf{q} \cdot \mathbf{r}} |\phi(\mathbf{q}, \mathbf{r}_c)\rangle$, where \mathbf{q} is the wave vector. The amplitude satisfies a simple 2×2 matrix equation, which can be solved easily to yield

$$\omega_c(\mathbf{q}, \mathbf{r}_c) = [H_1^2(\mathbf{q}) - H_2^2]^{1/2} - \mathbf{q} \cdot (\boldsymbol{\Omega} \times \mathbf{r}_c) \quad (3)$$

for the local quasiparticle energy, where $H_1(\mathbf{q}) = \mathbf{q}^2/2 + 2gn(\mathbf{r}_c) + V(\mathbf{r}_c) - \mu$, and

$$|\phi(\mathbf{q}, \mathbf{r}_c)\rangle = \frac{1}{2} \begin{pmatrix} \zeta + \zeta^{-1} \\ (\zeta - \zeta^{-1})e^{-2i\alpha(\mathbf{r}_c)} \end{pmatrix}$$

for the two-component amplitude of the local eigenstate, where $\zeta = (\frac{H_1 - H_2}{H_1 + H_2})^{1/4}$. We note that the wave vector \mathbf{q} and the wave packet center \mathbf{r}_c enter the local quasiparticle energy and wave function parametrically. The wave function is normalized in the sense that $\langle \phi(\mathbf{q}, \mathbf{r}_c) | \sigma_z | \phi(\mathbf{q}, \mathbf{r}_c) \rangle = 1$. We have also chosen the phase of the wave function such that it is smooth and single valued in the parameters $(\mathbf{q}, \mathbf{r}_c)$. We shall see that the parametric dependence of the eigenstates on the center position of the wave packet will manifest as Berry phase terms in the equations of motion.

We now turn our attention to the wave packet itself, which is to be constructed out of these eigenstates as

$$|\Phi\rangle = \int d^3q a(\mathbf{q}, t) e^{i\mathbf{q}\cdot\mathbf{r}} |\phi(\mathbf{q}, \mathbf{r}_c)\rangle, \quad (4)$$

where the superposition amplitude $a(\mathbf{q}, t)$ may be taken as a Gaussian in \mathbf{q} . The normalization is taken to be $\langle \Phi | \sigma_z | \Phi \rangle = \int d^3q |a(\mathbf{q}, t)|^2 = 1$. We assume that the Gaussian is centered at $\mathbf{q}_c = \int d^3q |a(\mathbf{q}, t)|^2 \mathbf{q}$ and has a width narrow compared to momentum scales of the energy dispersion and of the eigenstates. Microscopic calculation shows that $\mathbf{q}_c = \langle \Phi | \sigma_z \mathbf{p} | \Phi \rangle = \int \delta\psi^* \mathbf{p} \delta\psi d\mathbf{r}$, implying that it represents the mechanical momentum of the quasiparticle. To be self consistent, the wave packet must yield the preassigned center position $\mathbf{r}_c = \langle \Phi | \sigma_z \mathbf{r} | \Phi \rangle$.

The dynamics of the quasiparticle can be derived from a time-dependent variational principle for the Bogoliubov equation, where the action (defined as time integral of the Lagrangian) is extremized with respect to the quasiparticle wave function. Here the Lagrangian is given by

$$L = -\langle \Phi | \mathcal{Q} | \Phi \rangle + \left\langle \Phi \left| i\sigma_z \frac{d}{dt} \right| \Phi \right\rangle, \quad (5)$$

and the wave packet (4) is chosen as the variational wave function with time-dependent parameters \mathbf{r}_c and \mathbf{q}_c . We use d/dt to mean the derivative with respect to the time dependence of the wave function explicitly or implicitly through \mathbf{r}_c and \mathbf{q}_c . Under the previously discussed conditions that the wave packet is narrow both in position and momentum spaces, the Lagrangian (5) can be evaluated as a function of variational parameters \mathbf{r}_c and \mathbf{q}_c , and their time derivatives, independent of the width and shape of the wave packet in position or momentum.

Quasiparticle energy and the Berry phase.—The first term in Lagrangian (5) corresponds to the total energy of the quasiparticle wave packet, which is found to be

$$\omega = [H_1^2(\mathbf{q}_c) - H_2^2]^{1/2} - \mathbf{q}_c \cdot (\boldsymbol{\Omega} \times \mathbf{r}_c) + (1 - \rho^2) \mathbf{q}_c \cdot \mathbf{v}_s, \quad (6)$$

where $\mathbf{v}_s = \nabla\alpha(\mathbf{r}_c)$ is the local velocity of the vortex. The first two terms are the local quasiparticle energy Eq. (3) with momentum \mathbf{q}_c , stemming from the expectation value

$\langle \Phi | \mathcal{Q}_c | \Phi \rangle$ of the local Bogoliubov operator. The last term is the correction of the wave packet energy originating from the linear gradient expansion of the Bogoliubov operator $\Delta\mathcal{Q} = \frac{1}{2}((\mathbf{r} - \mathbf{r}_c) \cdot \frac{\partial\mathcal{Q}}{\partial\mathbf{r}_c} + \text{c.c.})$, and $\rho = \langle \Phi | \Phi \rangle$ is the total atomic mass contained in the quasiparticle wave packet.

To understand the quasiparticle energy expression, it will be instructive to consider the simpler situation of a uniformly flowing superfluid of velocity \mathbf{v}_s . If \mathbf{p}_0 is the quasiparticle momentum in the reference frame where the superfluid is static, then we have the relationship $\mathbf{q}_c = \mathbf{p}_0 + \rho\mathbf{v}_s$. In terms of \mathbf{p}_0 , the quasiparticle energy (6) can be expressed (in the limit of small \mathbf{v}_s) as $\omega = \varepsilon(\mathbf{p}_0) + \mathbf{p}_0 \cdot \mathbf{v}_s$, where $\varepsilon(\mathbf{p}_0) = [H_1^2(\mathbf{p}_0) - H_2^2]^{1/2} = [p_0^2(p_0^2/4 + gn)]^{1/2}$ is the energy dispersion in a static superfluid. This is just the standard Landau formula obtained using Galilean transformation [1].

The evaluation of the second term of the Lagrangian (5) is similar to that in Ref. [14], which yields

$$L = -\omega + (\mathbf{q}_c + \mathbf{A}) \cdot \dot{\mathbf{r}}_c. \quad (7)$$

Here the term $\mathbf{q}_c \cdot \dot{\mathbf{r}}_c$ comes from the time dependence of the superposition amplitude $a(\mathbf{q}, t)$ in the wave packet (4), while the vector potential $\mathbf{A} = i\langle \phi | \sigma_z | \partial\phi / \partial\mathbf{r}_c \rangle = -(\rho - 1)\nabla\alpha(\mathbf{r}_c)$ is obtained from the position dependence of the two-component local eigenvector, which is similar to the case of a spin in a position-dependent Zeeman field [8]. The vector potential, also called Berry connection, arises for a vortex because of the nonzero local velocity. Its line integral over a path \mathcal{C} gives a Berry phase $\Gamma(\mathcal{C}) = \oint_{\mathcal{C}} d\mathbf{r}_c \cdot \mathbf{A} = -\oint_{\mathcal{C}} (\rho - 1) d\alpha(\mathbf{r}_c)$ of the eigenvector [15]. For instance, the accumulated Berry phase for a quasiparticle moving around a vortex in an infinite uniform superfluid is $-2\pi(\rho - 1)$, where ρ represents the total atom number (notice that the atom mass m has been taken as unity) in the quasiparticle wave packet. Apart from a nonessential 2π , it means that the phase acquired by moving each atom around a vortex is 2π , which agrees with Feynman's argument [16].

We note that the vector potential vanishes at very high momenta, where $\rho \rightarrow 1$. Likewise, the energy correction due to superfluid flow [the last term of Eq. (6)] vanishes in this limit. This is quite reasonable, because the quasiparticle becomes a free particle and is decoupled from the condensate at high momenta. The condensate has influence on the quasiparticle only at low and intermediate momenta.

Semiclassical dynamics.—Following the standard procedure of analytical mechanics, we may introduce the canonical momentum $\mathbf{k}_c = \partial L / \partial \dot{\mathbf{r}}_c = \mathbf{q}_c + \mathbf{A}$ conjugate to the coordinate vector \mathbf{r}_c . We see that the vector potential makes the canonical and mechanical momenta different. The energy $\omega = \mathbf{k}_c \cdot \dot{\mathbf{r}}_c - L$ becomes the Hamiltonian and the equations of motion are

$$\dot{\mathbf{r}}_c = \frac{\partial\omega}{\partial\mathbf{k}_c}, \quad \dot{\mathbf{k}}_c = -\frac{\partial\omega}{\partial\mathbf{r}_c}, \quad (8)$$

where the quasiparticle energy ω is now recasted as a function of \mathbf{k}_c , $\omega = [H_1^2(\mathbf{k}_c) - H_2^2]^{1/2} - (\mathbf{k}_c - \mathbf{A}) \cdot (\boldsymbol{\Omega} \times \mathbf{r}_c) + \mathbf{k}_c \cdot \mathbf{A}$. We see that the vector potential \mathbf{A} modifies the energy expression in this canonical formulation.

In terms of the physical quantity of mechanical momentum \mathbf{q}_c , the quasiparticle equations of motion are dramatically altered:

$$\begin{aligned} \dot{\mathbf{r}}_c &= \frac{\partial \omega}{\partial \mathbf{q}_c} + \frac{\partial \rho}{\partial \mathbf{q}_c} (\dot{\mathbf{r}}_c \cdot \mathbf{v}_s), \\ \dot{\mathbf{q}}_c &= -\frac{\partial \omega}{\partial \mathbf{r}_c} - \dot{\mathbf{r}}_c \times \left(\frac{\partial \rho}{\partial \mathbf{r}_c} \times \mathbf{v}_s \right) + \left(\dot{\mathbf{q}}_c \cdot \frac{\partial \rho}{\partial \mathbf{q}_c} \right) \mathbf{v}_s. \end{aligned} \quad (9)$$

They follow directly from (8) by the transformation $\mathbf{k}_c = \mathbf{q}_c + (1 - \rho) \mathbf{v}_s$. Here ω is the original energy expression (6). We see that additional terms depending on superfluid velocity and total atom mass in the quasiparticle wave packet appear in both equations, therefore the equations of motion are no longer of canonical form. This is a result very different from the case of static superfluid studied in [5].

In the statistical treatment of quasiparticles, an important semiclassical quantity is the density of states of phase space. For the canonical variables \mathbf{k}_c and \mathbf{r}_c , this should be taken as the constant $1/(2\pi)^3$. Because of the vector potential, the position \mathbf{r}_c and mechanical momentum \mathbf{q}_c are no longer canonical variables, which leads to a modification of the density of states [17]

$$D(\mathbf{r}_c, \mathbf{q}_c) = \frac{1}{(2\pi)^3} \det \left(\mathbf{I} - \frac{\partial \rho}{\partial \mathbf{q}_c} \mathbf{v}_s \right), \quad (10)$$

where \mathbf{I} is the 3×3 unit matrix.

The thermodynamics properties of condensates are determined by the physical quantities of quasiparticles such as energy and density of states, therefore they are strongly affected by the Berry phase. For instance, the thermal depletion of the condensate density is given by

$$\delta n(\mathbf{r}_c, T) = \int d^3 \mathbf{q}_c D(\mathbf{r}_c, \mathbf{q}_c) \rho / (e^{\omega/k_B T} - 1) \quad (11)$$

according to our theory, which is different from that for a static condensate because of the modifications of energy ω and density of state $D(\mathbf{r}_c, \mathbf{q}_c)$. Notice that the integration of this distribution over position space yields the total number of thermal atoms.

Transverse force on a vortex.—The general form of the transverse force per unit length acting on a moving vortex with velocity \mathbf{v}_L in an infinite uniform superfluid can be written as $\mathbf{F} = \kappa(\mathcal{C}) \hat{\mathbf{z}} \times \mathbf{v}_L$, where $\kappa(\mathcal{C})$ is the momentum circulation along a path far away from the core of the vortex, and we assume the normal fluid velocity is zero [12]. In the past several decades, there has been some controversy about the expression of $\kappa(\mathcal{C})$, which was argued to be either $2\pi\hbar n_{\text{tot}}/m$ or $2\pi\hbar n_s/m$ through different approaches [9–11]. Here $2\pi\hbar/m$ is the quantum of circulation, n_{tot} is the total mass density, and n_s is the superfluid density.

According to our theory, the total momentum circulation $\kappa(\mathcal{C})$ contains two contributions: $2\pi\hbar[n_{\text{tot}} - \delta n(\mathbf{r}_c, T)]/m$ from the condensate and $\kappa_1(\mathcal{C}) = \oint_{\mathcal{C}} \mathbf{W} \cdot d\mathbf{r}_c$ from quasiparticles. \mathbf{W} is the momentum density of quasiparticles under equilibrium Bose-Einstein distribution and is obtained by summing over the mechanical momenta from all states,

$$\begin{aligned} \mathbf{W} &= \int d^3 \mathbf{q}_c D(\mathbf{r}_c, \mathbf{q}_c) \frac{\mathbf{q}_c}{e^{\omega(\mathbf{r}_c, \mathbf{q}_c)/k_B T} - 1} \\ &= \int \frac{d^3 \mathbf{k}_c}{(2\pi)^3} \frac{\mathbf{k}_c + (\rho - 1) \nabla \theta_c}{e^{\omega(\mathbf{r}_c, \mathbf{k}_c)/k_B T} - 1}, \end{aligned} \quad (12)$$

where T is the temperature, k_B is the Boltzman constant, $D(\mathbf{r}_c, \mathbf{q}_c) = \frac{1}{(2\pi)^3} [1 - \frac{1-\rho^2}{\omega_c r_c^2} (\mathbf{r}_c \times \mathbf{q}_c)_z]$, and $1/(2\pi)^3$ are the density of states for the noncanonical momentum \mathbf{q}_c and canonical momentum \mathbf{k}_c , respectively.

Since we choose the circular contour \mathcal{C} to be far away from the core of the vortex, we can expand the Bose-Einstein distribution $f(\mathbf{k}_c) \approx f_0(\mathbf{k}_c) - \Delta f$ with $f_0(\mathbf{k}_c) = \frac{1}{e^{\omega_c/k_B T} - 1}$, and $\Delta f = (1 - \rho) \mathbf{k}_c \cdot \nabla \theta_c e^{\omega_c/k_B T} / [k_B T (e^{\omega_c/k_B T} - 1)^2]$. Considering a closed orbit with fixed radius R and taking the limit of $R \rightarrow \infty$, we find the thermal depletion of the condensate $\delta n(\mathbf{r}_c, T) = \frac{1}{2\pi^2} \times \int_0^\infty f_0(\mathbf{k}_c) \rho k_c^2 dk_c$, and quasiparticle momentum circulation

$$\begin{aligned} \kappa_1(\mathcal{C}) &= \frac{1}{\pi} \int_0^\infty f_0(\mathbf{k}_c) \rho k_c^2 dk_c - \frac{1}{3\pi k_B T} \\ &\quad \times \int_0^\infty \frac{e^{\omega_c/k_B T} k_c^4 dk_c}{(e^{\omega_c/k_B T} - 1)^2}. \end{aligned} \quad (13)$$

The first term of $\kappa_1(\mathcal{C})$ stems from the difference between the canonical momentum \mathbf{k}_c and mechanical momentum \mathbf{q}_c , and cancels with $-2\pi\hbar\delta n(\mathbf{r}_c, T)/m$. The second term of $\kappa_1(\mathcal{C})$ comes from the deviation Δf in the distribution function, and is found to be $2\pi\hbar\rho_n/m$, where $\rho_n = 2\pi^2 k_B^4 T^4 / 45\hbar^3 s^5$ is the normal fluid density. Summing up the contributions from both condensate and quasiparticles, we find the total momentum circulation $\kappa(\mathcal{C}) = 2\pi\hbar(n_{\text{tot}} - \rho_n)/m = 2\pi\hbar n_s/m$, and the transverse force per unit length $\mathbf{F} = 2\pi\hbar n_s \hat{\mathbf{z}} \times \mathbf{v}_L/m$. This form of transverse force agrees with that obtained by Thouless *et al.* using general properties of superfluid order [9], and rules out the existence of the Iordanskii force that is proportional to the normal fluid density ρ_n , and is supposed to originate in the asymmetric scattering of quasiparticles by the vortex [10–12].

Experimental observation.—The above discussions show that the modifications of mechanical momentum, energy dispersion, and density of states of quasiparticles due to the Berry phase affect the thermodynamics of atoms. Our prediction of a transverse force proportional T^4 can in principle be measured in experiments. In the following, we propose an experiment for a direct observation of the local

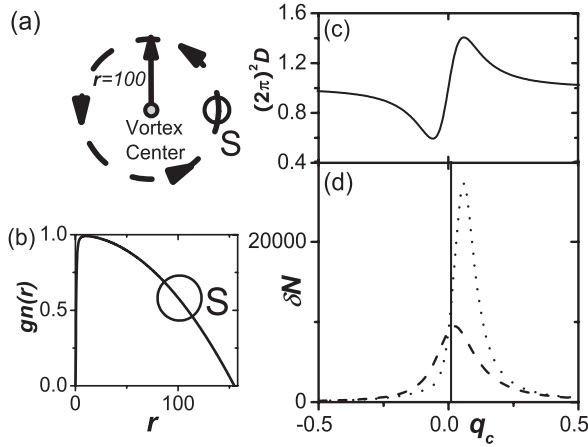


FIG. 1. Thermal-atom momentum distribution around a trapped vortex. Length scales are in units of $l = 0.73 \mu\text{m}$ for a Rb^{87} BEC in a trap specified in the text. (a) A schematic plot of experimental geometry. Measurement of thermal-atom momentum distribution is to be made in a region S at 100 unit lengths away from the vortex center with a radius of 20 unit lengths. This region contains about 9740 condensate and 1650 thermal atoms. (b) Condensate atom density (multiplied by the interaction strength g) in the presence of a trapped vortex. (c) Density of states for quasiparticles in the measurement region S as a function of quasiparticle momentum q_c along the circulation flow. (d) Momentum distribution of thermal atoms in the measurement region S . Dotted and dashed lines correspond to our Eq. (14) and the naive expression $\int_S d\mathbf{r}_c \rho / [\exp(\frac{\omega_c + \mathbf{q}_c \cdot \nabla \theta}{k_B T}) - 1]$, respectively. For reference, the momentum of condensate atoms is indicated by the vertical solid line.

thermal-atom momentum distribution around a vortex, in which the effect of the Berry phase can be seen clearly.

Figure 1(a) is a schematic plot of experimental geometry. The length unit is $l = \sqrt{\frac{\hbar}{m(\mu + \hbar\Omega)}}$, chosen for convenience. It corresponds to $l \approx 0.73 \mu\text{m}$ for a BEC of $N = 5 \times 10^5 \text{Rb}^{87}$ atoms in a quasi-two-dimensional magnetic trap [18] with axial and radial trapping frequencies $\omega_z = 2\pi \times 800 \text{ Hz}$, $\omega_r = 2\pi \times 2 \text{ Hz}$, in which a single vortex is created at a rotation frequency of $\Omega = 0.4\omega_r$ [19]. The condensate density profile is shown in Fig. 1(b), where one can see that the observation region S is chosen relatively far away from the vortex core, which was to make sure that the semiclassical approximation remains valid. We will consider a temperature of $T = 2\mu/k_B \approx 21.1 \text{ nK}$ which is about a third of the BEC transition temperature $T_c \approx 52.8 \text{ nK}$ for this system.

The atoms are supposed to be in a hyperfine state ($|1\rangle$). By focusing two copropagating Raman beams at region S one may drive both the thermal and condensate atoms there to another hyperfine state ($|2\rangle$). The remaining atoms in state $|1\rangle$ are to be rapidly expelled from the trap by applying radio-frequency radiation that flips them to an anti-trapped hyperfine state. Finally, the time of flight measurement [20] of atoms in state $|2\rangle$ can then determine their momentum distribution, which, according to our

theory, is given by

$$\delta n(\mathbf{p}_c, T) = \int_S d^2\mathbf{r}_c D(\mathbf{r}_c, \mathbf{q}_c) \rho / (e^{\omega/k_B T} - 1). \quad (14)$$

In Fig. 1(c), we see that there are more quasiparticles moving along the circulating superfluid flow than in the opposite direction and the deviation from the naive value of the density of states $1/(2\pi)^2$ is substantial. In Fig. 1(d), we find that the momentum distribution is quite different from the commonly believed one that does not take into consideration the Berry phase effects.

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