

One-dimensional topological chains with Majorana fermions in two-dimensional nontopological optical lattices

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The recent experimental realization of one-dimensional (1D) equal Rashba-Dresselhaus spin-orbit coupling (ERD-SOC) for cold atoms provides a disorder-free and highly controllable platform for the implementation and observation of Majorana fermions (MFs), analogous to the broadly studied solid-state nanowire-superconductor heterostructures. However, the corresponding 1D chains of cold atoms possess strong quantum fluctuation, which may destroy the superfluids and MFs. In this paper, we show that such 1D topological chains with MFs may be on demand generated in a two- or three-dimensional nontopological optical lattice with 1D ERD-SOC by modifying local potentials on target locations using experimentally already implemented atomic gas microscopes or patterned (e.g., double- or triple-well) optical lattices. All ingredients in our scheme have been experimentally realized, and the combination of them may pave the way for the experimental observation of MFs in a clean system.

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I. INTRODUCTION

Majorana fermions (MFs) [1] obey non-Abelian exchange statistics and are crucial for realizing fault-tolerant topological quantum computation [2–4]. Following initial theoretical proposals [5–10], some possible signatures of MFs have been observed recently in experiments [11–17] using one-dimensional (1D) nanowires or ferromagnetic atomic chains on top of an s -wave superconductor and with strong spin-orbit coupling (SOC). However, these signatures are not conclusive because of disorder and other complications in solid state [18–24]. In this context, the recent experimental realization of SOC [25–31] in ultracold atomic gases provides a disorder-free and highly controllable platform for observing MFs. In experiments, 1D equal Rashba-Dresselhaus SOC (ERD-SOC) and a tunable Zeeman field have been achieved, which, together with the s -wave superfluidity, makes it possible to observe MFs [32–40] in 1D atomic tubes or chains, similar to the nanowire systems.

However, unlike solid-state nanowire systems where s -wave superconducting pairs are induced from proximity effects, the superfluid pairing in the 1D atomic chain is self-generated from the s -wave contact interaction, leading to the strong quantum fluctuation that renders the long-range superfluid order impossible in the thermodynamic limit. To circumvent this obstacle, quasi-1D systems with multiple weakly coupled uniform chains [41–49] have been studied in both the solid state and cold atoms, where transverse tunneling was found to lift the zero-energy degeneracy of multiple MFs.

In this paper, we consider a truly two-dimensional (2D) nontopological fermionic optical lattice with the experimentally realized 1D ERD-SOC. We raise the question of whether single or multiple topological 1D chains supporting MFs can be on demand generated at target locations in such nontopological 2D systems. Generally, a 1D chain in a 2D lattice can be locally modified to satisfy the topological condition for MFs using

the recently experimentally realized single-site addressing (the atomic gas microscopes) [50–56] or patterned (e.g., double- or triple-well) optical lattices [57–59]. However, the atom chain is strongly coupled with neighboring chains through transverse tunneling in the 2D system; therefore a naive expectation is that the coupling may destroy the local topological properties and the associated MFs.

Here we show that 1D topological chains with MFs can indeed be generated on demand from 2D nontopological fermionic optical lattices with the experimentally achieved 1D ERD-SOC. Local addressing lasers in atomic gas microscopes can modify the effective local chemical potentials along single or multiple 1D chains, leading to a topological phase transition to generate discrete topological chains that are characterized by nonzero winding numbers and host MFs at chain ends. Multiple MFs in spatially separated multiple topological chains still couple, with the coupling-induced zero-energy splitting exponentially decaying with the distance of neighboring topological chains. We emphasize that these 1D topological chains are embedded in the true 2D background where the tunnelings along both the x and y directions are the same. This is different from previously studied quasi-1D systems where multiple 1D chains are weakly coupled along the transverse direction, which usually destroy MFs. Note that similar results apply also to three dimensions if 1D chains can be locally addressed in a three-dimensional (3D) optical lattice. In the weak transverse tunneling region (quasi-1D), the MF coupling is extremely small for two topological chains separated by one or two nontopological chains, making it possible to observe multiple MFs in 2D or 3D double- or triple-well optical lattices without requiring the single-site addressing.

II. MODEL HAMILTONIAN

We consider a spin-1/2 ultracold degenerate fermionic gas (spin \uparrow and \downarrow) in a 2D square lattice with lattice size $N = n_x \times n_y$. As shown in the schematic picture in Fig. 1, two Raman lasers couple two spin states to induce 1D ERD-SOC along

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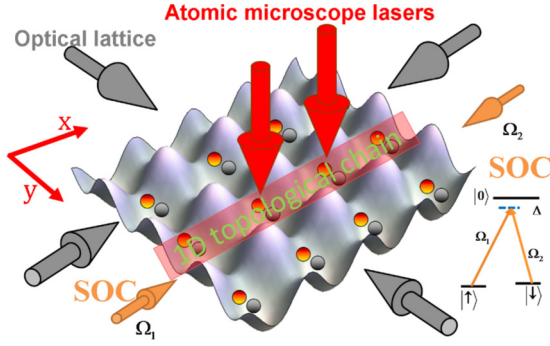


FIG. 1. Illustration of the proposed experimental setup. Gray arrows represent 2D square optical lattice lasers. The red tube demonstrates the 1D potential chain induced by local addressing lasers in atomic microscopes (red arrows). Two counterpropagating Raman lasers (orange arrows) couple two spin states, generating 1D ERD-SOC [25–31].

the x axis. The far-detuned local addressing lasers [50–56] can modify the local potential of the optical lattice along a 1D chain at target locations along the x direction. Multiple spatially well separated 1D chains can also be generated using additional local addressing lasers. In the 2D lattice, the tight-binding mean-field Bogoliubov–de Gennes (BdG) Hamiltonian is

$$H_{BdG} = H_L + H_{SOC} + H_D + H_\Delta, \quad (1)$$

where $H_L = -\sum_{i,\sigma,\eta} t_\eta (C_{i,\sigma}^\dagger C_{i+\eta,\sigma} + \text{H.c.}) - \sum_{i,\sigma} \bar{\mu} C_{i,\sigma}^\dagger C_{i,\sigma}$ is the bare Hamiltonian in the 2D lattice with $\eta = \{x, y\}$. The fermionic operator $C_{i,\sigma}^\dagger$ ($C_{i,\sigma}$) creates (annihilates) a particle with spin σ at site $\mathbf{i} = (i_x, i_y)$. We use $\bar{\mu} = \mu - 2t_x - 2t_y$ for the effective chemical potential to match that in the continuous model. t_x and t_y are the nearest-neighbor tunnelings along the x and transverse y directions, respectively. $H_{SOC} = \alpha \sum_i (C_{i,\uparrow}^\dagger C_{i-\hat{x},\downarrow} - C_{i,\uparrow}^\dagger C_{i+\hat{x},\downarrow} + \text{H.c.}) + h_z \sum_i (C_{i,\uparrow}^\dagger C_{i,\uparrow} - C_{i,\downarrow}^\dagger C_{i,\downarrow})$ describes the experimentally available 1D ERD-SOC, with the SOC strength α and the Zeeman field h_z . $H_D = \sum_{i,\sigma} V_T(i_y) C_{i,\sigma}^\dagger C_{i,\sigma}$ represents the 1D potential dip with the local potential $V_T(i_y)$, which is generated by local addressing lasers and is nonzero only at the i_y chain. $H_\Delta = -\sum_i \Delta_i (C_{i,\uparrow}^\dagger C_{i,\downarrow} + \text{H.c.})$ is the mean-field pairing Hamiltonian, with the order parameter $\Delta_i = -g \langle C_{i,\downarrow} C_{i,\uparrow} \rangle$ and the on-site interaction strength g . Hereafter, we take $t_x = t$ as the energy unit. We solve the corresponding BdG equation self-consistently with the pairing gap equation and fixed chemical potential, following the standard numerical procedure [60–64]. To find the ground state, the order parameter has no constraints and could be complex. We use the box boundary condition for the self-consistent calculation. In practical experiments, there is a weak harmonic confinement that may alter the locations of MFs but does not change the essential physics [64].

The above BdG Hamiltonian preserves particle-hole symmetry $\Xi H_{BdG} \Xi^{-1} = -H_{BdG}$, where $\Xi = \tau_x \mathcal{K}$, $\tau_x = \tilde{\tau}_x \otimes \tilde{\sigma}_0 \otimes \tilde{\rho}_0$, $\tilde{\tau}_i$ and $\tilde{\sigma}_i$ are 2×2 Pauli matrices acting on particle-hole and spin spaces, respectively, $\tilde{\rho}_0$ is an $N \times N$ identity matrix on the lattice site space, and \mathcal{K} is the complex-conjugate operator. If the order parameter Δ_i is real, the

Hamiltonian is also real, which preserves a time-reversal-like symmetry $\Theta H_{BdG} \Theta^{-1} = H_{BdG}$, with $\Theta = \mathcal{K}$, as well as a chiral symmetry $S H_{BdG} S^{-1} = -H_{BdG}$, with $S = \Theta \Xi = \tau_x$. In this case, the system belongs to the BDI topological class characterized by a \mathbb{Z} topological invariant [65,66].

III. ONE TOPOLOGICAL CHAIN

Since the superfluid order is not stable in strictly 1D systems due to the strong quantum fluctuations [67], the topological analysis fails to work for exact 1D cold-atom systems with interactions. This motivates us to extend previous calculations for 1D or quasi-1D systems [41–49] to truly higher dimensions. To better understand the topological criteria of our 2D system, we first consider a 2D lattice with no tunneling along the y axis ($t_y = 0$). Thus the 2D lattice is composed of individual x -direction 1D chains. At the central chain we add an extra constant potential $V_T(y_c) = V$, so that the central chain becomes topological, while other parts of the system are still in the nontopological region. Here the topological region is defined locally by the criteria $h_z \geq \sqrt{(\mu - V_T)^2 + \Delta^2}$, the same as the usual 1D topological chains [7,10]. In our numerical calculations, we take $n_x = 81$, $n_y = 9$, and $y_c = 5$. In the central topological chain, two MFs should exist at two ends. When t_y is increased from zero to t , the topological chain couples with neighboring nontopological chains, and the system changes from 1D to quasi-1D and finally to 2D. A natural question is whether MFs at the center chain will survive with the strong coupling.

Figure 2 demonstrates the existence of MFs even in a truly 2D region with $t_y = t$. The amplitude of the superfluid order parameter Δ_i is plotted in Fig. 2(a). We find that Δ_i is homogeneous along the x axis in the self-consistent calculation, except at the boundary. Furthermore, Δ_i has a constant phase across the whole 2D system; therefore we can choose it to be real without loss of generality for the discussion of the topological properties. Figure 2(b) shows

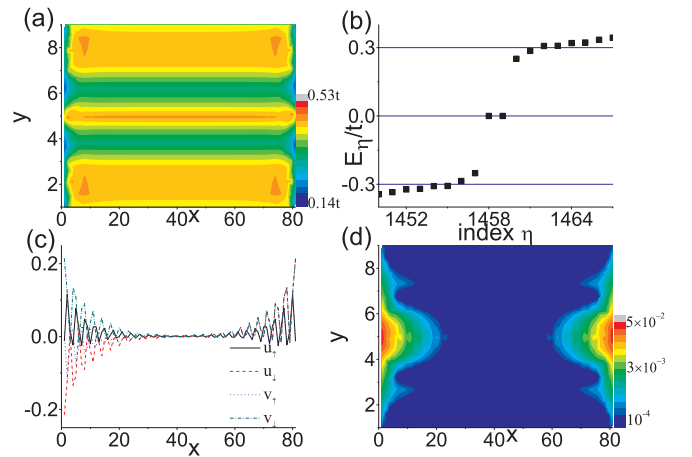


FIG. 2. MFs in a single 1D topological chain in the 2D optical lattice. (a) The amplitude of the order parameter $|\Delta_i|$. (b) The quasiparticle energy spectrum. (c) The zero-energy-mode wave function along the central chain. (d) The zero-energy-mode density (log scale). Parameters: $t_y = t$, $\alpha = 2t$, $g = -5.5t$, $h_z = 1.4t$, $V = -1.45t$, $\mu = -1.555t$.

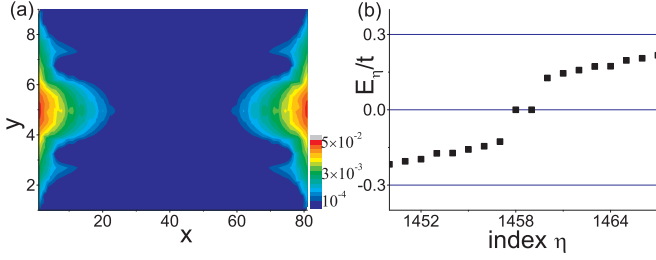


FIG. 3. (a) The zero-energy-mode density (log scale). (b) The quasiparticle energy spectrum. Here $h_y = 0.2t$; the other parameters are the same as in Fig. 2.

the quasiparticle energy spectrum, where we clearly see the existence of Majorana zero-energy modes with a tiny energy splitting $E \approx \pm 2 \times 10^{-5}t$ mainly due to the finite-size effect. The minigap energy, defined as the energy difference between the zero-energy mode and the next lowest quasiparticle state, is comparable to the amplitude of the order parameter Δ_i . Figure 2(c) shows the wave function of the zero-energy mode ($E \approx +2 \times 10^{-5}t$ state) along the central chain, which satisfies the relation for MFs: $u_\sigma = \lambda v_\sigma$, $\lambda = \pm 1$, indicating the central chain is still topological with two MFs at its ends. Figure 2(d) shows the density of the zero-energy mode, which is a square of the zero-energy-mode wave function for both spin-up and spin-down atoms in the 2D plane. The zero-energy mode still localizes at the ends of the central chain but slightly spreads to neighboring chains due to the transverse tunneling. Note that in practical experiments, there exists a finite detuning for the Raman coupling between two bare spin states, which corresponds to an in-plane Zeeman field $h_y \sigma_y$ in our notation. Such a nonzero in-plane Zeeman field is known to break the inversion symmetry and lead to the FF type of ground states with finite momentum pairing. We confirm that our results still hold for the FF state. The self-consistent BdG results are present in Fig. 3, which shows the existence of MFs in a single 1D topological chain in the 2D optical lattice with an additional in-plane Zeeman field h_y .

IV. TOPOLOGICAL CHARACTERIZATION

The emergence of MFs at the edges of the central chain originates from the bulk topological properties of the 2D optical lattice with the imprinted 1D topological chain. In the above self-consistent BdG calculations, both the order parameter and the atom density are almost homogeneous along the x axis; therefore it would be a good approximation to assume that the bulk is uniform in the x direction. With a periodic boundary condition along the x axis, the momentum k_x is a good quantum number. The 2D lattices can be taken as a series of individual 1D chains coupled through transverse tunneling, with the effective BdG Hamiltonian

$$H_{BdG}(k_x) = H_0(k_x)\rho_0 + (V\tau_z\sigma_0 + \Delta'\tau_y\sigma_y)\rho' - t_y\tau_z\sigma_0\rho_x, \quad (2)$$

where $H_0(k_x) = [-2t_x \cos k_x - \bar{\mu}]\tau_z\sigma_0 + 2\alpha \sin k_x \tau_z\sigma_y + h_z\tau_z\sigma_z + \Delta_0\tau_y\sigma_y$ describes the original uniform individual chains with the SOC and the Zeeman field. ρ spans the y -axis chain space, with ρ_0 being the identity matrix. $(\rho')_{ij} = 1$

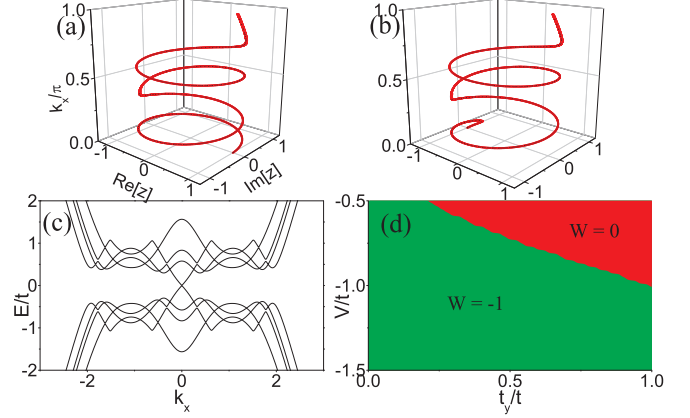


FIG. 4. Complex value z as a function of k_x in the region $(0, \pi)$ with (a) $V = -0.8t$ (nontopological) and (b) $V = -1.2t$ (topological). The other parameters are $t_y = t$, $\alpha = 2t$, $\Delta_0 = 0.4t$, $h_z = 1.4t$, $\Delta' = 0.1t$, $\mu = -1.555t$. (c) The band structure at the topological phase transition point $V = -t$. (d) The winding number as a function of V and t_y .

for $i = j = y_c$ and 0 otherwise ($i, j = 1 \dots n_y$). The ρ' part describes the potential and the order-parameter differences of the central chain from other chains. The ρ_x term describes the y -axis hopping between nearest-neighbor chains, with $(\rho_x)_{i,j} = 1$ for $|i - j| = 1$ and 0 for others.

The topological properties of the BdG Hamiltonian (2) can be characterized by the winding number W . For a single 1D chain in the 2D optical lattice, the BdG Hamiltonian is in the BDI topological class with a chiral symmetry $SH_{BdG}(k_x)S^{-1} = -H_{BdG}(k_x)$, where $S = \tau_x$. Therefore the BdG Hamiltonian can be transformed to be off-diagonal in the τ space,

$$UH_{BdG}U^+ = \begin{bmatrix} 0 & A(k_x) \\ A^T(-k_x) & 0 \end{bmatrix} = h(k_x)\tau_x + d\tau_y, \quad (3)$$

by a unitary transformation $U = e^{-i(\pi/4)\tau_y}$, where $A(k_x) = h(k_x) - id$, $h(k_x) = \{[-2t_x \cos k_x - \bar{\mu}]\sigma_0 + 2\alpha \sin k_x \sigma_y + h_z\sigma_z\}\rho_0 - t_y\sigma_0\rho_x + V\sigma_0\rho' + \Delta'\sigma_y\rho'$, and $d = \Delta_0\sigma_y\rho_0 + \Delta'\sigma_y\rho'$. The winding number is

$$W = -\frac{i}{\pi} \int_{k_x=0}^{\pi} \frac{dz}{z(k_x)}, \quad (4)$$

where $z(k_x) = \det A(k_x)/|\det A(k_x)|$ [68].

Figure 4 shows the change of the topological properties with the potential V along the central chain. When V is small, the whole lattice, including the center chain, is in the nontopological region. The corresponding complex value of z rotates when k_x changes from 0 to $k_x = \pi$, as shown in Fig. 4(a), indicating $W = 0$. When the potential depth $|V|$ increases beyond a threshold value $V_c \approx -t$, W changes to -1 . Across V_c , the band gap closes [Fig. 4(c)] and reopens, indicating a topological phase transition to a phase where the central chain becomes topological and hosts a pair of Majorana fermions, agreeing with the self-consistent calculation. This topological phase transition also relies on dimensionality, which is controlled by the transverse hopping strength. Figure 4(d) illustrates the winding number as a function of the potential depth V and transverse hopping strength t_y .

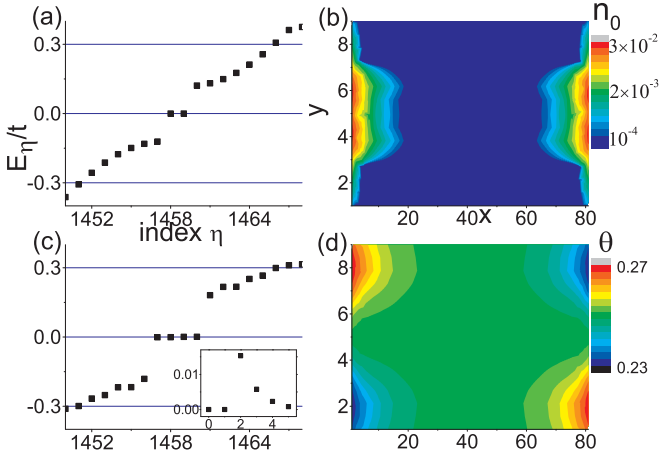


FIG. 5. MFs in two topological chains (a) and (b) at $y = 4$ and $y = 6$ or (c) and (d) at $y = 2$ and $y = 8$. (a) The quasiparticle energy spectrum for two topological chains separated by one nontopological chain. (b) The zero-energy-mode density (log scale). (c) The quasiparticle energy spectrum for two topological chains separated by five nontopological chains. The inset plots the lowest positive quasiparticle energy as a function of the number of nontopological chains in between. (d) The phase of the order parameter Δ_i . Here $t_y = t$; the other parameters are the same as in Fig. 2.

The larger transverse hopping is, the deeper the central chain potential needed to move into the topological region with a finite winding number is.

V. MULTIPLE TOPOLOGICAL CHAINS

Multiple topological chains may be generated using additional local addressing lasers to obtain multiple MFs. We first consider two topological chains separated by one nontopological chain, with the extra potential V_T added at $y = 4$ and $y = 6$. The energy spectrum from the self-consistent BdG calculation is plotted in Fig. 5(a), showing one zero-energy mode, although there are two topological chains. This is due to the strong coupling between the two chains, leading to the winding number $W = -1$, instead of -2 . Therefore there is only one pair of MFs when two chains are close. Figure 5(b) shows the density of the zero-energy mode, which widely spreads along the y axis from $y = 4$ to $y = 6$.

When two topological chains are further separated by more than one nontopological chain, the analysis of the bulk topological properties based on the constant order-parameter phase shows that $W = -2$, indicating two pairs of MFs. The effect of adding more nontopological chains between topological ones on the winding number is similar to decreasing the transverse hopping strength. Both situations give winding number $W = -2$ and have multiple degenerate MFs within the assumption of a constant value of the order-parameter phase. However, in the self-consistent calculation, the order-parameter phase is no longer uniform due to the interaction between two MFs at the same end, leading to the splitting of the zero-energy states. In Fig. 5(c), we plot the quasiparticle energy spectrum for two topological chains located at $y = 2$ and $y = 8$ obtained from the self-consistent calculation. Figure 5(d) shows the phase $\theta(x, y)$ of the order parameter $\Delta_i = |\Delta|e^{i\theta(x, y)}$, which has an

antisymmetric structure between two topological chains. The phase difference between two ends of one topological chain is opposite to the one on the other topological chain.

In principle there should be no zero-energy modes left due to the interaction between MFs, which splits the energy away from zero to a finite value [44,45]. In practice, due to the large distance between two topological chains, the coupling strength between two MFs on different topological chains is extremely small, and the energy splitting becomes negligible. For instance, the five-chain separation in Fig. 5(c) leads to an energy splitting $E \approx 8 \times 10^{-4}t$. The inset in Fig. 5(c) shows the change of the splitting with the number of nontopological chains between two topological ones. When the number is 0 or 1, $W = -1$, and the splitting is almost zero since there is only one MF at each end. When the number is 2 or more, the winding number becomes -2 , and the interaction of two MFs induces a splitting. The energy splitting decreases exponentially with the distance between two topological chains and approaches almost zero for the five-lattice separation. The above discussion illustrates that the MF coupling issue in weakly coupled topological chains [43–45] is well resolved by separating the topological chains with nontopological ones. Therefore, with experimentally already realized single-site addressing techniques, we can look for MFs in a truly 2D system, although the underlying spin-orbit coupling is only 1D.

VI. MULTIPLE MFS IN SUPERLATTICES

The interaction between MFs in topological chains can also be significantly reduced by decreasing the tunneling along the transverse direction, which makes the system quasi-1D,

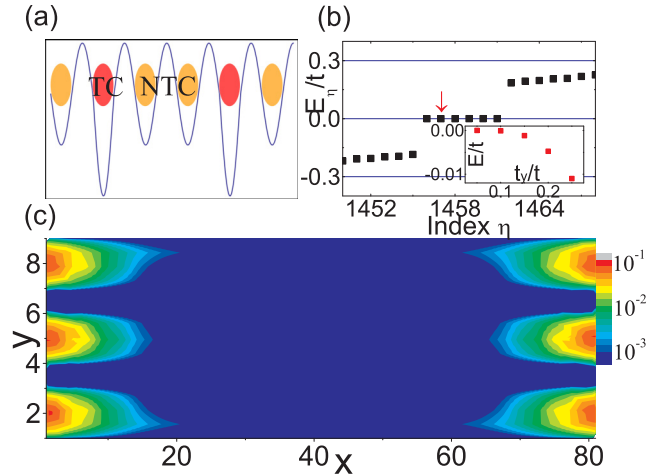


FIG. 6. MFs in quasi-1D triple-well superlattices. (a) Illustration of the triple-well superlattices along the y axis. Topological chains (TC) are at $y = 2, y = 5$, and $y = 8$, separated by nontopological chains (NTC). (b) The quasiparticle energy spectrum showing three pairs of zero-energy modes. The inset shows the energy of the second MF mode (pointed out by the red arrow) as a function of transverse hopping strength t_y . (c) The zero-energy-mode density (log scale). Other parameters are $t_y = 0.1t$, $\alpha = 0.75t$, $g = -3.5t$, $h_z = 0.7t$, $V = -0.55t$, $\mu = -0.55t$.

instead of 2D. In this case, no large separation between neighboring topological chains is needed, making it possible to generate multiple MFs using patterned optical superlattices along the y axis. In experiments, optical superlattices such as double-well or triple-well lattices can be generated using the superposition of different lattice beams [57–59], which are much easier than the single-site addressing. Figure 6(a) shows a triple-well optical lattice along the y axis with one of the triple wells in the topological region (i.e., two neighboring topological chains separated by two nontopological chains). With a small transverse tunneling $t_y = 0.1t$, the energy splitting for the MF zero-energy state is as tiny as $E \approx 5 \times 10^{-5}t$, as shown in Fig. 6. In our calculation, we put three topological chains at $y = 2$, $y = 5$, and $y = 8$ and find one pair of MFs formed at each topological chain end. However, if the transverse hopping strength t_y is big, these three topological chains will couple strongly with each other. The inset in Fig. 6(b) shows the energy splitting of the second pair of MFs with increasing t_y . In the large transverse hopping case ($t_y > 0.1t$), the system leaves only one pair of MFs [45]. We also confirm that similar physics occurs if another triple-well superlattice is applied

along the z axis to form a 3D lattice with weak tunneling along both the y and z directions.

VII. CONCLUSION

We showed that, with the assistance of atomic gas microscopes or patterned optical superlattices, 2D nontopological optical lattices with experimentally achieved 1D ERD-SOC can host noncoupled 1D topological chains with MFs. We emphasize that although we illustrated the idea using a 2D geometry, the same physics also applies to 3D nontopological optical lattices, provided selected 1D chains can be locally modified or patterned optical superlattices are used. All ingredients in our proposed schemes are available in current experiments, and the scheme may lead to an unambiguous experimental signature of the long-sought MFs in a clean cold atomic system.

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