

In-Plane Zeeman-Field-Induced Majorana Corner and Hinge Modes in an s -Wave Superconductor Heterostructure

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Second-order topological superconductors host Majorana corner and hinge modes in contrast to conventional edge and surface modes in two and three dimensions. However, the realization of such second-order corner modes usually demands unconventional superconducting pairing or complicated junctions or layered structures. Here we show that Majorana corner modes could be realized using a 2D quantum spin Hall insulator in proximity contact with an s -wave superconductor and subject to an in-plane Zeeman field. Beyond a critical value, the in-plane Zeeman field induces opposite effective Dirac masses between adjacent boundaries, leading to one Majorana mode at each corner. A similar paradigm also applies to 3D topological insulators with the emergence of Majorana hinge states. Avoiding complex superconductor pairing and material structure, our scheme provides an experimentally realistic platform for implementing Majorana corner and hinge states.

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Introduction.—Majorana zero energy modes in topological superconductors and superfluids [1–4] have attracted great interest in the past two decades because of their non-Abelian exchange statistics and potential applications in topological quantum computation [5,6]. A range of physical platforms [7–18] in both solid state and ultracold atomic systems have been proposed to host Majorana modes. In particular, remarkably experimental progress has been made recently to observe Majorana zero energy modes in s -wave superconductors in proximity contact with materials with strong spin-orbit coupling, such as semiconductor thin films and nanowires, topological insulators, etc., [19–24]. In such topological superconductors and superfluids, Majorana zero energy modes usually localize at 2D vortex cores or 1D edges, where the Dirac mass in the low-energy Hamiltonian changes sign.

Recently, a new class of topological superconductors, dubbed as higher-order topological superconductors, has been proposed [25–38]. In contrast to conventional topological superconductors, r th-order ($r \geq 2$) topological superconductors in d dimensions host $(d - r)$ -dimensional Majorana bound states, rather than $d - 1$ -dimensional gapless Majorana excitations. For example, in 2D second-order topological superconductors, the edge modes manifest themselves as 0D Majorana excitations localized at the corners, instead of 1D edges, giving rise to Majorana corner modes (MCMs). A variety of schemes have been proposed recently to implement MCMs, such as p -wave superconductors under magnetic field [26], 2D topological insulators in proximity to high temperature superconductors (d -wave or s_{\pm} -wave pairing) [29–31], π -junction Rashba layers [34] in contact with s -wave superconductors.

However, those schemes demand either unconventional superconducting pairings or complicated junction or lattice structures, which are difficult to implement with current experimental technologies.

In this Letter, we propose that MCMs can be realized with a simple and experimentally already realized heterostructure [39–42] composing of an s -wave superconductor in proximity contact with a quantum spin Hall insulator (QSHI) and subject to an in-plane Zeeman field, as sketched in Fig. 1. Here we consider a simple square lattice. At each edge of the 2D QSHI, there are two helical edge states with opposite spins and momenta, which thus support a proximity-induced s -wave superconducting pairing, resulting in a quasiparticle band gap for the helical edge mode spectrum [39–42].

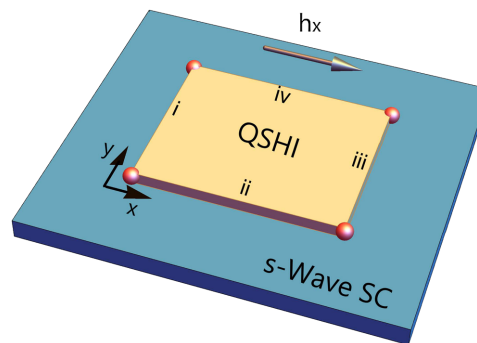


FIG. 1. Illustration of a heterostructure composing of a quantum spin Hall insulator on top of an s -wave superconductor and subject to an in-plane Zeeman field. The spheres at four corners represent four Majorana zero energy modes.

Because of different spin-orbit coupling at adjacent edges, an in-plane Zeeman field induces quite different effects on adjacent edges. Across a critical Zeeman field, the quasiparticle band gap along one edge first closes then reopens, indicating a topological phase transition, but remains unaffected for adjacent edges. Before the phase transition, the Dirac mass term in the low-energy effective Hamiltonian for the helical edge states has the same sign for two adjacent edges. After the topological phase transition, the Dirac mass term reverses its sign at the corner connecting two edges, resulting in MCMs. In contrast, the corner Dirac mass sign change in previous schemes originates from the sign change of the pairing order through unconventional superconducting pairing. There is only one MCM at each corner due to the time-reversal symmetry breaking, instead of Majorana Kramers pairs [29,30].

Applying similar physics to three dimensions, we find that second-order topological superconductor can be implemented in a 3D strong topological insulator, where the interplay between s -wave pairing and Zeeman field (not necessarily in-plane) gives rise to four domain walls on the edges between two neighboring surfaces, yielding Majorana hinge modes.

Physical system and model Hamiltonian.—Consider a QSHI in proximity contact with an s -wave superconductor and subject to a Zeeman field \mathbf{h} (see Fig. 1). The four edges of a square sample are labeled by i, ii, iii, iv. The physics of the heterostructure can be described by an effective Hamiltonian [29]

$$H(\mathbf{k}) = 2\lambda_x \sin k_x \sigma_x s_z \tau_z + 2\lambda_y \sin k_y \sigma_y \tau_z + (\xi_{\mathbf{k}} \sigma_z - \mu) \tau_z + \Delta_0 \tau_x + \mathbf{h} \cdot \mathbf{s}, \quad (1)$$

under the basis $\hat{C}_{\mathbf{k}} = (c_{\mathbf{k}}, -is_y c_{-\mathbf{k}}^\dagger)^T$ with $c_{\mathbf{k}} = (c_{\mathbf{k},a,\uparrow}, c_{\mathbf{k},b,\uparrow}, c_{\mathbf{k},a,\downarrow}, c_{\mathbf{k},b,\downarrow})^T$. Here λ_i is the spin-orbit coupling strength, Δ_0 denotes s -wave superconducting order parameter induced by proximity effect, $\xi_{\mathbf{k}} = \epsilon_0 - 2t_x \cos k_x - 2t_y \cos k_y$ with $2\epsilon_0$ being the crystal-field splitting energy and t_i the hopping strength on the square lattice, and μ is the chemical potential. Three Pauli matrices σ , s and τ act on orbital (a , b), spin (\uparrow , \downarrow) and particle-hole degrees of freedom, respectively. For simplicity of the presentation, we focus on the $\mu = 0$ case, where simple analytic results for edge modes can be obtained.

In the absence of superconducting pairing and Zeeman field, the Hamiltonian (1) is invariant under the time-reversal $\mathcal{T} = is_y K$ and space-inversion $\mathcal{I} = \sigma_z$ operations, where K is the complex-conjugation operator. Here the band topology can be characterized by a Z_2 topological index protected by \mathcal{T} symmetry or an equivalent Z index for the spin Chern number [43]. The system is a QSHI in the band inverted region $[\epsilon_0^2 - (2t_x + 2t_y)^2][\epsilon_0^2 - (2t_x - 2t_y)^2] < 0$. With the open boundary condition, there are two helical edge states

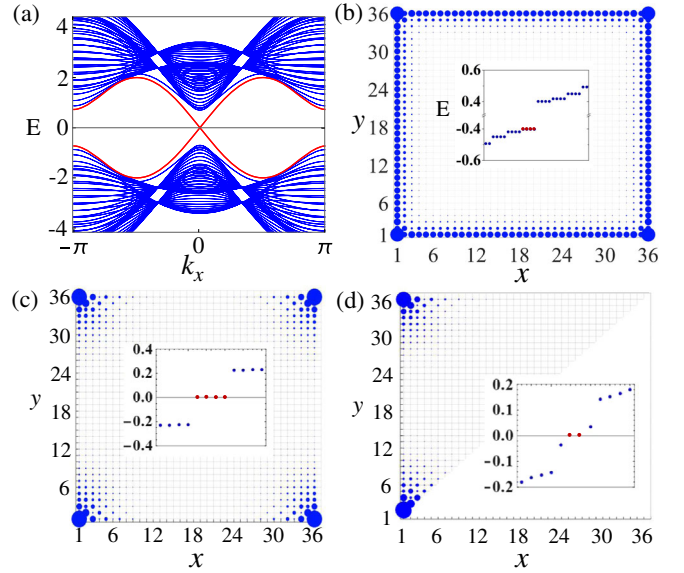


FIG. 2. (a) Quasiparticle bands with edge spectrum (red lines) for open boundary conditions along the y direction. The gap for edge spectrum closes at $h_x = \Delta_0 = 0.4$. (b) Density distributions of the edge bound states (red dots in the inset) for a trivial superconductor, where we have chosen $h_x = 0.0$, $\Delta_0 = 0.4$. (c)–(d) Density distributions of MCMs in different geometries. The radii of the blue disks are proportional to local density. The insets show the energy levels for $h_x = 0.8$ and $\Delta_0 = 0.4$. In both Fig. 2 and Fig. 3, $t_x = t_y = \lambda_x = \lambda_y = 1.0$ and $\epsilon_0 = 1.0$.

with opposite spins and momenta propagating along each edge in the QSHI phase [1,2].

Topological phase diagram and MCMs.—In the presence of Δ_0 , a finite quasiparticle energy gap is opened in the edge spectrum for two helical edge states due to the s -wave pairing. The in-plane Zeeman field h_x has different effects on the single particle edge spectra (i.e., $\Delta_0 = 0$) along the x and y directions: it can (cannot) open the gap along the k_x (k_y) direction [44]. Such anisotropic effect of h_x leads to very different physics when $\Delta_0 \neq 0$. Along the k_x direction, the quasiparticle band gap first closes [Fig. 2(a)] at the critical point $h_{xc} = \Delta_0$ and then reopens with increasing h_x , indicating a topological phase transition. While along the k_y direction, the quasiparticle band gap does not close [44]. The difference between the edge spectra drives the heterostructure to a second-order topological superconductor.

The emergence of MCMs after the topological phase transition is confirmed by the numerical simulation of corresponding lattice tight-binding model in real space, as shown in Figs. 2(b), 2(c). Before the topological phase transition [Fig. 2(b)], there are no zero energy bounded states localized at edges. After the in-plane Zeeman field exceeds the critical point h_{xc} , four zero energy MCMs emerge at each corner of the square sample [Fig. 2(c)]. The emergence of MCMs is independent of the underlying geometry of the sample. For example, in Fig. 2(d), a similar result is observed in an equilateral right triangular sample.

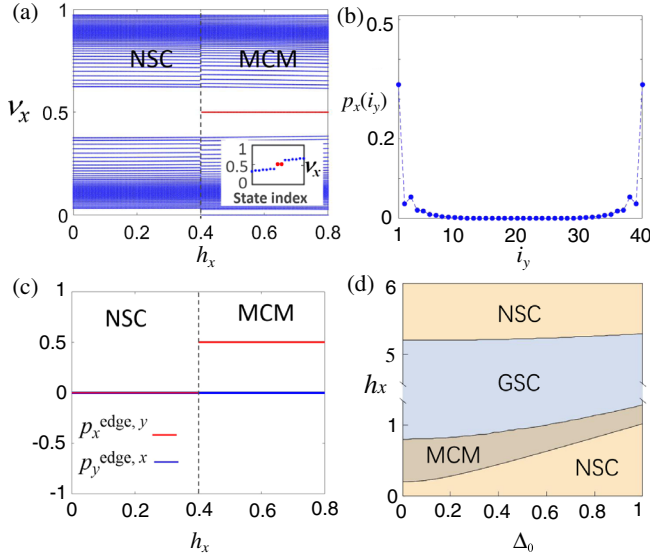


FIG. 3. (a) Wannier spectra ν_x versus h_x . The inset showcases Wannier centers for different state indexes with $h_x = 0.8$. (b) Majorana polarization distribution $p_x(i_y)$ versus lattice index i_y . (c) Majorana edge polarizations $p_x^{\text{edge},y}$ and $p_y^{\text{edge},x}$ along y -normal and x -normal edges, respectively. In (a)–(c), $\Delta_0 = 0.4$ and $\mu = 0.0$ are used. (d) Phase diagram with $\mu = 0.2$. NSC denotes a trivial superconductor, and GSC denotes a gapless superconductor. MCM and NSC phases are separated by the phase boundary $h_x = \sqrt{\mu^2 + \Delta_0^2}$.

In this case, there are only two MCMs at two left corners due to the orientation of the hypotenuse edge that leads to different effects of the in-plane Zeeman field.

To examine the topological characterization of MCMs, we further calculate the Majorana edge polarizations $p_x^{\text{edge},y}$ and $p_y^{\text{edge},x}$ using the Wilson loops on a cylindrical geometry [45,46]. Majorana edge polarization at the y -normal edge is defined by $p_x^{\text{edge},y} = \sum_{i_y=1}^{N_y/2} p_x(i_y)$, where N_y is the number of unit cells along y , and the polarization distribution is $p_x(i_y) = (1/N_x) \sum_{j,k_x,\beta,n} |[u_{k_x}^n]^{i_y,\beta} [\nu_{k_x}^j]^n|^2 \nu_x^j$. Here, $[\nu_{k_x}^j]^n$ represents the n th component of the j th eigenvector corresponding to the Wannier center ν_x^j of the Wannier Hamiltonian $\mathcal{H}_{\mathcal{W}_x} = -i \ln \mathcal{W}_x$ with \mathcal{W}_x the Wilson loop operator [44]. $[u_{k_x}^n]^{i_y,\beta}$ is the (i_y, β) -th component of occupied state $|u_{k_x}^n\rangle$ with i_y and β being the site index and the internal degrees of freedom, respectively. Similarly, we can define Majorana edge polarization $p_y^{\text{edge},x}$. In the MCM phase, only the Wannier spectra ν_x contain two half-quantized Wannier values, as shown in Fig. 3(a), implying that the edge polarizations occur only along the y -normal edges but not the x -normal edges. Such an observation has been numerically verified by distributions of localized edge polarization along y [see Fig. 3(b)], and zero edge polarization distributions along x . This further leads to half quantization of $p_x^{\text{edge},y}$ and vanishing $p_y^{\text{edge},x}$, as demonstrated in Fig. 3(c).

We remark that the above topological characterizations show that the MCM phase in our system falls into the class of extrinsic higher-order topological phases distinguished by gap closings of the edge spectra [45] on a cylindrical geometry, instead of bulk spectra on a torus geometry for intrinsic higher-order phases. However, the MCMs cannot be annihilated by perturbations without closing the edge energy gap [44].

Low-energy theory on edges.—All above numerical results can be explained by developing an effective low-energy theory on edges. With both Δ_0 and h_x , the Hamiltonian $H(\mathbf{k})$ possesses both inversion symmetry and particle-hole symmetry $\mathcal{P}H(\mathbf{k})\mathcal{P}^{-1} = -H(-\mathbf{k})$, but breaks the time-reversal symmetry, where $\mathcal{P} = \tau_x K$. Without loss of generality, we assume a positive in-plane Zeeman field applied along x direction, i.e., $h_x > 0$ and $h_y = h_z = 0$. The eigenenergies of $H(\mathbf{k})$

are $E(\mathbf{k}) = \pm \sqrt{(2\lambda_x \sin k_x)^2 + (\zeta \pm h_x)^2}$, where $\zeta = \sqrt{\xi_k^2 + (2\lambda_y \sin k_y)^2 + \Delta_0^2}$ and each of them is twofold degenerate. For large h_x , the system must be a normal superconductor, which becomes gapless for moderate h_x . When h_x is small, the low-energy effective Hamiltonian can be obtained through the lowest order expansion with respect to \mathbf{k} at Γ point

$$H_{\text{eff}}(\mathbf{k}) = (\epsilon + t_x k_x^2 + t_y k_y^2) \sigma_z \tau_z + 2\lambda_x k_x \sigma_x s_z \tau_z + 2\lambda_y k_y \sigma_y \tau_z + \Delta_0 \tau_x + h_x s_x, \quad (2)$$

where $\epsilon = \epsilon_0 - 2t_x - 2t_y < 0$ is assumed for topologically nontrivial QSHI.

Assuming an open-boundary condition along the x direction for edge i , we can replace k_x with $-i\partial_x$ and rewrite $H_{\text{eff}}(\mathbf{k}) = H_0(-i\partial_x) + H_p(k_y)$ with $H_0 = (\epsilon - t_x \partial_x^2) \sigma_z \tau_z - 2i\lambda_x \partial_x \sigma_x s_z \tau_z$, and $H_p = t_y k_y^2 \sigma_z \tau_z + 2\lambda_y k_y \sigma_y \tau_z + \Delta_0 \tau_x + h_x s_x$. When Δ_0 is small comparing to the energy gap, we can treat H_p as a perturbation and solve H_0 to derive the effective edge Hamiltonian for edge i . Assume that Ψ_a is a zero energy solution for H_0 bounded at edge i , $\sigma_y s_z \tau_z \Psi_a$ is also the eigenstate for H_0 due to $\{H_0, \sigma_y s_z\} = 0$. We choose the basis vector ζ_β for Ψ_a satisfying $\sigma_y s_z \zeta_\beta = -\zeta_\beta$, where $\zeta_1 = |-, +, +\rangle$, $\zeta_2 = |+, -, +\rangle$, $\zeta_3 = |-, +, -\rangle$, $\zeta_4 = |+, -, -\rangle$ are eigenstates of $\sigma_y s_z \tau_z$. Under this basis, the effective low-energy Hamiltonian for the edge becomes $H_{\text{edge},i} = 2i\lambda_y s_z \tau_z \partial_y + \Delta_0 \tau_x$ with the topology characterized by a Z invariant [47]. Similarly, we obtain the low-energy Hamiltonian for every edge

$$H_{\text{edge},j} = -i\lambda_j s_z \tau_z \partial_l_j + \Delta_0 \tau_x + h_j s_x. \quad (3)$$

Here the parameters are $\lambda_j = \{-2\lambda_y, 2\lambda_x, 2\lambda_y, -2\lambda_x\}$, $l_j = \{y, x, y, x\}$, and $h_j = \{0, h_x, 0, h_x\}$ for $j = \text{i-iv}$ edges.

From the effective edge Hamiltonian (3), we see that the superconducting order induces quasiparticle gaps for all helical edge states regardless of Zeeman fields since $\{s_z\tau_z, \tau_x\} = 0$. On the other hand, Eq. (3) indicates that the in-plane Zeeman field h_x only opens a gap on two parallel edges (ii and iv), but keeps two perpendicular edges (i and iii) untouched [44].

When $\Delta_0 = 0$, the low-energy edge Hamiltonian possesses two zero-energy bound states on edge i: $\Psi_1(x) = A_1(\sin \alpha x)e^{-(\lambda_x/t_x)x}(\zeta_1 + \zeta_2)$ and $\Psi_2(x) = A_2(\sin \alpha x)e^{-(\lambda_x/t_x)x}(\zeta_3 + \zeta_4)$, where $\alpha = \sqrt{-(\lambda_x^2/t_x^2 + h_x/t_x + \epsilon/t_x)}$ and $A_1(A_2)$ is the normalization constant. Similarly, there are two zero energy bound states localized at edge iii, which are confirmed by real space numerical simulation [44].

After a unitary transformation $U = 1 \oplus (-is_y)$, the edge Hamiltonian reads

$$H'_{\text{edge},j} = -i\lambda_j s_z \partial_{l_j} + \Delta_0 s_x \tau_z + h_j s_x, \quad (4)$$

on the rotated basis $\chi_1 = | +1 \rangle | +1 \rangle$, $\chi_2 = | +1 \rangle | -1 \rangle$, $\chi_3 = | -1 \rangle | +1 \rangle$, $\chi_4 = | -1 \rangle | -1 \rangle$, which are eigenstates of $s_z \tau_y$. For edge i, the Hamiltonian $H'_{\text{edge},i}$ has two decoupled diagonal blocks with Dirac masses $\Delta_0 + h_x$ and $h_x - \Delta_0$, respectively. While for edge ii, Dirac masses are the same Δ_0 for two blocks. When $(\Delta_0 - h_x)\Delta_0 < 0$ (i.e., $h_x > \Delta_0$), the Dirac masses on edges i and ii have opposite signs, leading to the emergence of a localized mode at the intersection of two edges, which is the MCM observed numerically in Fig. 2(c). At the corner between edges i and ii, the MCM can be obtained from the zero-energy wave function

$$\Phi(x, y) \propto \begin{cases} e^{-(|\Delta_0 - h_x|/2\lambda_y)|y - y_0|}(\chi_3 - i\chi_4) & (\text{edge i}), \\ e^{-(\Delta_0/2\lambda_x)|x - x_0|}(\chi_3 - i\chi_4) & (\text{edge ii}), \end{cases} \quad (5)$$

where the corner locates at (x_0, y_0) . We see that MCMs could have different density distributions along different directions when $|\Delta_0 - h_x|/\lambda_y \neq \Delta_0/\lambda_x$.

For the triangle geometry in Fig. 2(d), the effect of the in-plane Zeeman field on the hypotenuse edge can be studied by projecting it to the direction of the Zeeman field, which shows that the Zeeman field acts uniformly on the hypotenuse and upper edges. Consequently, there is no kink of Dirac mass at that corner, i.e., no MCM. Generally, such an argument applies to all geometric configurations with odd edges (e.g., a square with a small right triangle removed at a corner), which is consistent with bulk spectra because the particle-hole symmetry demands that zero-energy modes must be lifted pairwise.

For a general form of the in-plane Zeeman field, MCMs emerge in the region $\sqrt{h_x^2 + h_y^2} > \Delta_0$. However, an out-of-plane Zeeman field ($h_z s_z$ term) does not induce MCMs because the helical edge states of QSHIs remain gapless for

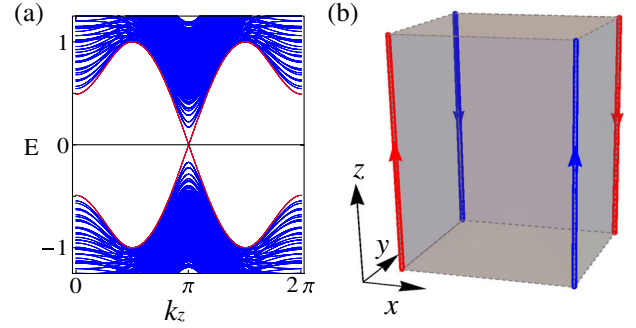


FIG. 4. (a) Quasiparticle spectrum along k_z with open boundary conditions along the x and y directions. (b) Majorana hinge excitations in a 3D second-order topological superconductor. Parameters are $t_x = t_y = t_z = \lambda_x = \lambda_y = \lambda_z = 1.0$, $\Delta_0 = 0.3$, $h_x = 0.6$, and $m_0 = 2.0$.

any h_x and h_z affects each edge in the same way [44]. Finally, for a nonzero chemical potential $\mu \neq 0$, the spectrum is more complicated [44], and MCMs still exist for $h_x > \sqrt{\mu^2 + \Delta_0^2}$ with bulk spectrum being gapped. The phase diagram with a finite μ is shown in Fig. 3(d).

Majorana hinge modes in three dimensions.—Similar physics also applies to three dimensions. Consider a 3D topological insulator described by the Hamiltonian $H_T(\mathbf{k}) = \xi_{\mathbf{k}}' \sigma_z s_0 + \sum_i \lambda_i \sin k_i \sigma_x s_i$ with $\xi_{\mathbf{k}}' = m_0 + \sum_i t_i \cos k_i$, which respects both time reversal and inversion symmetries. For $1 < |m_0| < 3$, $H_T(\mathbf{k})$ represents a 3D topological insulator that possesses surface Dirac cones with gapped bulk spectrum protected by \mathcal{T} and \mathcal{I} symmetries. In the presence of an s -wave superconducting order Δ_0 and a Zeeman field,

$$H_{3D}(\mathbf{k}) = \xi_{\mathbf{k}}' \sigma_z \tau_z + \lambda_x \sin k_x \sigma_x s_x \tau_z + \lambda_y \sin k_y \sigma_y s_y \tau_z + \lambda_z \sin k_z \sigma_z s_z \tau_z + \Delta_0 \tau_x + \mathbf{h} \cdot \mathbf{s}. \quad (6)$$

For $\Delta_0 \neq 0$, $|\mathbf{h}| = 0$, the surface states are gapped and the system is a trivial superconductor. When $h_x > 0$, $h_y = h_z = 0$, the in-plane Zeeman field h_x breaks the time reversal symmetry in the x direction, generating a class-D superconductor. Tuning $h_x > \Delta_0$, we observe the gapless chiral Majorana hinge modes propagating along the z direction as shown in Fig. 4. Such a 3D second-order topological superconductor can be characterized by a Z invariant [47].

Figure 4(a) shows the energy spectrum with open boundary conditions along x and y directions, where the chiral Majorana hinge modes (each twofold degenerate) emerge in the bulk energy gap. The combination of the Zeeman field and the superconductor order gives rise to four domain walls at which the Dirac mass sign changes. Because of the inversion symmetry, the chiral modes at diagonal hinges propagate along opposite directions, as illustrated in Fig 4(b). We remark that the requirement of in-plane Zeeman field can be released in three dimensions

and the direction of the Zeeman field can be used to control the directionality of the hinge modes. Specifically, when the Zeeman field lies along the y (z) direction and $h_y > \Delta_0$ ($h_z > \Delta_0$), the chiral Majorana hinge modes propagate along the x (y) direction with periodic boundary conditions.

Discussion and conclusion.—InAs/GaSb quantum wells are 2D Z_2 QSHIs with large bulk insulating gaps up to ~ 50 meV, and significant experimental progress has been made [48–51] recently to observe their helical edge states. Superconducting proximity effects in InAs/GaSb quantum wells were also observed in experiments [39–41]. In particular, edge-mode superconductivity due to proximity contact with an s -wave superconductor has been detected through transport measurement [40], and giant supercurrent states have been observed [41]. The in-plane Zeeman field could be realized using an in-plane magnetic field due to the relatively large g factor for InAs/GaSb quantum wells [52]. By engineering a suitable quantum device, zero-bias peaks for MCMs should be observable in transport or STM types of experiments.

Another potential material is the monolayer WTe_2 [53] that has been confirmed as a QSHI in recent experiments [54,55]. When proximate to superconductors, a proximity-induced superconducting gap of the order of ~ 0.7 meV [42] emerges. To achieve a comparable spin Zeeman splitting, an in-plane magnetic field $H \sim 0.3\text{--}3$ T is required, given the Landé g factor ranges from 4.5 [54,56] to larger than 44 [57] in WTe_2 , depending on the direction of the applied fields. Based on the aforementioned parameters, s -wave superconductor NbN can be used for the device fabrication, given its both high transition temperature, $T_c \sim 12$ K, and high critical field, for example, $H_c > 12$ T at 0.5 K [58]. For Majorana hinge modes in three dimensions, effective Zeeman fields could be induced by doping magnetic impurities into 3D topological insulators.

In conclusion, we have shown that a heterostructure composing of QSHI/ s -wave superconductor can become a second-order topological superconductor with MCMs in the presence of an in-plane Zeeman field. Because neither exotic superconducting pairings nor complex junction structures are required, our scheme provides a simple and realistic platform for the experimental study of the non-Abelian Majorana corner and hinge modes.

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