



# Tunable Spin-orbit Coupling and Quantum Phase Transition in a Trapped Bose-Einstein Condensate

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Spin-orbit coupling (SOC), the intrinsic interaction between a particle spin and its motion, is responsible for various important phenomena, ranging from atomic fine structure to topological condensed matter physics. The recent experimental breakthrough on the realization of SOC for ultra-cold atoms provides a completely new platform for exploring spin-orbit coupled superfluid physics. However, the SOC strength in the experiment is not tunable. In this report, we propose a scheme for tuning the SOC strength through a fast and coherent modulation of the laser intensities. We show that the many-body interaction between atoms, together with the tunable SOC, can drive a *quantum phase transition* (QPT) from spin-balanced to spin-polarized ground states in a harmonic trapped Bose-Einstein condensate (BEC), which resembles the long-sought Dicke QPT. We characterize the QPT using the periods of collective oscillations of the BEC, which show pronounced peaks and damping around the quantum critical point.

SOC plays a major role in many important condensed matter phenomena and applications, including spin and anomalous Hall effects<sup>1</sup>, topological insulators<sup>2</sup>, spintronics<sup>3</sup>, spin quantum computation, *etc.* In the past several decades, there has been tremendous efforts for developing new materials with strong SOC and new methods for tuning SOC with high accuracy for spin-based device applications<sup>4,5</sup>. However, the SOC strength in typical solid state materials (e.g.,  $\sim 10^4$  m/s in semiconductors) is generally much smaller than the Fermi velocity of electrons ( $\sim 10^6$  m/s), and its tunability is also limited and inaccurate.

On the other hand, the recent experimental breakthrough on the realization of SOC for ultra-cold atoms<sup>6</sup> provides a completely new platform for exploring SOC physics in both BEC<sup>7–12</sup> and degenerate Fermi gases<sup>13–16</sup>. In a degenerate Fermi gas, such SOC strength can be at the same order as (or even larger than) the Fermi velocity of atoms. Because of the strong SOC, spins are not conserved during their motion and new exotic superfluids may emerge. For instance, new ground state phases (e.g., stripes, phase separation, *etc.*) may be observed in spin-orbit coupled BEC<sup>8–12</sup> and new topological excitations (e.g., Weyl<sup>14</sup> and Majorana<sup>13</sup> fermions) may appear in spin-orbit coupled Fermi gases. The observation and applications of these exciting phenomena require fully tunable SOC for cold atoms to characterize the effects of SOC in various phases. Unfortunately, the strength of the SOC in the experiment<sup>6</sup> and other theoretical proposals<sup>17–20</sup> is not tunable because the SOC strength is determined by the directions and wavelengths, not the intensities, of the applied lasers.

In this report, we propose a scheme for generating tunable SOC for cold atoms through a fast and coherent modulation of the Raman laser intensities<sup>21</sup>, which can be easily implemented in experiments. Such tunable SOC for cold atoms provides a powerful tool for exploring new exotic Bose and Fermi superfluid phenomena. Here we focus on a *quantum phase transition* (QPT)<sup>22</sup> in a harmonic trapped BEC induced by the many-body interaction between atoms and the tunable SOC strength. With the increasing SOC strength, there is a sharp transition for the ground state of the BEC from a spin balanced (*i.e.*, equally mixed) phase to a spin fully polarized phase beyond a critical SOC strength (*i.e.*, the quantum critical point). By mapping the spin-orbit coupled interacting BEC to the well-known quantum Dicke model<sup>23,24</sup>, we obtain analytic expressions for the quantum critical point and the corresponding scaling behaviors for the QPT, which agree well with the numerical results obtained from the mean-field Gross-Pitaevskii (G-P) equation for the BEC.

The realization of QPT in the Dicke model using the spin-orbit coupled BEC opens the door for many significant applications in quantum optics, quantum information, and nuclear physics<sup>25–27</sup>. Previously the Dicke model has been studied in several experimental systems<sup>28–30</sup>, especially atoms confined in an optical cavity.



However the coupling between atoms and optical cavity fields is very weak, and the experimental observation of the QPT in the Dicke model only occurred recently using the momentum eigenstates for a BEC confined in a cavity<sup>31</sup>. Compared with the cavity scheme, the spin-orbit coupled BEC utilizes the many-body interaction between atoms and has the advantage of essentially no dissipation, fully tunable parameters, very strong coupling, and the use of atom internal states, thus provides an excellent platform for exploring Dicke model related applications.

Finally, the QPT is characterized using collective oscillations of the BEC, such as the center of mass (COM) motion and the scissors mode, where the oscillation periods show pronounced peaks at the quantum critical point. Furthermore, the oscillations of the BEC have regular periodic patterns in both spin balanced and polarized phases, but show strong damping in the transition region.

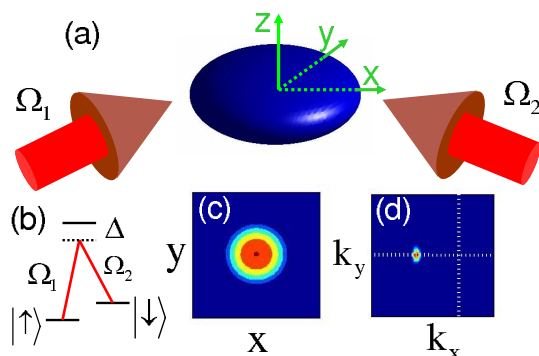
## Results

**System and hamiltonian.** The harmonic trapped BEC in consideration is similar as that in the recent benchmark experiment<sup>6</sup>. For simplicity, we consider a two-dimensional (2D) BEC in the  $xy$  plane with a strong confinement (with a trapping frequency  $\omega_z$ ) along the  $z$  direction. Such 2D setup does not affect the essential physics because the  $z$  direction is not coupled with the SOC. Two hyperfine ground states  $|\uparrow\rangle \equiv |F=1, m_F=-1\rangle$  and  $|\downarrow\rangle \equiv |F=1, m_F=0\rangle$  of <sup>87</sup>Rb atoms define the spins of atoms, which are coupled by two Raman lasers (with Rabi frequencies  $\Omega_1$  and  $\Omega_2$ ) incident at a  $\pi/4$  angle from the  $x$  axis, as illustrated in Figs. 1a and 1b.

The dynamics of the BEC are governed by the nonlinear G-P equation

$$i\hbar\partial\Phi/\partial t = (p^2/2m + V(\mathbf{r}) + H_S + H_I)\Phi, \quad (1)$$

under the dressed state basis  $|\tilde{\uparrow}\rangle = \exp(i\mathbf{k}_1 \cdot \mathbf{r})|\uparrow\rangle$ ,  $|\tilde{\downarrow}\rangle = \exp(i\mathbf{k}_2 \cdot \mathbf{r})|\downarrow\rangle$ , where  $\mathbf{k}_1$  and  $\mathbf{k}_2$  are the wavevectors of the lasers.  $\Phi = (\Phi_\uparrow, \Phi_\downarrow)^T$  is the wavefunction on the dressed state basis and satisfies the normalization condition  $\int dx dy (|\Phi_\uparrow|^2 + |\Phi_\downarrow|^2) = 1$ . The harmonic trapping potential  $V(\mathbf{r}) = \frac{1}{2}m\omega_y^2(\eta^2 x^2 + y^2)$ , where  $\omega_y$  is the trapping frequency in the  $y$  direction, and  $\eta = \omega_x/\omega_y$  is the ratio of the trapping frequencies.  $H_S = \gamma p_x \sigma_z + \hbar\Omega\sigma_x/2$  is the coupling term induced by the two Raman lasers with  $\sigma_z$  and  $\sigma_x$  as the Pauli matrices. The SOC strength  $\gamma = \hbar k_L/m$ ,  $k_L = |\mathbf{K}_1 - \mathbf{K}_2|/2 = \sqrt{2}\pi/\lambda$ , and  $\gamma$  is the wavelength of the Raman lasers. The Raman coupling constant  $\Omega = \Omega_1\Omega_2^*/2\Delta$  with  $\Delta$  as the detuning from the excited state. The mean field nonlinear interaction term  $H_I = \text{diag}(g_{\uparrow\uparrow}|\Phi_\uparrow|^2 + g_{\uparrow\downarrow}|\Phi_\downarrow|^2, g_{\downarrow\uparrow}|\Phi_\uparrow|^2 + g_{\downarrow\downarrow}|\Phi_\downarrow|^2)$ , where the inter- and intraspin interaction constants  $g_{\uparrow\uparrow} = g_{\downarrow\downarrow} = 4\pi\hbar^2 N(c_0 + c_2)/ma_z$  and  $g_{\uparrow\downarrow} = 4\pi\hbar^2 Nc_0/ma_z$ ,  $c_0$  and



**Figure 1 | An illustration of the experimental scheme for realizing SOC for cold atoms<sup>6</sup>.** (a) Laser setup. (b) Atom laser coupling. (c) A typical density distribution of one spin component of the BEC in the ground state. (d) Corresponding momentum distribution. The vertical and horizontal dotted lines are  $k_x = 0$  and  $k_y = 0$  respectively.

$c_2$  describe the corresponding  $s$ -wave scattering lengths<sup>32</sup>,  $N$  is the atom number, and  $a_z = \sqrt{2\pi\hbar/m\omega_z}$ .

Because the SOC strength  $\gamma$  is determined by the laser wavevector  $k_L$ , the SOC energy can be comparable to or even larger than other energy scales (e.g., the Raman coupling  $\Omega$ ) in the BEC. In a Fermi gas,  $\gamma$  can be larger than the Fermi velocity of atoms. Unfortunately, due to the same reason,  $\gamma$  cannot be easily adjusted in experiments, which significantly restricts the applications of the SOC in cold atoms. Note that although theoretically it may be possible to tune the SOC strength by varying the angle between two Raman beams, experimentally it is impractical because of many limitations of the experimental setup.

**Tunable SOC for cold atoms.** We propose a scheme for tuning the SOC strength  $\gamma$  through a fast and coherent modulation of the Raman coupling  $\Omega = \Omega_0 + \tilde{\Omega}\cos(\omega t)$  that can be easily realized in experiments by varying the Raman laser intensities. For  $\tilde{\Omega} > \Omega_0$ ,  $\Omega$  changes sign at certain time, which can be achieved by applying a  $\pi$  phase shift on one Raman laser. Here the modulation frequency  $\omega$  is chosen to be much larger than other energy scales in Eq. (1). In this case, the Hamiltonian in Eq. (1) can be transformed to a time-independent one using a unitary transformation  $\psi = \exp[i\tilde{\Omega}\sin(\omega t)\sigma_x/(2\omega)]\Phi$ . After a straightforward calculation with the elimination of the fast time-varying part in the Hamiltonian<sup>33,34</sup>, the nonlinear G-P equation (1) becomes

$$i\hbar\partial\psi/\partial t = [p^2/2m + V(\mathbf{r}) + \bar{H}_S + \bar{H}_I]\psi, \quad (2)$$

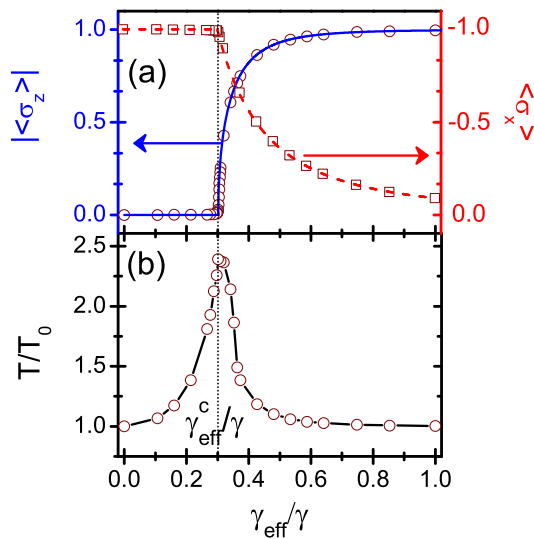
where the Raman coupling becomes  $\bar{H}_S = \gamma_{\text{eff}}p_x\sigma_z + \hbar\Omega_0\sigma_x/2$  with the effective SOC strength

$$\gamma_{\text{eff}} = \gamma J_0(\tilde{\Omega}/\omega). \quad (3)$$

Here  $J_0$  is the zero order Bessel function. Clearly,  $\gamma_{\text{eff}}$  can be tuned from the maximum  $\gamma$  without the modulation to zero with a strong modulation. The mean field interaction term  $\bar{H}_I = \alpha(|\psi_\uparrow|^2 + |\psi_\downarrow|^2)\psi + \beta\Gamma\psi$ ,  $\alpha = g_{\uparrow\uparrow}$ ,  $\beta = (g_{\downarrow\downarrow} - g_{\uparrow\uparrow})/2$  and  $\Gamma$  is a  $2 \times 2$  matrix whose elements are given by  $\Gamma_{11} = |\psi_\uparrow|^2 \left[ \frac{3}{4} - J_0\left(\frac{\tilde{\Omega}}{\omega}\right) + \frac{1}{4}J_0\left(\frac{2\tilde{\Omega}}{\omega}\right) \right] + |\psi_\downarrow|^2 \left[ \frac{1}{4} - \frac{1}{4}J_0\left(\frac{2\tilde{\Omega}}{\omega}\right) \right]$ ,  $\Gamma_{22} = |\psi_\uparrow|^2 \left[ \frac{1}{4} - \frac{1}{4}J_0\left(\frac{2\tilde{\Omega}}{\omega}\right) \right] + |\psi_\downarrow|^2 \left[ \frac{3}{4} + J_0\left(\frac{\tilde{\Omega}}{\omega}\right) + \frac{1}{4}J_0\left(\frac{2\tilde{\Omega}}{\omega}\right) \right]$ , and  $\Gamma_{12} = \Gamma_{21}^* = -\frac{1}{4} \left[ 1 - J_0\left(\frac{2\tilde{\Omega}}{\omega}\right) \right] \left[ \psi_\uparrow^* \psi_\downarrow - \psi_\uparrow \psi_\downarrow^* \right]$ .

**Quantum phase transition.** The tunable SOC, in combination with the many-body interaction between atoms, can drive a quantum phase transition between different quantum ground states in a harmonic trapped BEC. Here the ground state of the BEC is obtained numerically through an imaginary time evolution of the G-P equation (2). A typical density profile of the ground state is shown in Fig. 1c, which has a Thomas-Fermi shape, similar as that in a regular BEC. However, the momentum distribution of the BEC has a peak around the single particle potential minimum located at  $(K_x, K_y) = (-K_{\text{min}}, 0)$  (see Fig. 1d), where  $K_{\text{min}} = \sqrt{\gamma_{\text{eff}}^2 - \Omega_0^2/4\gamma_{\text{eff}}^2}$  and the degeneracy between  $\pm K_{\text{min}}$  is spontaneously broken.

To characterize the ground state of the spin-orbit coupled BEC, we calculate the spin polarization  $|\langle\sigma_z\rangle| = \left| \int d\mathbf{r} (|\psi_\uparrow|^2 - |\psi_\downarrow|^2) \right|$ , and  $\langle\sigma_x\rangle = 2\text{Re} \int d\mathbf{r} \psi_\uparrow^* \psi_\downarrow$ . Here we choose the absolute value of  $\langle\sigma_z\rangle$  because the two degenerate ground states at  $\pm K_{\text{min}}$  have opposite  $\langle\sigma_z\rangle$  due to the spinmomentum locking term  $p_x\sigma_z$  and they are spontaneously chosen in experiments. In Fig. 2a, we plot  $|\langle\sigma_z\rangle|$  and  $\langle\sigma_x\rangle$  with respect to  $\gamma_{\text{eff}}$ . For a small  $\gamma_{\text{eff}}$ , the spin up and down atoms have an equal population, thus  $\langle\sigma_z\rangle = 0$ ,  $\langle\sigma_x\rangle = -1$ , i.e., the spin balanced



**Figure 2 | Quantum phase transition with tunable  $\gamma_{\text{eff}}$ .**  $\Omega_0 = 16$ ,  $\gamma = 9.37$  is the bare SOC strength without modulation. (a) Plot of the spin polarization  $|\langle \sigma_z \rangle|$  and  $\langle \sigma_x \rangle$  in the ground state. The blue and red lines are from the prediction of the Dicke Hamiltonian. The circles and squares are from the numerical simulation of the G-P equation (2). (b) Plot of the COM motion period  $T$ . The shift of the harmonic trap  $D = 1$ ,  $T_0 = 2\pi/\omega_y$ .

phase. Beyond a critical point  $\gamma_{\text{eff}}^c$ , the spin population imbalance increases dramatically and reaches the spin polarized phase ( $\langle \sigma_z \rangle = 1$  ( $\langle \sigma_x \rangle = 0$ )) within a small range of  $\gamma_{\text{eff}}$ . The spin balanced and spin polarized phases at small and large  $\gamma_{\text{eff}}$  can be understood from the single particle Hamiltonian  $\hat{H}_s$ , where the Raman coupling  $\Omega_0\sigma_x/2$  and SOC  $\gamma_{\text{eff}}P_x\sigma_z$  dominate at the small and large  $\gamma_{\text{eff}}$  respectively. More numerical results show that the critical transition point occurs at  $\gamma_{\text{eff}}^c = \sqrt{\Omega_0/2}$ .

The QPT from spin-balanced to spin polarized phases can be understood by mapping the spin-orbit coupled BEC to the quantum Dicke model. For an interacting BEC in a harmonic trap with a large atom number  $N$  (so that the mean field theory works), all atoms are forced to occupy the same many-body ground state (i.e., the state in Fig. 1c). Therefore the energy variation for the change of the spin (e.g., spin flip) of one atom need be determined by the coupling between the atom spin and the many-body ground state mode. This is very different from a non-interacting BEC where atoms do not affect each other, but the same as that for many atoms interacting with a single photon mode in an optical cavity<sup>23</sup>. Treating the interacting many-body ground state as a single mode composed of different harmonic trap modes, we can map the Hamiltonian for the spin-orbit coupled BEC to

$$H = \hbar\omega_x Na^\dagger a + \hbar\Omega_0 S_x + \frac{\gamma_{\text{eff}}\sqrt{m\hbar\omega_x}}{\sqrt{2}}(a^\dagger - a)(S_+ - S_-), \quad (4)$$

which is similar to the Dicke model for two-level atoms coupled with a cavity field<sup>23</sup>.  $S_{x,y,z}$  are the large spins for all atoms,  $S_+ = S_y + iS_z$ ,  $S_- = S_y - iS_z$ ,  $a^\dagger a$  is a harmonic trap mode,  $a = \sqrt{m\omega_x/2\hbar}(x + ip_x/m\omega_x)$ . In the mapping to the Dicke Hamiltonian (4), we neglect two minor effects of the interaction terms in Eq. (2): 1) a constant mean field background energy term that does not affect the dynamics; 2) a small term  $\kappa S_z^2/N^2$  with  $\kappa \sim g_{\perp\perp} - g_{\uparrow\uparrow}$ . The second term can shift the Dicke phase transition point. However, for realistic experimental parameters for <sup>87</sup>Rb,  $\kappa$  is very small and the effects of  $\kappa S_z^2/N^2$  can be neglected, as we confirm in the numerical simulation of the G-P equation. Nevertheless, these minor effects cannot be captured by the non-interacting BEC. The QPT from spin-balanced to spin-polarized phases in the spin-orbit coupled BEC is similar to

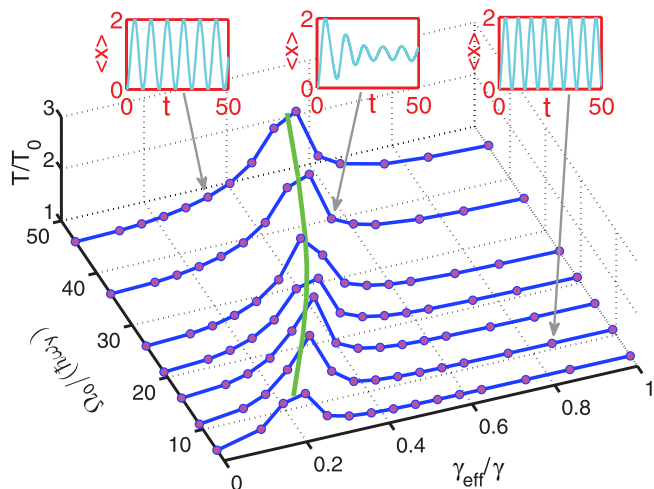
that from normal to superradiant phases in the Dicke model. The critical point for the QPT can be derived from the standard mean-field approximation<sup>24</sup>, yielding the relation  $\gamma_{\text{eff}}^c = \sqrt{\Omega_0/2}$ , which is exactly the same as that from numerically simulating the G-P equation (2). Clearly, the QPT can also be driven by varying  $\Omega_0$  for a fixed  $\gamma_{\text{eff}}$ . Just beyond the critical point  $\gamma_{\text{eff}}^c$ , the Dicke model predicts that the scaling of the order parameters is  $|\langle \sigma_z \rangle| = 2|S_z|/N = \sqrt{1 - (\gamma_{\text{eff}}^c/\gamma_{\text{eff}})^4}$ ,  $\langle \sigma_x \rangle = 2S_x/N = -(\gamma_{\text{eff}}^c/\gamma_{\text{eff}})^2$  for  $\gamma_{\text{eff}} \geq \gamma_{\text{eff}}^c$ , and  $|\langle \sigma_z \rangle| = 0$ ,  $\langle \sigma_x \rangle = -1$  for  $\gamma_{\text{eff}} < \gamma_{\text{eff}}^c$ . Such scaling behaviors are confirmed in our numerical simulation of the G-P equation (see Fig. 2a). The perfect match between numerical results from the G-P equation and the predictions of the Dicke Hamiltonian shows the validity of the mapping to the Dicke model.

We emphasize that the many-body interaction between atoms in Eq. (2) plays a critical role in the QPT by forcing all atoms in a single spatial mode. For a non-interacting BEC, the atoms can occupy both  $\pm K_{\text{min}}$  in the momentum space with an artificial ratio because these two states are energetically degenerate and there is no correlation between atoms. The resulting spatial distribution of the BEC is thus artificial and the above single spatial mode approximation in Eq. (4) does not apply. Our numerical simulation of the G-P equation without interactions also shows  $\langle \sigma_z \rangle = 0$  or other random values in certain region of  $\gamma_{\text{eff}} > \gamma_{\text{eff}}^c$ , which disagrees with the prediction of the Dicke model. This disagreement confirms that atoms in a non-interacting BEC do not respond to the change of  $\gamma_{\text{eff}}$  collectively, although non-interacting and interacting BECs share the same transition for the energy spectrum at  $\gamma_{\text{eff}}^c$ , which changes from one single minimum at  $K_x = 0$  to two minima at  $\pm K_{\text{min}}$ . While for interacting BECs with large atom numbers  $N = 4 \times 10^4$  and  $10^6$ , we obtain exactly the same results as that in Fig. 2a, which further confirm the validity of our mapping to the Dicke model in the large  $N$  limit.

**Collective dynamics in BEC: the signature of QPT.** It is well-known that various physical quantities may change dramatically around the quantum critical point (i.e., critical phenomena), which provides additional experimental signatures of the QPT. We focus on two types of collective dynamics of the ground state of the BEC: the COM motion and the scissors mode induced by a sudden shift or rotation of the harmonic trapping potential, respectively. In a regular BEC without SOC, the COM motion is a standard method to calibrate the harmonic trapping frequency because the oscillation period depends only on the trapping frequency<sup>35</sup> and is not affected by other parameters such as nonlinearity, shift direction and distance, etc.

We numerically integrate the G-P equation (2) and calculate the COM  $\langle \mathbf{r}(t) \rangle = \int dx dy (|\psi_1(t)|^2 + |\psi_{\perp}(t)|^2) \mathbf{r}(t)$ . The COM motion strongly depends on the direction of the shift  $\vec{D}$  of the harmonic trap. When  $\vec{D}$  is along the  $y$  direction, the period of the COM motion along the  $y$  direction is  $T_0 = 2\pi/\omega_y$  and not affected by  $\gamma_{\text{eff}}$ , while the COM motion in the  $x$  direction disappears (i.e.,  $\langle x \rangle = 0$ ). Here the COM period  $T$  is obtained through the Fourier analysis of  $\langle \mathbf{r}(t) \rangle$ . The physics is very different when  $\vec{D}$  is along the  $x$  direction, where  $\langle y(t) \rangle = 0$  as expected, but  $\langle x(t) \rangle$  depends strongly on  $\gamma_{\text{eff}}$ , as shown in Fig. 2b. In Fig. 3, we also plot  $T$  as a function of  $\gamma_{\text{eff}}$  and  $\Omega_0$ . Without SOC ( $\gamma_{\text{eff}} = 0$ ),  $T = T_0$ , the period for a regular BEC, as expected.  $T$  increases with  $\gamma_{\text{eff}}$  in the spin-balanced phase, but decreases when spin starts to be polarized, leading to a sharp peak at the quantum critical point  $\gamma_{\text{eff}}^c$ . The oscillation of  $\langle x(t) \rangle$  in the spin balanced phase is completely dissipationless, while a strong damping occurs in a small range of  $\gamma_{\text{eff}}$  beyond  $\gamma_{\text{eff}}^c$  (see the inset in Fig. 3). Far beyond  $\gamma_{\text{eff}}^c$ , the oscillation becomes regular again with the period  $T = T_0$  because the ground state has only one component in this region. The peak and the damping of the oscillation around  $\gamma_{\text{eff}}^c$  provide clear experimental signatures for the QPT. Moreover,  $T$  also depends on





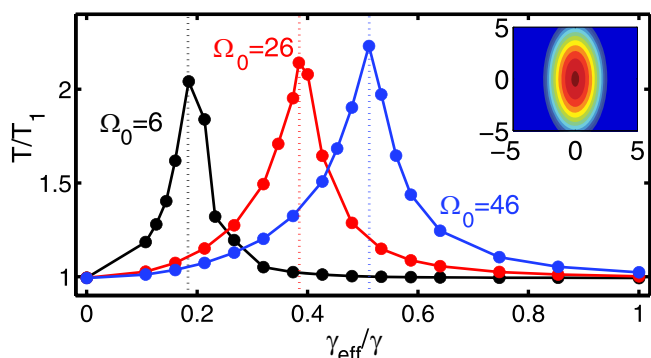
**Figure 3** | Plot of the COM motion period  $T$  versus  $\gamma_{\text{eff}}$  and  $\Omega_0$ . The insets show the corresponding  $\langle x(t) \rangle$ . The green line is the theoretical prediction  $\gamma_{\text{eff}}^c = \sqrt{\Omega_0}/2$  from the Dicke model. The circles are the numerical results from the G-P equation (2).  $T_0 = 2\pi/\omega_y$ ,  $D = 1$ .

the magnitude  $D$  of the shift near the critical point  $\gamma_{\text{eff}}^c$ : the larger  $D$ , the smaller  $T$ .

Another collective dynamics, the scissors mode<sup>36</sup>, shows a similar feature as the COM motion. The scissors mode can be excited by a sudden rotation of the asymmetric trapping potential (i.e.,  $\eta \neq 1$ ) by an angle  $\theta$ , which induces an oscillation of the quantity  $\langle xy \rangle = \int dx dy (|\psi_{\uparrow}(t)|^2 + |\psi_{\downarrow}(t)|^2) xy$ . Without SOC, the period of the scissors mode is  $T_1 = 2\pi / \sqrt{\omega_x^2 + \omega_y^2}$ <sup>36</sup>, as observed in experiments<sup>37</sup>. In Fig. 4, We plot the oscillation period  $T$  with respect to  $\gamma_{\text{eff}}$  for three different  $\Omega_0$ . We have confirmed that the same QPT occurs for  $|\langle \sigma_z \rangle|$  and  $\langle \sigma_x \rangle$  of the ground state in this asymmetric potential with the quantum critical point  $\gamma_{\text{eff}}^c = \sqrt{\Omega_0}/2$ , as predicted by the Dicke model. Similar as the COM motion, we observe the peak and damping of the oscillation around  $\gamma_{\text{eff}}^c$ . Far beyond  $\gamma_{\text{eff}}^c$ , the oscillation period is  $T_1$ . Similar as the dependence of the COM motion on the shift distance  $D$ , the angle  $\theta$  also influences the period of the scissors mode near  $\gamma_{\text{eff}}^c$ : the smaller  $\theta$ , the larger  $T$ .

## Discussion

In summary, we show that the SOC strength in the recent breakthrough experiment for realizing SOC for cold atoms can be tuned through a fast and coherent modulation of the applied laser intensities. Such tunable SOC provides a powerful tool for exploring



**Figure 4** | Plot of the scissors mode oscillation period  $T$  with respect to  $\gamma_{\text{eff}}$  for three different  $\Omega_0$ .  $T_1 = 2\pi / \omega_y \sqrt{\eta^2 + 1}$  is the oscillation period without SOC,  $\eta = \sqrt{5}$ ,  $\theta = 4^\circ$ . The inset is a typical density distribution of the ground state of the BEC. The dotted lines show the corresponding quantum critical points predicted by the Dicke model.

spin-orbit coupled superfluid physics in future experiments. By varying the SOC strength, the many-body interaction between atoms can drive a QPT from spin balanced to spin polarized ground states in a harmonic trapped BEC, which realizes the long-sought QPT from normal to superradiant phases in the quantum Dicke model and may have important applications in quantum information and quantum optics.

## Methods

We choose the physical parameters to be similar as those in the experiment<sup>4</sup>:  $(\omega_x, \omega_z) = 2\pi \times (40, 400)$  Hz,  $\eta = 1$ ,  $\lambda = 804.1$  nm,  $c_0 = 100.86a_B$ ,  $c_2 = -0.46a_B$ <sup>38</sup> with the Bohr radius  $a_B$ ,  $N = 1 \times 10^3$ ,  $\omega = 2\pi \times 4.5$  kHz. For the numerical simulation, we need a dimensionless G-P equation that is obtained by choosing the units of the energy, length and time as  $\hbar\omega_y$ ,  $\sqrt{\hbar/(m\omega_y)} = 1.7\mu\text{m}$ , and  $1/\omega_y = 4$  ms, respectively. The dimensionless parameters in the G-P equation become  $\gamma = \sqrt{\hbar/(m\omega_y)}k_l = 9.37$ ,  $\alpha = 2N\sqrt{2\pi m\omega_z/\hbar}(c_0 + c_2) = 495$  and  $\beta = -N\sqrt{2\pi m\omega_z/\hbar}c_2 = 1.14$ .

**Note added.** After our manuscript was initially posted at arXiv (arXiv:1111.4778), our proposed peaks of the dipole oscillation periods (Figs. 2b and 3) were observed experimentally<sup>39</sup>, and more detailed theoretical studies were also performed<sup>40</sup>.

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## Author contributions

Y.Z., G.C. and C.Z. conceived the idea, Y.Z., G.C., C.Z. performed the calculation, Y.Z., G.C., C.Z. wrote the manuscript, C.Z. supervised the whole research project.

## Additional information

**Competing financial interests:** The authors declare no competing financial interests.

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