REVIEW FOR MATH 106 FIRST MIDTERM (2007)

1) Find the following limits:

a) $\lim_{x\to 0} \frac{5\tan^4(2x^2)}{x^8}$

b) $\lim_{x\to 1} \frac{x^3-1}{x^4-1}$

c) $\lim_{x\to\infty} \frac{3x+2\sin x+\sqrt{x}}{5x-3\cos x}$

d) $\lim_{x\to-\infty} \frac{3x+2}{\sqrt{x^2+5}}$

e) $\lim_{x\to\infty} \sqrt{x^2+x+5}-x$

f) $\lim_{x\to\infty} x.\sin\frac{1}{x}$

g) $\lim_{x\to 0} \frac{\cos x-1}{x^2}$

h) $\lim_{x\to\infty}(\sqrt{2x+1}-\sqrt{3x-1})$

2) Find dy/dx. Do NOT simplify your answers.

a) $y = x^{\pi}(\sin x)\sqrt{\tan^{2e}(x)}$

b) $y = \csc^2(sin^2(x^{2/3}))$

c) $y^2(2-x) = (x^3-y)\sin y$

3) a) Find all the points on the curve $x^2+xy+y^2=7$ where the tangent line is parallel to the x-axis.

b) If $x^{1/3}+y^{1/3}=4$, then find d^2y/dx^2 at the point $(8,8)$ using implicit differentiation.

4) a) Prove that $f(x)=|x-1|$ is NOT differentiable at $x=1$ using the formal definition of the derivative.

b) Prove that $f(x)=|x-1|$ is differentiable at $x=1.001$ using the formal definition of the derivative. Calculate $f'(1.001)$.

5) For what values of a and b will the function

$$g(x)=\begin{cases} ax+b & x\le -1 \\ ax^3+x+2b, & x>-1 \end{cases}$$

be differentiable for all values of x?

6) Show that the function $f(x)=x+\sin^2(x/3)-8$ has exactly one zero in $(-\infty,\infty)$.

7) Find the point (x, y) on the curve $y = \sqrt{x}$ on the curve which is nearest to $(2, 0)$.

8) Let $f : [0, 1] \to [0, 1]$ be a continuous function. Prove that there exists a number $c \in [0, 1]$ such that $f(c) = c$.

9) Find $\frac{d^{999}}{dx^{999}} \cos x$

10) Prove the generalized product rule $(uvw)' = uvw' + uv'w + u'vw$.

11) Find the dimensions of the right circular cylinder of maximum volume inscribed in a sphere of radius 10cm.

12) Show that there is no differentiable function $f(x)$ with domain $\mathbf{R}$ such that $f(x)$ is increasing and concave up for all real numbers, and $\lim_{x\to\infty} f(x) = 10$ ($y = 10$ line is a horizontal asymptote).

13) Sketch the graph of the rational function $f(x) = \frac{(x+1)^2}{x^2+1}$.

14) Sketch the graph of the function $f(x) = 8x^2 - x^4$.

15) Sketch the graph of the function $f(x) = x - 3\sqrt[3]{x}$.

16) Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y = 8 - x^2$.

17) Use the Mean Value Theorem to prove the inequality

$$|\sin x - \sin y| \leq |x - y| \qquad \text{for any } x \text{ and } y.$$

18) Find all the critical points of the function $f(x) = |x^2 - 2x - 3|$ on the interval (-2,4). Determine the absolute minimum and absolute maximum values of $f(x)$ on (-2,4), if exists.

19) Find all points on the curve $y = x^{1/3}(x - 4)$
a) where the tangent line is parallel to the x-axis
b) where the tangent line is parallel to the y-axis

20) Given the parametric equations

$$x \sin t + 2x = t \text{ and } tsint - 2t = y$$

Find dy/dx at $t = \pi$.