REVIEW FOR MATH 106 SECOND MIDTERM (2007)

1) Find the following limit:

$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}(4+\frac{3i}{n})^5$

2) Find $g'(x)$

a) $g(x) = \int_{x^2}^{\cos x} \ln t dt$

b) $g(x) = \int_0^1 \frac{\sin t}{t} dt$

3) Find the area between the following curves.

a) $y = \sqrt[3]{x}$ and $y = x$

b) $x = y^2 - 4y$ and $x = 2y - y^2$

4) Compute the following integrals:

a) $\int_3^5 \frac{dx}{x.\ln x}$

b) $\int_0^\pi \sin^7 x dx$

c) $\int_{-1}^3 \sin(x-1)^3 dx$

d) $\int_0^1 \frac{e^{\sqrt{x}} dx}{\sqrt{x}}$

5) Find y'.

a) $y = x^{\sin x}$

b) $y = \frac{(x+2)^7 (x^2-8)^{1/4}}{(x^3+7)^{14}}$

6) Find the following limits.

a) $\lim_{x\to\infty} x^{\frac{1}{x}}$

b) $\lim_{x\to 0^+} x^x$

7) Find the volume of the solids obtained by rotating the defined regions about the given axis.

a) Region: $y = \frac{1}{x}$, $y = 0$, $x = 1$, $x = 3$ rotate about the axis $y = -1$

b) Region: $y = x^2$, $x = y^2$ rotate about the axis $x = -1$

c) Region: $y = e^{-x^2}$, $y = 0$, $x = 0$, $x = 1$ about y-axis.

d) Region: $x = 0$, $x = y^2 - 1$ rotate about the axis $y = 7$

8) Find the length of the curve $x = \frac{y^{(3/2)}}{3} - y^{(1/2)}$ from $y = 1$ to $y = 9$.

9) Find the following indefinite integrals.

a) $\int e^x . \sin x dx$

b) $\int \sin \sqrt{x}\ dx$

c) $\int \frac{\sqrt{4-x^2}}{x}\ dx$

d) $\int \ln(x^2 + x)\ dx$

e) $\int (x^4 + 1)^{1/3} x^7 dx$

f) $\int \sqrt{\frac{x-1}{x^5}} dx$

10) Find the area of the region enclosed by the curves $y = |x^2 - 4|$ and $y = (x^2/2) + 4$. (Sketch the graph of the curves and indicate the enclosed region)

11) The disk $(x-3)^2 + (y-4)^2 \leq 4$ is revolved about the line $y = -4$ to generate a solid. Find its volume using the washer method of integration.

12) Find the $\lim_{x\to 0} \frac{(\int_2^{2+x} \sqrt{t^3+1}\ dt)}{x^2-x}$.

13) Suppose that f is a continuous function on $[a, b]$. Prove that

$$F(x) = \int_a^x f(t)dt$$

is differentiable at every point of x in $[a, b]$ and find its derivative.

14) Which one is bigger, π^e or e^π ? Give a proof.

15) Find the $\lim_{x\to 0^+} (sinx)^{tanx}$ if it exists.

16) Determine whether the improper integral

$$\int_0^\infty \frac{16 \arctan x}{1 + x^2}\ dx$$

is convergent or divergent. Find its value if it is convergent.

17) Find the length of the curve

$$y = \int_0^x \sqrt{\cos 2t}\ dt \quad 0 \leq x \leq \pi/4.$$