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KOÇ UNIVERSITY

MATH 106 - CALCULUS

Midterm II (B)                      December 6, 2004

**Duration of Exam: 90 minutes**

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**INSTRUCTIONS:** No calculators may be used on the test. No books, no notes, no questions, and talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below. GOOD LUCK!**

Surname, Name: \_\_\_\_\_

Student ID no: \_\_\_\_\_

Signature: \_\_\_\_\_

Section (Check One):                      Section 1: Prof. Toma Albu                      \_\_\_\_\_  
    Section 2: Prof. Ali Mostafazadeh                      \_\_\_\_\_  
    Section 3: Prof. Tolga Etgü                      \_\_\_\_\_  
    Section 4: Prof. Özlem Keskin                      \_\_\_\_\_

PROBLEM	POINTS	SCORE
1	20	
2	20	
3	20	
4	15	
5	15	
6	10	
<b>TOTAL</b>	<b>100</b>	

**Name:**

**Problem 1 (20 pts)** Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be the function defined by  $f(x) = 2x^4 - 4x^2 + 3$ .

(1.a) Determine the intervals in  $\mathbb{R}$  where  $f$  is increasing and intervals where it is decreasing.

(1.b) Determine the local extremum values of  $f$ .

(1.c) Determine the intervals in  $\mathbb{R}$  where  $f$  is concave up and intervals where it is concave down.

(1.d) Determine the inflection points of  $f$ .

(1.e ) Plot the graph of  $f$ .

**Name:**

**Problem 2 (20 pts)** Evaluate the following integrals.

(2.a)  $\int \frac{5 \sin x}{3 + 7 \cos x} dx$

(2.b)  $\int \frac{dx}{x^2 - 4x + 5}$

(2.c)  $\int \frac{x+2}{\sqrt{1-x^2}} dx$

(2.d)  $\int \tanh x \, dx$

**Name:**

**Problem 3**

(3.a) **(15 pts)** Let  $f$  be a continuous function on an interval  $[a, b]$ . Prove that the function  $\int_a^x f(t) dt$  has a derivative at every  $x \in [a, b]$  and

$$\frac{d}{dx} \int_a^x f(t) dt = f(x).$$

(3.b) **(5 pts)** Evaluate

$$\frac{d}{dx} \int_{1+\sin^4 x}^3 \frac{dt}{\ln t}.$$

**Name:**

**Problem 4 (15 pts)** Calculate the area of the region bounded by the graphs of the functions  $f(x) = -x^3 + x^2$  and  $g(x) = -x^2 + x$ .

**Name:**

**Problem 5 (15 pts)** Calculate the length of the curve defined by

$$y(x) = \frac{2 \ln x - x^2 + 1}{4}, \quad 1 \leq x \leq 2.$$

**Name:**

**Problem 6 (10 pts)** Knowing that  $\sinh$  is a one-to-one function, its inverse  $\sinh^{-1}$  exists. Show that it satisfies

$$\sinh^{-1}(x) = \ln(x + \sqrt{x^2 + 1}), \quad \text{for every real number } x.$$