KOÇ UNIVERSITY

MATH 106 - CALCULUS

Midterm II (A) December 6, 2004

Duration of Exam: 90 minutes

INSTRUCTIONS: No calculators may be used on the test. No books, no notes, no questions, and talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below. GOOD LUCK!**

Surname, Name: _____

Student ID no: _____

Signature: _____

Section (Check One):

Section 1: Prof. Toma Albu ___
Section 2: Prof. Ali Mostafazadeh ___
Section 3: Prof. Tolga Etgü ___
Section 4: Prof. Özlem Keskin ___

PROBLEM	POINTS	SCORE
1	20	
2	20	
3	20	
4	15	
5	15	
6	10	
TOTAL	**100**	

Name:

Problem 1 (20 pts) Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) = x^4 - 2x^2 + 3$.

(1.a) Determine the intervals in $\mathbb{R}$ where f is increasing and intervals where it is decreasing.

(1.b) Determine the local extremum values of f.

(1.c) Determine the intervals in $\mathbb{R}$ where f is concave up and intervals where it is concave down.

(1.d) Determine the inflection points of f.

(1.e) Plot the graph of f.

Name:

Problem 2 (20 pts) Evaluate the following integrals.

(2.a) $\displaystyle\int \frac{7\sin x}{3+5\cos x}\,dx$

(2.b) $\displaystyle\int \frac{dx}{x^2+4x+5}$

(2.c) $\displaystyle\int \frac{x+1}{\sqrt{1-x^2}}\,dx$

(2.d) $\displaystyle\int \tanh x\,dx$

Name:

Problem 3

(3.a) **(15 pts)** Let f be a continuous function on an interval $[a, b]$. Prove that the function $\int_a^x f(t)\,dt$ has a derivative at every $x \in [a, b]$ and

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x).$$

(3.b) **(5 pts)** Evaluate

$$\frac{d}{dx}\int_{1+\sin^2 x}^{3} \frac{dt}{\ln t}.$$

Name:

Problem 4 (15 pts) Calculate the area of the region bounded by the graphs of the functions $f(x) = x^3 - x^2$ and $g(x) = x^2 - x$.

Name:

Problem 5 (15 pts) Calculate the length of the curve defined by

$$y(x) = \frac{2\ln x - x^2 + 3}{4}, \qquad 1 \leq x \leq 2.$$

Name:

Problem 6 (10 pts) Knowing that sinh is a one-to-one function, its inverse $\sinh^{-1}$ exists. Show that it satisfies

$$\sinh^{-1}(x) = \ln(x + \sqrt{x^2 + 1}), \qquad \text{for every real number } x.$$