KOÇ UNIVERSITY

MATH 106 - CALCULUS

Midterm I (B) November 1, 2004

Duration of Exam: 90 minutes

INSTRUCTIONS: No calculators may be used on the test. No books, no notes, no questions, and talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below. GOOD LUCK!**

Surname, Name: _____

Signature: _____

Section (Check One):

Section 1: Prof. Toma Albu ___
Section 2: Prof. Ali Mostafazadeh ___
Section 3: Prof. Tolga Etgü ___
Section 4: Prof. Özlem Keskin ___

PROBLEM	POINTS	SCORE
1	20	
2	10	
3	20	
4	20	
5	15	
6	15	
TOTAL	**100**	

Name:

Problem 1 Calculate the following limit or show that it does not exist:

(a) (5 pts) $\lim_{x\to-\infty} \dfrac{5x^7 - 2x^2 + 9}{-3x^7 - 4x + 5}$

(b) (5 pts) $\lim_{x\to-\infty} \dfrac{\cos(x^2 + 3)}{5x^2 - 2x + 3}$

(c) (5 pts) $\lim_{x\to 0} \dfrac{x + \sin x}{x + \tan x}$

(d) (5 pts) $\lim_{x\to+\infty} x \sin(\frac{1}{x})$

Name:

Problem 2 Find the derivative of the following functions.

(a) (5 pts) $f(x) = 15(x^2 - 3)^{1/5}(x + 4)^{-1/3}$

(b) (5 pts) $f(x) = \dfrac{\cos(2x)}{\cos x + \sin x}$

Name:

Problem 3 Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x^2 - 1 & \text{for } x < 1 \\ \tan^2(x^2 - 1) & \text{for } x \geq 1 \end{cases}$$

(a) (8 pts) Is f continuous at $x = 1$? Why?

b) (12 pts) Is f differentiable at $x = 1$? Why?

Name:

Problem 4

(a) (10 pts) State and prove the product rule for differentiation (This is the rule that you use to find the derivative of the product of two functions.)

(b) (10 pts) Let $u, v : \mathbb{R} \to \mathbb{R}$ be differentiable functions at x such that $v(x) \neq 0$, and let $y = \dfrac{u}{v}$. Use the product rule for differentiation and the equation $v(x)y(x) = u(x)$ to show that $y'(x) = \dfrac{u'(x)v(x) - v'(x)u(x)}{v(x)^2}$.

Name:

Problem 5 (15 pts) Find the equation for the line in the x-y plane that is tangent to the curve described by

$$x(t) = \frac{t^2 + 2t}{2}, \qquad y(t) = \frac{t^3 + 3t}{3}$$

at the point corresponding to $t = 0$.

Name:

Problem 6 (15 pts) Use implicit differentiation to find $\dfrac{dy}{dx}$ if

$$y^2 \cos\left(\frac{1}{y}\right) = 3x^2 + 2y^3.$$