KOÇ UNIVERSITY
MATH 106 - CALCULUS
Midterm I (B)   November 1, 2004
Duration of Exam: 90 minutes

INSTRUCTIONS: No calculators may be used on the test. No books, no notes, no questions, and talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below. GOOD LUCK!**

Surname, Name: ————————————————————

Signature: ————————————————————

Section (Check One):

Section 1: Prof. Toma Albu	——
Section 2: Prof. Ali Mostafazadeh	——
Section 3: Prof. Tolga Etgü	——
Section 4: Prof. Özlem Keskin	——

PROBLEM	POINTS	SCORE
1	20	
2	10	
3	20	
4	20	
5	15	
6	15	
TOTAL	**100**	

Name:

Problem 1 Calculate the following limit or show that it does not exist:

(a) (5 pts) $\quad \lim\limits_{x \to -\infty} \dfrac{3x^5 - 2x^2 + 11}{-2x^5 + 3x - 17}$

(b) (5 pts) $\quad \lim\limits_{x \to -\infty} \dfrac{\cos(x^2 + 3)}{5x^2 - 2x + 3}$

(c) (5 pts) $\quad \lim\limits_{x \to 0} \dfrac{x + \sin x}{x + \tan x}$

(d) (5 pts) $\quad \lim\limits_{x \to +\infty} x \sin(\tfrac{1}{x})$

Name:

Problem 2 Find the derivative of the following functions.

(a) (5 pts) $f(x) = 15(x^2 - 3)^{1/5}(x + 4)^{-1/3}$

(b) (5 pts) $f(x) = \dfrac{\cos(2x)}{\cos x + \sin x}$

Name:

Problem 3 Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x^2 - 1 & \text{for } x < 1 \\ \tan^2(x^2 - 1) & \text{for } x \geq 1 \end{cases}$$

(a) (8 pts)   Is f continuous at $x = 1$? Why?

b) (12 pts)   Is f differentiable at $x = 1$? Why?

Name:

Problem 4

(a) (10 pts)   State and prove the product rule for differentiation, i.e., the rule that you use to find the derivative of the product of two functions.

(b) (10 pts)   Let $u, v : \mathbb{R} \to \mathbb{R}$ be differentiable functions at x such that $v(x) \neq 0$ and $y = \dfrac{u}{v}$. Use the product rule for differentiation and the equation $v(x)y(x) = u(x)$ to show that $y'(x) = \dfrac{u'(x)v(x) - v'(x)u(x)}{v(x)^2}$, where a prime means a derivative.

Name:

Problem 5 (15 pts) Find the equation for the line in the x-y plane that is tangent to the curve described by

$$x(t) = \frac{t^2 + 2t}{2}, \qquad\qquad y(t) = \frac{t^3 + 3t}{3}$$

at the point corresponding to $t = 0$.

Name:

Problem 6   (15 pts)   Use implicit differentiation to find $\frac{dy}{dx}$ if

$$y^2 \cos\left(\frac{1}{y}\right) = 3x^2 + 2y^3.$$