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KOÇ UNIVERSITY  
MATH 106 - CALCULUS

Midterm I (A)

November 1, 2004

**Duration of Exam: 90 minutes**

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**INSTRUCTIONS:** No calculators may be used on the test. No books, no notes, no questions, and talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS)** and **sign your name**, and indicate your section below. **GOOD LUCK!**

Surname, Name: \_\_\_\_\_

Signature: \_\_\_\_\_

Section (Check One):

Section 1: Prof. Toma Albu \_\_\_\_\_  
Section 2: Prof. Ali Mostafazadeh \_\_\_\_\_  
Section 3: Prof. Tolga Etgü \_\_\_\_\_  
Section 4: Prof. Özlem Keskin \_\_\_\_\_

PROBLEM	POINTS	SCORE
1	20	
2	10	
3	20	
4	20	
5	15	
6	15	
<b>TOTAL</b>	<b>100</b>	

**Name:**

**Problem 1** Calculate the following limit or show that it does not exist:

(a) (5 pts)  $\lim_{x \rightarrow +\infty} \frac{3x^5 - 2x^2 + 11}{-2x^5 + 3x - 17}$

(b) (5 pts)  $\lim_{x \rightarrow -\infty} \frac{\cos(x^2 + 2)}{3x^2 - 11x - 1}$

(c) (5 pts)  $\lim_{x \rightarrow 0} \frac{x - \sin x}{x + \tan x}$

(d) (5 pts)  $\lim_{x \rightarrow -\infty} x \sin\left(\frac{1}{x}\right)$

**Name:**

**Problem 2** Find the derivative of the following functions.

(a) (5 pts)  $f(x) = 15(x^2 - 3)^{1/3}(x + 4)^{-1/5}$

(b) (5 pts)  $f(x) = \frac{\cos(2x)}{\cos x - \sin x}$

**Name:**

**Problem 3** Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be defined by

$$f(x) = \begin{cases} 1 - x^2 & \text{for } x < 1 \\ \tan^2(x^2 - 1) & \text{for } x \geq 1 \end{cases}$$

(a) (8 pts) Is  $f$  continuous at  $x = 1$ ? Why?

b) (12 pts) Is  $f$  differentiable at  $x = 1$ ? Why?

**Name:**

**Problem 4**

(a) (10 pts) State and prove the product rule for differentiation. (This is the rule that you use to find the derivative of the product of two functions.)

(b) (10 pts) Let  $u, v : \mathbb{R} \rightarrow \mathbb{R}$  be differentiable functions at  $x$  such that  $v(x) \neq 0$ , and let  $y = \frac{u}{v}$ . Use the product rule for differentiation and the equation  $v(x)y(x) = u(x)$  to show that  $y'(x) = \frac{u'(x)v(x) - v'(x)u(x)}{v(x)^2}$ .

**Name:**

**Problem 5** (15 pts) Find the equation for the line in the x-y plane that is tangent to the curve described by

$$x(t) = \frac{t^2 - 2t}{2}, \quad y(t) = \frac{t^3 - 3t}{3}$$

at the point corresponding to  $t = 0$ .

**Name:**

**Problem 6** (15 pts) Use implicit differentiation to find  $\frac{dy}{dx}$  if

$$y^2 \sin\left(\frac{1}{y}\right) = 3x^2 + 2y^3.$$