KOÇ UNIVERSITY

MATH 106 - CALCULUS

Final Exam (A) January 13, 2005

Duration of Exam: 135 minutes

INSTRUCTIONS: No calculators may be used on the test. No books, no notes, no questions, and talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below. GOOD LUCK!**

Surname, Name: ______________________________

Student ID no: ______________________________

Signature: ______________________________

Section (Check One):

Section 1: Prof. Toma Albu —
Section 2: Prof. Ali Mostafazadeh —
Section 3: Prof. Tolga Etgü —
Section 4: Prof. Özlem Keskin —

PROBLEM	1	2	3	4	5	6	7	TOTAL
POINTS	25	15	15	10	15	10	10	**100**
SCORE								

Name:

Problem 1 Let $f(x) = \frac{x}{x^3+1}$.

(1.a) **(1 pt)** Find the domain of f.

(1.b) **(4 pts)** Find

(i) $\lim_{x \to -1^+} f(x)$

(ii) $\lim_{x \to -1^-} f(x)$

(iii) $\lim_{x \to +\infty} f(x)$

(iv) $\lim_{x \to -\infty} f(x)$

(1.c) **(5 pts)** Determine the intervals on which f is decreasing and increasing.

(1.d) **(3 pts)** Plot the graph of f.

(1.e) **(12 pts)** Calculate the volume of the solid of revolution of $f(x)$ about the x-axis for $x \in [0, \infty)$.

Name:

Problem 2 (15 pts) A rectangle is to be inscribed in a semicircle of radius 3. What is the largest area the rectangle can have, and what are the lengths of the sides of such rectangle with the largest area?

Name:

Problem 3

(3.a) **(7 pts)** Let $f, g : \mathbb{R} \to \mathbb{R}$ be differentiable functions and df and dg be their differentials. Show that

$$\int f\ dg = f \cdot g - \int g\ df$$

(3.b)**(6 pts)** Find $\int e^{-\sqrt{x}} dx$.

(3.c) **(2 pts)**Evaluate $\int_0^\infty e^{-\sqrt{x}} dx$.

Name:

Problem 4 (10 pts) Evaluate the following integral.

$$\int \frac{1}{1-x^4}\, dx$$

Name:

Problem 5

(5.a) **(10 pts)** Prove or disprove the following statement.

If the series $\sum_{n=0}^{\infty} a_n$ converges, then $\lim\limits_{n\to\infty} a_n = 0$.

(5.b) **(5 pts)** Prove or disprove the following statement.

If $\{a_n\}$ is a sequence with $\lim\limits_{n\to\infty} a_n = 0$, then the series $\sum_{n=0}^{\infty} a_n$ converges .

Name:

Problem 6

(6.a) **(6 pts)** Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(x+2)^n}{3^n \sqrt{n}} \ .$$

(6.b) **(4 pts)** Find the interval of convergence of the power series above.

Name:

Problem 7 (10 pts) Find the Taylor series generated by $f(x) = \ln x$ at $x = 1$.