SAMPLE QUESTIONS (CHAPTER 2)

Section 2.1:

(17) Find the solution of the given initial value problem.

$$y' - 4y = e^{4t}, \quad y(0) = 2$$

$p(t) = -4 \qquad \mu(t) = e^{\int p(t)\,dt} = e^{-4t}$

Multiplying the equation with $\mu(t)$,

$$e^{-4t} \cdot y' - 4e^{-4t} y = 1.$$

$(y \cdot e^{-4t})' = 1$, integrating both sides,

$$y(t) \cdot e^{-4t} = t + c, \quad c \in \mathbb{R}$$

$$y(t) = t \cdot e^{4t} + c \cdot e^{4t}$$

$y(0) = 0 + c = 2$, so we have the solution of the initial value problem,

$$\boxed{y(t) = (t+2)\, e^{4t}}$$

(30) Find the value of y_0 for which the solution of the initial value problem

$$y' - y = 1 + 3\sin t, \quad y(0) = y_0$$

remains finite as $t \to \infty$.

Clearly, $\mu(t) = e^{-t}$

$$(y \cdot e^{-t})' = e^{-t} + 3e^{-t}\sin t \text{ , integrating,}$$

$$y \cdot e^{-t} = -e^{-t} + 3\int e^{-t}\sin t \, dt$$

We use integration by parts twice to calculate the integral,

$$(\star) \quad \int e^{-t}\sin t \, dt = -e^{-t}\sin t + \int e^{-t}\cos t \, dt$$

$u = \sin t \quad dv = e^{-t}dt$

$du = \cos t \, dt \quad v = -e^{-t}$

$$(\star\star) \quad \int e^{-t}\cos t \, dt = -e^{-t}\cos t - \int e^{-t}\sin t \, dt$$

$u = \cos t \quad dv = e^{-t}dt$

$du = -\sin t \, dt \quad v = -e^{-t}$

Subsituting $(\star\star)$ in $(\star)$, we get

$$\int e^{-t}\sin t = -e^{-t}\frac{(\cos t + \sin t)}{2} + c, \quad c \in \mathbb{R}$$

$$y(t).e^{-t} = -e^{-t} - 3e^{-t}\frac{(\cos t + \sin t)}{2} + d, \quad d \in \mathbb{R}$$

$$y(t) = -1 - \frac{3}{2}(\cos t + \sin t) + d.e^{t}$$

Placing $t=0$,

$$y(0) = -1 - \frac{3}{2} + d = y_0$$

$$d = y_0 + 5/2$$

Here is our unique solution to the initial value problem

$$y(t) = -1 - \frac{3}{2}(\cos t + \sin t) + (y_0 + 5/2)\, e^{t},$$

as $t \to \infty$, $\cos t$ & $\sin t$ remain bounded, but e^t diverge to positive infinity. So if we want finite values, we need to eliminate this term, i.e, choose $\boxed{y_0 = -5/2}$.

Section 2.2:

(6) Solve the given differential equation. $xy' = (1-y^2)^{1/2}$

First, we notice that the equation is seperable.

$(1-y^2)^{-1/2}\, dy = 1/x\, dx$, integrating both sides,

$\arcsin y = \ln x + c, \quad c \in \mathbb{R}$

So that $y(x) = \sin(\ln x + c), \quad c \in \mathbb{R}$

(19) $\sin 2x\, dx + \cos 3y\, dy = 0$, $y(\pi/2) = \pi/3$

$\cos 3y\, dy = -\sin 2x\, dx$, the equation is seperable,

integrating both sides,

$$\frac{\sin 3y}{3} = \frac{\cos 2x}{2} + c, \qquad c \in \mathbb{R}$$

Using the initial condition,

$$\frac{\sin \pi}{3} = \frac{\cos \pi}{2} + c \Rightarrow c = \frac{1}{2}$$

$$\sin 3y = \frac{3}{2}\cos 2x + \frac{3}{2}$$

$$y(x) = \frac{\arcsin\left(\frac{3}{2}(\cos 2x + 1)\right)}{3}.$$

(29) Solve the equation

$$\frac{dy}{dx} = \frac{ay+b}{cy+d},$$

where a, b, c, d are constants.

See that we have seperability,

$$\frac{cy+d}{ay+b}\,dy = 1.\,dx$$

$$\left(\frac{cy}{ay+b} + \frac{d}{ay+b}\right) dy = 1\,dx,$$ integrating both sides,

$$\frac{c}{a^2}\cdot\left(ay - b.\ln(ay+b)\right) + \frac{d}{a}\ln(ay+b) = x + r, \quad r\in\mathbb{R}$$

Hence we have the implicit form of the solution.

Section 2.4:

(15) Solve the given initial value problem and determine how the interval in which the solution exists depends on the initial value y_0.

$$y' + y^3 = 0, \qquad y(0) = y_0$$

———— ∘ ————

$$\frac{dy}{dx} = -y^3 \qquad -y^{-3}\,dy = 1.dx$$

$$\frac{1}{2}y^{-2} = x + c, \quad c\in\mathbb{R}$$

$$\frac{1}{2x+d} = y^2, \quad d\in\mathbb{R},$$

Putting the initial condition, $d = \frac{1}{y_0^2}$, moreover

$$y(x) = \frac{y_0}{\sqrt{2y_0^2 x + 1}}$$, and it exists as long as

$$2y_0^2 x + 1 > 0$$

$$2y_0^2 x > -1$$

If $y_0 = 0 \Rightarrow$ Solution exists for all $x \in \mathbb{R}$. (It's $y(x) = 0$.)

If $y_0 \neq 0 \Rightarrow$ Solution exists for all $x > \frac{-1}{2y_0^2}$

(27) Bernoulli Equations: $y' + p(t)y = q(t)y^n$, $n \in \mathbb{N}$

(a) Solve Bernoulli's equation when $n=0$; when $n=1$.

$n = 0 \Rightarrow y' + p(t)y = q(t)$, i.e., usual linear 1st order O.D.E.

$n = 1 \Rightarrow y' + (p(t) - q(t))y = 0$, i.e, usual linear 1st order O.D.E.

(b) Show that if $n \neq 0, 1$, then the subsitution $v = y^{1-n}$ reduces Bernoulli's equation to a linear equation. (Leibniz, 1696)

$$v = y^{1-n} \Rightarrow v' = (1-n) \cdot y^{-n} \cdot y' \Rightarrow y' = \frac{v' \cdot y^n}{1-n}$$

$$v = y^{1-n} \Rightarrow y = v \cdot y^n$$

Substituting, we have

$$y' + p(t) y = q(t) y^n$$

$$\frac{v' \cdot y^n}{(1-n)} + p(t) \cdot v \cdot y^n = q(t) \cdot y^n$$, or, equivalently,

$$v' + (1-n) \cdot p(t) \cdot v = (1-n) q(t),$$ a first order linear equation with respect to v.

(30) Solve the Bernoulli equation:

$$y' = \varepsilon y - \sigma y^3, \quad \varepsilon > 0, \; \sigma > 0.$$

$$y' - \varepsilon y = -\sigma y^3 \quad , \quad n = 3, \text{ so let } v = y^{1-n} = y^{-2}$$

$$v' = -2 \cdot y^{-3} \cdot y' \quad \Rightarrow \quad y' = \frac{-v' \cdot y^3}{2} \qquad y = v \cdot y^3$$

Substituting,

$$\frac{-v' \cdot y^3}{2} - \varepsilon \cdot v \cdot y^3 = -\sigma y^3$$, or, equivalently,

$$v' + 2\varepsilon v = 2\sigma \quad , \quad \mu(t) = e^{2\varepsilon t}$$

$$\left(v \cdot e^{2\varepsilon t} \right)' = e^{2\varepsilon t} \, 2\sigma$$, integrating both sides

$$v \cdot e^{2\varepsilon t} = \frac{\sigma}{\varepsilon} \cdot e^{2\varepsilon t} + c, \quad c \in \mathbb{R}$$

$$v(t) = \frac{\sigma}{\varepsilon} + c \cdot e^{-2\varepsilon t}$$ Recall that $v = y^{-2}$, i.e,

$$v(t) = \frac{\sigma \cdot e^{2\varepsilon t} + \varepsilon \cdot c}{\varepsilon \cdot e^{2\varepsilon t}} \qquad y = \sqrt{1/v}$$

So $$y(t) = \sqrt{\frac{\varepsilon \cdot e^{2\varepsilon t}}{\sigma \cdot e^{2\varepsilon t} + \varepsilon \cdot c}}$$, where $\varepsilon, \sigma > 0$, $c \in \mathbb{R}$.

Section 2.6:

(11) Solve the following equation.

$$(y/x + 6x)\,dx + (\ln x - 2)\,dy = 0, \quad x > 0$$

Let $M(x,y) = y/x + 6x \quad N(x,y) = \ln x - 2$

$$\frac{\partial M}{\partial y} = \frac{1}{x} = \frac{\partial N}{\partial x}$$, so the equation is exact.

There exists a function $\psi(x,y)$ such that $\frac{\partial \psi}{\partial x} = M$, $\frac{\partial \psi}{\partial y} = N$ and $\psi = c$, $c \in \mathbb{R}$ gives the solution.

$$\psi = \int M\,dx = y \cdot \ln x + 3x^2 + g(y)$$, g is a fn. of y.

$$\frac{\partial \psi}{\partial y} = \ln x + g'(y) = N = \ln x - 2$$ So $g(y) = -2y + d$, $d \in \mathbb{R}$

Try

$$\frac{d\mu}{dx} = \underbrace{\frac{(x+2)\cos y - \cos y}{x \cdot \cos y}}_{\frac{x+1}{x}} \mu$$

Solve the 1^{st} order o.d.e,

$$\mu' - \left(\frac{x+1}{x}\right)\mu = 0 \quad , \text{ the integrating factor}, \lambda,$$

$$\lambda = e^{-\int \frac{x+1}{x} dx} = e^{-\int 1\, dx - \int \frac{1}{x} dx} = x^{-1} e^{-x} = \frac{1}{x e^x}$$

$$\left(\mu \cdot \frac{1}{x \cdot e^x}\right)' = 0 \qquad \boxed{\mu(x) = x \cdot e^x} \quad \left(\begin{array}{c}\text{without loss} \\ \text{of generality} \\ \text{choose } c = 1\end{array}\right.$$

Multiplying with μ, the equation becomes

$$\underbrace{x(x+2)\, e^x \sin y}_{M(x,y)}\, dx + \underbrace{x^2 e^x \cos y}_{N(x,y)}\, dy = 0$$

$$M_y = (x^2 + 2x) e^x \cos y = N_x = (x^2 e^x + 2x e^x) \cos y$$

Equation is exact now. So there is such Ψ.

$\Psi = \int N dy = x^2 e^x \sin y + g(x)$, g a fn. of only x.

$\Psi_x = (x^2 + 2x) e^x \cdot \sin y + g'(x)$, so $g'(x) = 0 \Rightarrow g(x) = c,\ c \in \mathbb{R}$

Thus we have the solution given by

Hence we have

$$\Psi(x,y) = y \cdot \ln x + 3x^2 - 2y + d, \quad d \in \mathbb{R}$$

and we have

$$y \cdot \ln x + 3x^2 - 2y = c, \quad c \in \mathbb{R}$$

$$y(\ln x - 2) = c - 3x^2$$

$$\boxed{y(x) = \frac{c - 3x^2}{\ln x - 2}}$$

(22) Solve the following equation.

$$(x+2)\sin y \, dx + x \cdot \cos y \, dy = 0$$

Let $M(x,y) = (x+2)\sin y$ $\quad N(x,y) = x \cdot \cos y$

$$\frac{\partial M}{\partial y} = (x+2)\cos y \neq \frac{\partial N}{\partial x} = \cos y \text{ , not exact.}$$

We will try to find an integrating factor to make this exact. If the integrating factor will depend only on x, it will satisfy

$$\frac{d\mu}{dx} = \frac{M_y - N_x}{N}\mu,$$

if it will depend only on y, it will satisfy

$$\frac{d\mu}{dy} = \frac{N_x - M_y}{M}\mu.$$

$$x^2 e^x \cdot \sin y = d, \quad d \in \mathbb{R}$$

$$\sin y = d x^{-2} e^{-x}$$

$$\boxed{y(x) = \arcsin\left(\frac{d}{x^2 e^x}\right), \quad d \in \mathbb{R}}$$

is the solution.

SAMPLE QUESTIONS (CHAPTER 3)

Section 3.1:

(15) Find the solution of the given initial value problem, and describe its behavior as t increases.

$$y'' + 8y' - 9y = 0 \quad , \quad y(1) = 1, \quad y'(1) = 0.$$

—— o ——

$$y = e^{rt} \quad , \quad y' = r.e^{rt} \quad , \quad y'' = r^2 e^{rt}$$

Placing in the equation, we obtain

$$r^2 e^{rt} + 8r e^{rt} - 9e^{rt} = 0 \quad , \text{ i.e,}$$

$$e^{rt} \left(r^2 + 8r - 9 \right) = 0$$

e^{rt} is nonzero for any $t \in \mathbb{R}$, so we obtain the characteristic equation:

$$r^2 + 8r - 9 = 0$$

$$\begin{matrix} r & +9 \\ r & -1 \end{matrix}$$

two roots:

$$(r-1)(r+9) = 0 \quad \begin{cases} r_1 = 1 \\ r_2 = -9 \end{cases}$$

So we have two solutions:

$$y_1(t) = e^t \quad , \quad y_2(t) = e^{-9t}$$

Solutions have the general form:

$$y(t) = c_1 . e^{t} + c_2 . e^{-9t}$$

But we have initial conditions, so there'll be a unique solution.

$$y(1) = c_1 . e^{1} + c_2 . e^{-9} = 1$$

$$y'(t) = c_1 e^{t} - 9 c_2 e^{-9t}$$

$$y'(1) = c_1 . e^{1} - 9 c_2 e^{-9} = 0$$

So $10 c_2 e^{-9} = 1 \Rightarrow c_2 = \frac{e^9}{10}$

and $c_1 = \frac{9}{10e}$

So that

$$y(t) = \frac{9}{10} e^{t-1} + \frac{1}{10} e^{9(1-t)}$$

As t increases, second part of the function will get closer to 0, and the first part will get bigger and bigger. As $t \to \infty$, $y(t)$ diverges.

Section 3.2:

(11) Determine the longest interval in which the given initial value problem is certain to have a unique twice differentiable solution.

$$(x-3)\,y'' + xy' + (\ln|x|)\,y = 0, \quad y(1) = 0, \; y'(1) = 1.$$

——— ∘ ———

First we write the equation in the form:

$$y'' + \frac{x}{x-3}\,y' + \frac{\ln|x|}{x-3}\,y = 0$$

See that $\ln|x|$ does not exist for $x = 0$, also for $x = 3$, we have discontinuity for coefficient functions.

As the initial condition point, $t_0 = 1$ is in the interval $(0,3)$, we choose that interval, for the unique, twice differentiable solution.

(26) Verify that the functions y_1 and y_2 are solutions of the given differential equation. Do they constitute a fundamental set of solutions?

$$x^2 y'' - x(x+2)\,y' + (x+2)\,y = 0, \quad x > 0;$$

$$y_1(x) = x, \qquad y_2(x) = x e^x$$

——— ∘ ———

$y_1(x) = x$

$y_1'(x) = 1$

$y_1''(x) = 0$

Subsituting,

$x^2 . 0 - x(x+2) . 1 + (x+2)x = -x^2 - 2x$

$+ x^2 + 2x = 0$ ✓ (y_1 is a solution)

$y_2(x) = xe^x$

$y_2'(x) = e^x + xe^x$

$y_2''(x) = 2e^x + xe^x$

Subsituting,

$2x^2e^x + x^3e^x - (x^2+2x)(e^x + xe^x) +$

$(x+2)xe^x = 2x^2e^x + x^3e^x - x^2e^x - x^3e^x - 2xe^x$

$- 2x^2e^x + x^2e^x + 2xe^x = 0$

✓ (y_2 is a solution)

Now we'll check their Wronskian,

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$$

$$= x(e^x + xe^x) - x . e^x . 1$$

$$= xe^x + x^2e^x - xe^x$$

$$= x^2 e^x$$

For $x > 0$, $W(x) = x^2e^x \neq 0$. So we have

y_1 & y_2 constituting a fundamental set of

solutions.

Section 3.3:

19. Find the solution of the given initial value problem.

$$y'' - 6y' + 13y = 0, \quad y(\pi/2) = 0, \quad y'(\pi/2) = 2.$$

The equation above has the characteristic equation:

$r^2 - 6r + 13 = 0$, which has the roots:

$$r_{1,2} = \frac{6 \mp \sqrt{36 - 4.13.1}}{2} = 3 \mp 2i$$

$$r_1 = 3 + 2i \qquad r_2 = 3 - 2i$$

See that $\lambda = 3$, $\mu = 2$.

$$y(t) = c_1 . e^{3t} . \cos 2t + c_2 . e^{3t} \sin 2t$$

$$y(\pi/2) = c_1 . e^{3\pi/2} . \cos\pi + c_2 . e^{3\pi/2} \sin\pi$$

$$-c_1 e^{3\pi/2} = 0, \quad \boxed{c_1 = 0}$$

$$y'(t) = c_2 (3e^{3t} \sin 2t + 2e^{3t} \cos 2t)$$

$$y'(\pi/2) = c_2 . (3e^{3\pi/2} \sin\pi + 2e^{3\pi/2} \cos\pi)$$

$$-2 \cdot c_2 \cdot e^{3\pi/2} = 2$$

$$\boxed{c_2 = -e^{-3\pi/2}}$$

So that

$$y(t) = e^{3t - 3\pi/2} \cdot \sin 2t$$

Section 3.4:

(13) Solve the given initial value problem.

$$9y'' - 12y' + 4y = 0, \quad y(0) = 2, \quad y'(0) = -1$$

See that the characteristic equation is

$$9r^2 - 12r + 4 = 0$$

$$\begin{array}{cc} 3r & -2 \\ 3r & -2 \end{array}$$

and we have

$$r_1 = r_2 = \frac{2}{3}$$

Thus the general solution

$$y(t) = c_1 \cdot e^{2t/3} + c_2 \cdot t \cdot e^{2t/3}$$

$$y(0) = c_1 + c_2 \cdot 0 = 2 \quad , \text{ i.e, } \quad c_1 = 2$$

$$y'(t) = \frac{2c_1}{3} e^{2t/3} + c_2 \left(e^{2t/3} + \frac{2t}{3} e^{2t/3} \right)$$

$$y'(0) = \frac{2c_1}{3} + c_2 = -1 \text{ , i.e, } c_2 = \frac{-7}{3}$$

And the unique solution to the initial value problem is:

$$\boxed{y(t) = 2e^{2t/3} - \frac{7}{3} t . e^{2t/3}}$$

(26) Use the method of reduction of order to find a second solution of the given differential equation.

$$t^2 . y'' - t(t+2) y' + (t+2) y = 0 \quad , t > 0;$$

and $\quad y_1(t) = t.$

First, we set $\quad y_2(t) = v(t) . t .$

$$y_2'(t) = v(t) + t . v'(t)$$

$$y_2''(t) = 2v'(t) + t . v''(t)$$

Substituting ,

$$2t^2 v'(t) + t^3 v''(t) - t^2 v(t) - t^3 v'(t) - 2t v(t) - 2t^2 v'(t)$$

$$+ t^2 v(t) + 2v(t) t = 0$$

and that leads to

$$t^3 v''(t) - t^3 v'(t) = 0, \quad \text{or, equivalently,}$$

$$v''(t) - v'(t) = 0 \quad (\text{recall that } t>0).$$

Let $w = v'$, then

$$w' - w = 0$$

this has the immediate solution $w(t) = e^t$

so that, again, $v(t) = e^t$.

Therefore $y_2(t) = t \cdot v(t) = t \cdot e^t$

Section 3.5:

Find the solution of the given initial value problem.

13) $y'' + y' - 2y = 2t, \quad y(0) = 0, \quad y'(0) = 1$.

First, we solve the homogenous equation,

$$r^2 + r - 2 = 0 \quad = (r+2)(r-1)$$

$$r_1 = 1 \quad r_2 = -2$$

$$y_c(t) = c_1 \cdot e^t + c_2\, e^{-2t} \quad (\text{complementary solution})$$

Set $Y(t) = At + B$

$Y'(t) = A$

$Y''(t) = 0$

Subsituting,

$$0 + A - 2(At + B) = 2t$$

$$(-2A)t + (A - 2B) = 2t$$

$$A = -1$$

$$B = -1/2$$

$$Y(t) = -t - 1/2$$

General solution of the nonhomogenous equation

$$y(t) = c_1 e^t + c_2 e^{-2t} - t - 1/2$$

$$y'(t) = c_1 e^t - 2c_2 e^{-2t} - 1$$

$$y(0) = c_1 + c_2 - 1/2 = 0 \Rightarrow c_1 + c_2 = 1/2$$

$$y'(0) = c_1 - 2c_2 - 1 = 1 \Rightarrow c_1 - 2c_2 = 2$$

$$c_2 = -1/2$$

$$c_1 = 1$$

So, the solution of the initial value problem

$$\boxed{y(t) = e^t - \frac{1}{2} e^{-2t} - t - 1/2}$$

(14) $y'' - 2y' + y = te^t + 4, \quad y(0) = 1, \quad y'(0) = 1$

$r^2 - 2r + 1 = 0 \qquad r_1 = r_2 = 1$

$(r-1)(r-1) = 0$

So we have the solutions

$y_1(t) = e^t \qquad y_2(t) = te^t$

te^t is a solution for the homogerous equation, so we set:

$Y_1(t) = At^3 e^t + Bt^2 e^t$

$Y_1'(t) = 3At^2 e^t + At^3 e^t + 2Bt e^t + Bt^2 e^t$

$Y_1''(t) = 6Ate^t + 6At^2e^t + At^3e^t + 2Be^t + 4Bte^t + Bt^2e^t$

Substituting,

$At^3e^t + 6At^2e^t + Bt^2e^t + 6Ate^t + 4Bte^t + 2Be^t - 6At^2e^t - 2At^3e^t$
$- 4Bte^t - 2Bt^2e^t + At^3e^t + Bt^2e^t = te^t$

$6Ate^t + 2Be^t = t.e^t$

$A = 1/6 \quad B = 0$

$Y_1(t) = \dfrac{t^3e^t}{6}$

$$\left.\begin{aligned} Y_2(t) &= A, \quad A \in \mathbb{R} \\ Y_2'(t) &= 0 \\ Y_2''(t) &= 0 \end{aligned}\right\} \text{ Subsituting,}$$

$$0 - 2.0 + A = 4$$

$$A = 4$$

$$Y_2(t) = 4, \quad \text{therefore}$$

$$Y(t) = \frac{t^3 e^t}{6} + 4$$

Thus

$$y(t) = c_1 e^t + c_2 t e^t + \frac{t^3 e^t}{6} + 4$$

$$y(0) = c_1 + 4 = 1 \quad \Rightarrow \quad c_1 = -3$$

$$y'(t) = c_1 e^t + c_2 e^t + c_2 t e^t + \frac{t^2 e^t}{2} + \frac{t^3 e^t}{6}$$

$$y'(0) = c_1 + c_2 = 1$$

$$\Rightarrow \quad c_2 = 4$$

So the solution of the initial value problem:

$$\boxed{y(t) = -3e^t + 4t e^t + \frac{t^3 e^t}{6} + 4}$$

(15) $y'' + 4y = t^2 + 3e^t, \quad y(0) = 0, \quad y'(0) = 2$

$r^2 + 4 = 0 \qquad r_{1,2} = 0 \mp 2i$

$r^2 = -4$

$\lambda = 0 \qquad \mu = 2$

$y_c(t) = C_1 . \cos 2t + C_2 . \sin 2t$

$$\left.\begin{array}{l} Y_1(t) = At^2 + Bt + C \\ Y_1'(t) = 2At + B \\ Y_1''(t) = 2A \end{array}\right\} \qquad 2A + 4At^2 + 4Bt + 4C = t^2$$

$B = 0 \qquad A = 1/4 \qquad C = -1/8$

$Y_1(t) = t^2/4 - 1/8$

$Y_2(t) = de^t = Y_2'(t) = Y_2''(t)$

$de^t + 4d\,e^t = 5de^t = 3e^t \Rightarrow d = \frac{3}{5}$

$Y_2(t) = \frac{3}{5}e^t$

So the general solution of the nonhomogeneous equation,

$y(t) = C_1 \cos 2t + C_2 \sin 2t + 3e^t/5 + t^2/4 - 1/8$

$y(0) = C_1 + \frac{3}{5} - \frac{1}{8} = 0 \Rightarrow \boxed{C_1 = \frac{-19}{40}}$

$y'(t) = -2C_1 \sin 2t + 2C_2 \cos 2t + 3e^t/5 + t/2$

$y'(0) = 2C_2 + 3/5 = 2 \Rightarrow \boxed{C_2 = \frac{7}{10}}$

Hence $y(t) = \frac{-19}{40} \cos 2t + \frac{7}{10} \sin 2t + \frac{3}{5}e^t + \frac{t^2}{4} - \frac{1}{8}$

(17) $y'' + 4y = 2\sin 2t, \qquad y(0) = 2, \quad y'(0) = -1$

$r^2 + 4 = 0 \Rightarrow y_c(t) = C_1 \cos 2t + C_2 \sin 2t$

Let $Y(t) = A \cos 2t + B \sin 2t$ won't work, as these two are solutions of the homogenous equation.

Set $Y(t) = At \cos 2t + Bt \sin 2t$

$Y'(t) = -2At \sin 2t + A \cos 2t + 2Bt \cos 2t + B \sin 2t$

$Y''(t) = -4At \cos 2t - 4A \sin 2t - 4Bt \sin 2t + 4B \cos 2t$

$-4At \cos 2t - 4Bt \sin 2t - 4A \sin 2t + 4B \cos 2t + 4At \cos 2t + 4Bt \sin 2t = 2 \sin 2t$

$-4A \sin 2t + 4B \cos 2t = 2 \sin 2t$

$B = 0$

$A = -1/2$

So $Y(t) = -t \cos 2t / 2$

$y(t) = C_1 \cos 2t + C_2 \sin 2t - t \cos 2t / 2$

$C_1 = 2 \qquad C_2 = -1/4$

$$\boxed{y(t) = 2\cos 2t - \sin 2t / 4 - t \cdot \cos 2t / 2}$$