COMPLEX 6

(15) Let $f(z) = (z-1)(z-4)^2$.

Find lines thru $z=2$ on which $|f(z)|$ has a relat. max or relat. min at $z=2 = 2+0i$

$y = mx + b$ and $0 = 2m + b$ $\Rightarrow$ $\boxed{y = m(x-2)}$ $\qquad z = (x, y)$

$|f(z)| = |z-1|\,|z-4|^2$ and $z = x + m(x-2)i$ here

$$\Rightarrow \quad |f(z)| = \left(\sqrt{(x-1)^2 + m^2(x-2)^2}\right) \cdot \left[(x-4)^2 + m^2(x-2)^2\right]$$

We want: $\boxed{\frac{d|f|}{dx} = 0 \quad \text{at} \quad x = 2}$

$$\frac{d|f|}{dx} = \frac{1}{2} \frac{2(x-1) + 2m^2(x-2)}{\sqrt{(x-1)^2 + m^2(x-2)^2}} \left[(x-4)^2 + m^2(x-2)^2\right] + \sqrt{(x-1)^2 + m^2(x-2)^2} \cdot \left[2(x-4) + 2m^2(x-2)\right]$$

$$\left.\frac{d|f|}{dx}\right|_{x=2} = \frac{1}{\cancel{2}} \frac{\cancel{2}}{1} \cdot [4] + 1 \cdot [4] = 5 \neq 0 \quad \Longrightarrow \quad \boxed{m \notin \mathbb{R}}$$

The only remaining possibility is the line $x = 2$. Try this. $z = 2 + yi$ now.

$$|f| = \sqrt{1^2 + y^2} \cdot (2^2 + y^2)$$

$$\frac{d|f|}{dy} = \frac{1}{\cancel{2}} \frac{\cancel{2}y}{\sqrt{y^2+1}} (4 + y^2) + \sqrt{1+y^2}\, 2y \quad . \text{ See if } \left.\frac{d|f|}{dy}\right|_{y=0} = 0.$$

$$\left.\frac{d|f|}{dy}\right|_{y=0} = 0 \quad \checkmark$$

It's obvious that $|f|$ has a global min. at $y = 0$.

$\Longrightarrow$ $|f(z)|$ has a relative <u>min</u> on the line <u>$x=2$</u> at $z = 2$, and does not have a relat. min / max at $z=2$ on any other line passing through $z = 2$.