Chapter 5

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Chapter 5

5.1-1 $\omega_c = 10^8$, $k_f = 10^5$, $k_p = 25$

FM: The instantaneous frequency is $\omega_i = 10^8 + 10^5 m(t)$, $(\omega_i)_{\text{min}} = 10^8 - 10^5 = 9.99 \times 10^7$ rad/s, $(\omega_i)_{\text{max}} = 10^8 + 10^5 = 1.001 \times 10^8$ rad/s. Figure S5.1-1 shows that the cycle is split into four equal parts of length $10^{-3}/4 = 2.5 \times 10^{-4}$ second. The instantaneous frequency increases linearly from $(\omega_i)_{\text{min}}$ to $(\omega_i)_{\text{max}}$, stays there, then decreases linearly back to $(\omega_i)_{\text{min}}$, and stays there for the last quarter-cycle.

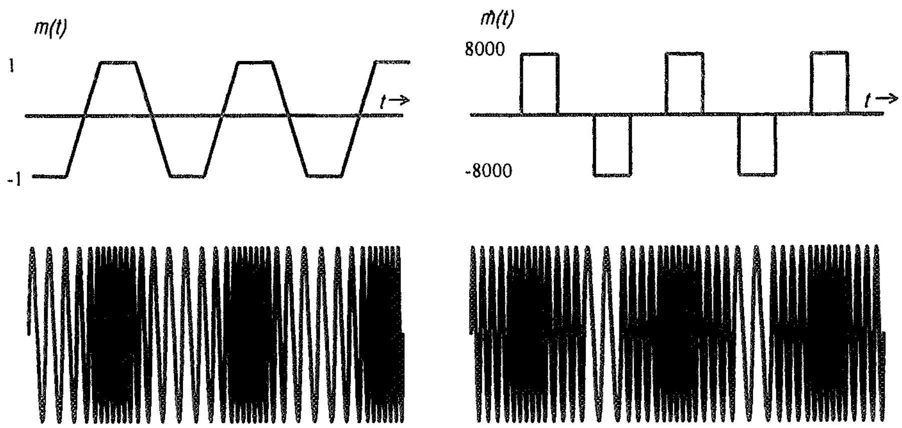

Fig. S5.1-1

PM: In this case $\omega_i = 10^8 + 25\dot{m}(t)$ with $(\dot{m}(t))_{\text{max}} = 8000$, $(\dot{m}(t))_{\text{min}} = -8000$, $(\omega_i)_{\text{min}} = 10^8 - 2 \times 10^5 = 9.98 \times 10^7$ rad/s, $(\omega_i)_{\text{max}} = 10^8 + 2 \times 10^5 = 1.002 \times 10^8$ rad/s. Figure S5.1-1 shows that the waveform in this case stays at $(\omega_i)_{\text{max}}$ for one quarter-cycle, shifts to ω_c, shifts to $(\omega_i)_{\text{min}}$, and then shifts back to ω_c for the last quarter cycle.

5.1-2

(a) $\omega_c = 2\pi \times 10^6$, $k_f = 2000\pi$, $k_p = \pi/2$.

FM: The instantaneous frequency is $f_i = 10^6 + 1000m(t)$, $(f_i)_{\text{min}} = 10^6 - 1000 = 999$ kHz, $(f_i)_{\text{max}} = 10^6 + 1000 = 1001$ kHz. As shown in Fig. S5.1-2 the instantaneous frequency of the FM wave increases linearly from 999 to 1001 kHz over 10^{-3} s, then switches back to 999 kHz and repeats.

PM: Since $m(t)$ has jump discontinuities, the direct approach will be used. When one cycle of the sawtooth is centered on the origin, $m(t) = 2000t$ over that cycle. Hence,

$$
\begin{aligned}
\varphi_{\text{PM}}(t) &= \cos\left[2\pi \times 10^6 t + \frac{\pi}{2} m(t)\right] \\
&= \cos\left[2\pi \times 10^6 t + \frac{\pi}{2} 2000t\right] \\
&= \cos\left[2\pi \left(10^6 + 500\right) t\right].
\end{aligned}
$$

As shown in Fig. S5.1-2, at the discontinuity there is a jump of $2k_p = \pi$, otherwise the carrier frequency is constant at $10^6 + 500$ Hz.

(b) This is equivalent to another PM signal with $f_c = 1000.5$ kHz and periodic rectangular message that switches from 1 to -1, as shown in the example at the beginning of the chapter. It is necessary to keep k_p less than π since those periodic signal jumps at those points of discontinuity $\Delta = 2$; otherwise a larger k_p will give rise to phase ambiguity when $k_p\Delta > 2\pi$.

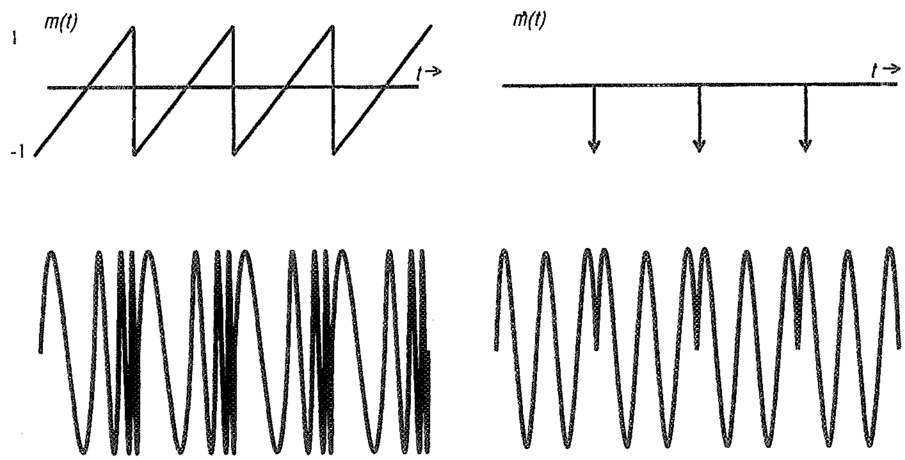

Fig. S5.1-2

5.1-3 $\omega_c = 2\pi \times 10^3$ rad/s.

(a) $k_f = 20\pi$, $f_i = 10^3 + 10m(t)$, $(f_i)_{\min} = 10^3 - 10(3) = 970$ Hz, and $(f_i)_{\max} = 10^3 + 10(3) = 1030$ Hz. As shown in Fig. S5.1-3, the cycle of the message is split into five equal parts of length 0.05 s. Starting from the origin, the instantaneous frequency decreases linearly from $(f_i)_{\max}$ to $(f_i)_{\min}$ over parts 1 and 2. Then f_i increases linearly from $(f_i)_{\min}$ to f_c over part 3, remains there for part 4, and then increases linearly again back to $(f_i)_{\max}$ over part 5.

(b) $k_p = \pi/2$. Since there are no jump discontinuities, $f_i = 10^3 + \dot{m}(t)/4$ Hz. Additionally, the signal has $\dot{m}(t)$ which takes on the values of zero, $(\dot{m}(t))_{\max} = 60$, and $(\dot{m}(t))_{\min} = -60$. Thus, f_i takes on the values of $f_c = 1$ kHz, $(f_i)_{\max} = 10^3 + 15 = 1015$ Hz, and $(f_i)_{\min} = 10^3 - 15 = 985$ Hz. As shown in Fig. S5.1-3, the cycle of the message is split into five equal parts of length 0.05 s. Starting from the origin, the instantaneous frequency remains at $(f_i)_{\min}$ for parts 1 and 2, switches to $(f_i)_{\max}$ for part 3, switches to f_c for part 4, and then switches to $(f_i)_{\max}$ for part 5.

5.1-4 We are given $\omega_c = 10000\pi$, and that over $|t| \le 1$,

$$\varphi_{\rm EM}(t) = 10 \cos 13,000\pi t$$

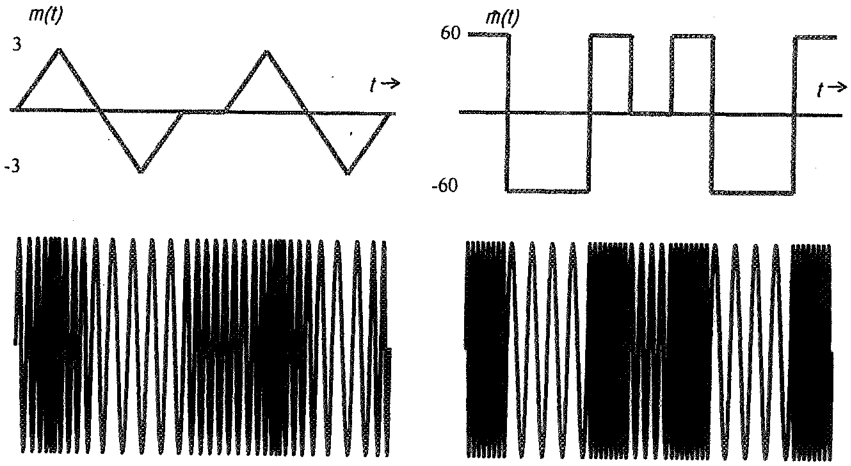

Fig. S5.1-3

(a) If this were a PM signal with $k_p = 1000$, we would have

$$\begin{aligned}\varphi_{PM}(t) &= 10\cos 13000\pi t = 10\cos\left[\omega_c t + k_p m(t)\right] \\ &= 10\cos\left[10000\pi t + 1000 m(t)\right]\end{aligned}$$

Clearly, $m(t) = 3\pi t$ over this interval.

(b) For an FM signal with $k_f = 1000$,

$$\varphi_{FM}(t) = A\cos\left[\omega_c t + k_f\int_0^t m(\alpha)\,d\alpha\right] = 10\cos\left[10,000\pi t + 1000\int_0^t m(\alpha)d\alpha\right].$$

Therefore,

$$\int_0^t m(\alpha)d\alpha = 3\pi t$$

and $m(t) = 3\pi$ over the interval.

5.1-5 $\omega_c = 4\pi \times 10^3$ rad/s.

(a) For an FM signal with $k_f = 500\pi$, $f_i = 2\times 10^3 + 250m(t)$. $(f_i)_{\text{min}} = 2000 - 1000 = 1$ kHz, $(f_i)_{\text{max}} = 2000 + 1000 = 3$ kHz. As shown in Fig. S5.1-5, the cycle of the periodic message consists of four segments of length 0.01 s. In the first segment, the instantaneous frequency of the FM wave increases linearly from $(f_i)_{\text{min}}$ to $(f_i)_{\text{max}}$. Next, it remains there for segment 2. Over segments 3 and 4, the instantaneous frequency of the FM wave decreases linearly from $(f_i)_{\text{max}}$ to $(f_i)_{\text{min}}$, and the cycle repeats.

(b) For a PM signal with $k_p = 0.25\pi$, $f_i = 2\times 10^3 + \dot{m}(t)/8$. In this case, $\dot{m}(t)$ takes on the values of zero, $(\dot{m}(t))_{\text{max}} = 8/(0.01) = 800$, and $(\dot{m}(t))_{\text{min}} = -8/(0.02) = -400$. So $(f_i)_{\text{min}} = 2000 - 50 = 1.95$ kHz and $(f_i)_{\text{max}} = 2000 + 80 = 2.08$ kHz. Again, as shown in Fig. S5.1-5, the cycle of the periodic message consists of 4 segments of length 0.01 s. In the first segment the instantaneous frequency of the PM wave is $(f_i)_{\text{max}}$ and next it switches to f_c for segment 2. For segments 3 and 4, the instantaneous frequency of the PM wave switches to $(f_i)_{\text{min}}$, and the cycle repeats.

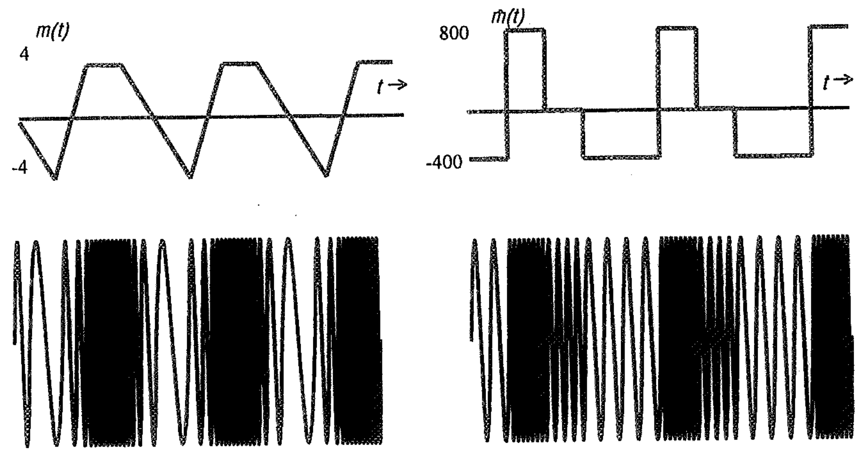

Fig. S5.1-5

5.2-1 The signal bandwidth of $m(t)$ is to be approximated by its own fifth harmonic frequency, $B = 5/(0.25) = 20$ Hz.

(a) $k_f = 20\pi$, $m_p = 3$, $\Delta f = k_f m_p/(2\pi) = 30$ Hz. Therefore, $B_{FM} = 2(\Delta f + B) = 2(30 + 20) = 100$ Hz.

(b) $k_p = \pi/2$, $\dot{m}_p = 3/0.05 = 60$, $\Delta f = k_p \dot{m}_p/(2\pi) = 15$ Hz. Therefore, $B_{\text{PM}} = 2(\Delta f + B) = 2(15 + 20) = 70$ Hz.

5.2-2 The signal bandwidth of $m(t)$ is to be approximated as $B = 200$ Hz.

(a) $k_f = 500\pi$, $m_p = 4$, $\Delta f = k_f m_p/(2\pi) = 1000$ Hz. Therefore, $B_{FM} = 2(\Delta f + B) = 2(1000 + 200) = 2400$ Hz.

(b) $k_p = \pi/4$. In this case, $(\dot{m}(t))_{\max} = 8/(0.01) = 800$, and $(\dot{m}(t))_{\min} = -8/(0.02) = -400$.

$$\Delta f = \frac{k_p}{2\pi} \frac{(\dot{m}(t))_{\max} - (\dot{m}(t))_{\min}}{2} = 600/8 = 75 \text{ Hz}$$

Therefore, $B_{\text{PM}} = 2(\Delta f + B) = 2(75 + 200) = 550$ Hz.

5.2-3 $m(t) = 2\cos 1000t + 9\cos 2000\pi t$ so $B = 1$ kHz.

(a) $A = 10$, $\omega_c = 10^6$, $k_f = 1000\pi$, $k_p = 1$. Therefore,

$$\begin{aligned}\varphi_{\text{PM}}(t) =& A\cos(\omega_c t + k_p m(t)) \\ =& 10\cos((10^6)t + 2\cos 1000t + 9\cos 2000\pi t)\end{aligned}$$

$\int_{-\infty}^{t} m(\alpha)d\alpha = 2\sin 1000t/1000 + 9\sin 2000\pi t/(2000\pi)$, so

$$\begin{aligned}\varphi_{\text{FM}}(t) =& A\cos\left(\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha\right)\\ =& 10\cos((10^6)t + 2\pi\sin 1000t + 4.5\sin 2000\pi t)\end{aligned}$$

(b) **PM:** $\dot{m}(t) = -2000\sin 1000t - 18000\pi\sin 2000\pi t$, $\dot{m}_p = 2000 + 18000\pi$,

$$\Delta f = \frac{k_p \dot{m}_p}{2\pi} = 1000/\pi + 9000 = 9.32 \text{ kHz}$$

Hence, $B_{\text{PM}} = 2(\Delta f + B) = 2(9.32 + 1) = 20.64$ kHz

FM: $m_p = 11$, $\Delta f = k_f m_p/(2\pi) = (500)(11) = 5.5$ kHz, and $B_{\text{FM}} = 2(\Delta f + B) = 2(5.5 + 1) = 13$ kHz.

5.2-4 $\omega_c = 2\pi \times 10^6$, $\varphi_{\text{EM}}(t) = 10\cos(\omega_c t + 0.1\sin 2000\pi t)$, $B = 1$ kHz.

(a) $P = 10^2/2 = 50$

(b) $\theta(t) = \omega_c t + 0.1\sin 2000\pi t$, $\omega_i(t) = \omega_c + 200\pi\cos 2000\pi t$, $\Delta w = 200\pi$, $\Delta f = 100$ Hz.

(c) $\Delta\phi = 0.1$ rad.

(d) $B_{\text{EM}} = 2(\Delta f + B) = 2(0.1 + 1) = 2.2$ kHz.

5.2-5 $\omega_c = 2\pi \times 10^6$, $\varphi_{\text{EM}}(t) = 5\cos(\omega_c t + 20\sin 1000\pi t + 10\sin 2000\pi t)$, $B = 1$ kHz.

(a) $P = 5^2/2 = 12.5$.

(b) $\theta(t) = \omega_c t + 20\sin 1000\pi t + 10\sin 2000\pi t$, $\omega_i(t) = \omega_c + 20,000\pi\cos 1000\pi t + 20,000\pi\cos 2000\pi t$. We need to find the maximum and the minimum of

$$20,000\pi(\cos 1000\pi t + \cos 2000\pi t)$$

Using calculus, we can find that the derivative of $\cos x + \cos 2x$ is $\sin x(1 + 4\cos x)$. Hence, it has maximum of 2 at $x = 0$ but minimum of $-(1+\sqrt{3})/4$. Thus, $\Delta\omega = 31250\pi$, $\Delta f = 15.625$ kHz.

(c) We need to find the maximum and the minimum of $2\sin 1000\pi t + \sin 2000\pi t) = 2\sin x + sin2x$. Applying calculus, its derivative equals

$$2(\cos x + 2\cos^2 x - 1)$$

Thus, the maximum $3\sqrt{3}/2$ occurs at $x = \pi/3$ and the minimum $-3\sqrt{3}/2$ occurs at $x = -\pi/3$. $\Delta\phi = 15\sqrt{3}$ rad.

(d) $B_{\text{EM}} = 2(\Delta f + B) = 2(15.625 + 1) = 33.25$ kHz.

5.2-6 In this case, $B = 3 \times 10^3 = 3$ kHz.

PM: $k_p = 25$, $\dot{m}_p = 2/(10^{-3}/4) = 8000$

$$\Delta f = \frac{k_p \dot{m}_p}{2\pi} = \frac{(25)(8,000)}{2\pi} = 31.84 \text{ kHz}$$

$B_{\text{PM}} = 2(\Delta f + B) = 2(31.83 + 3) = 69.66$ kHz.

FM: $k_f = 10^5$, $m_p = 1$,

$$\Delta f = \frac{k_f m_p}{2\pi} = 10^5/2\pi = 15.92 \text{ kHz}$$

Thus, $B_{\text{FM}} = 2(\Delta f + B) = 2(15.92 + 3) = 37.84$ kHz.

5.2-7 $m(t) = \sin 2000\pi t$, $B = 1$ kHz, $k_f = 200000\pi$, and $k_p = 10$.

(a) **PM:** $\dot{m}(t) = 2000\pi \cos 2000\pi t$, $\dot{m}_p = 2000\pi$,

$$\Delta f = \frac{k_p \dot{m}_p}{2\pi} = \frac{(10)(2,000\pi)}{2\pi} = 10\,\text{kHz}$$

$B_{\text{PM}} = 2(\Delta f + B) = 2(10 + 1) = 22$ kHz.

FM: $m_p = 1$, $\Delta f = k_f m_p/(2\pi) = 200000\pi/(2\pi) = 100$ kHz. Thus, $B_{\text{FM}} = 2(\Delta f + B) = 2(100 + 1) = 202$ kHz.

(b) $m(t) = 2 \sin 2000\pi t$, $B = 1$ kHz. **PM:** $\dot{m}(t) = 4000\pi \cos 2000\pi t$, $\dot{m}_p = 4000\pi$;

$$\Delta f = \frac{k_p \dot{m}_p}{2\pi} = \frac{(10)(4,000\pi)}{2\pi} = 20\,\text{kHz}$$

$B_{\text{PM}} = 2(\Delta f + B) = 2(20 + 1) = 42$ kHz.

FM: $m_p = 2$, $\Delta f = k_f m_p/(2\pi) = 400000\pi/(2\pi) = 200$ kHz. Thus, $B_{\text{FM}} = 2(\Delta f + B) = 2(200 + 1) = 402$ kHz.

(c) $m(t) = \sin 4000\pi t$, $B = 2$ kHz. **PM:** $\dot{m}(t) = 4000\pi \cos 4000\pi t$, $\dot{m}_p = 4000\pi$;

$$\Delta f = \frac{k_p \dot{m}_p}{2\pi} = \frac{(10)(4,000\pi)}{2\pi} = 20\ \text{kHz}$$

$B_{\text{PM}} = 2(\Delta f + B) = 2(20 + 2) = 44$ kHz.

FM: $m_p = 1$, $\Delta f = k_f m_p/(2\pi) = 200,000\pi/2\pi) = 100$ kHz; Thus, $B_{\text{FM}} = 2(\Delta f + B) = 2(100 + 2) = 204$ kHz.

(d) Doubling the amplitude of $m(t)$ roughly doubles the bandwidth of both FM and PM. Doubling the frequency of $m(t)$ (i.e., expanding the spectrum of $M(\omega)$ by a factor of 2) has hardly any effect on the FM bandwidth. However, it roughly doubles the bandwidth of PM, indicating that the PM spectrum is sensitive to the shape of the baseband spectrum. The FM spectrum is relatively insensitive to the nature of the spectrum $M(\omega)$.

5.2-8 $m(t) = e^{-t^2/100}$, $f_c = 10^4$ Hz, $k_f = 6000\pi$, and $k_p = 8000\pi$.

(a) **FM:** $m_{\max} = 1$ and $m_{\min} = 0$, $\Delta f = k_f(m_p - 0)/(2 \times 2\pi) = 6000\pi/(2 \times 2\pi) = 1.5$ kHz.
PM:

$$\dot{m}(t) = \frac{-t}{50} e^{-t^2/100}$$

We need to find the peak of $\dot{m}(t)$ as $\dot{m}_p$. we set

$$\ddot{m}(t) = \frac{-1}{50} e^{-t^2/100} + \frac{2t^2}{5,000} e^{-t^2/100} = 0$$

This gives $\ddot{m}(\sqrt{50}) = 0$. Therefore, $\dot{m}_p = |\dot{m}(\sqrt{50}))| = 0.0858$, and

$$\Delta f = \frac{k_p \dot{m}_p}{2\pi} = \frac{(8000\pi)(0.0858)}{2\pi} = 343\,\text{Hz}$$

(b) Using the transform pair

$$e^{-t^2/2\sigma^2} \Longleftrightarrow \sigma\sqrt{2\pi} e^{-\sigma^2\omega^2/2}$$

we obtain $M(\omega) = 10\sqrt{\pi} e^{-25\omega^2}$. This is also a Gaussian waveform in frequency domain that decays rapidly. Its 3 dB bandwidt h is about 0.0187 Hz, which is extremely small in comparison to Δf for FM. Also,

$$\mathcal{F}\{\dot{m}(t)\} = j2\pi f M(f)$$

which is narrow enough such that the 3 dB bandwidth can be considered small versus Δf for the PM. Thus, for both FM and PM, the overall bandwidth can be approximated by $2\Delta f$.

FM: $B_{\text{FM}} = 2\Delta f \approx 3$ kHz. **PM:** $B_{\text{PM}} = 2\Delta f \approx 686$ Hz.

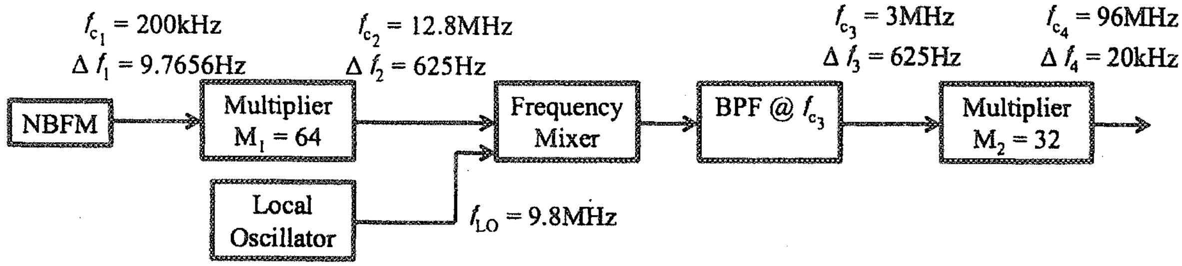

Fig. S5.3-1

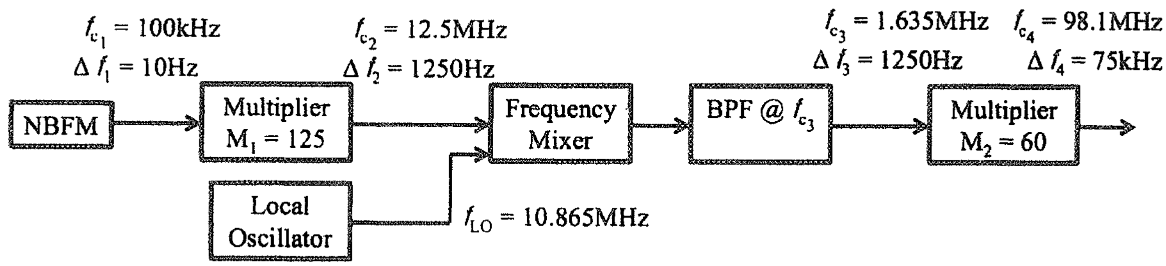

Fig. S5.3-2

(a) The design is shown in Fig. S5.3-2. In this case, the NBFM generator generates $f_{c_1} = 100$ kHz, and $\Delta f_1 = 10$ Hz. The final WBFM should have $f_{c_4} = 98.1$ MHz, and $\Delta f_4 = 75$ kHz. The total factor of frequency multiplication needed is $M_1 \cdot M_2 = \frac{\Delta f_4}{\Delta f_1} = 7500$. With this, we can use $M_1 = 125$, $M_2 = 60$.

To find f_{LO}, there are three possible relationships: $f_{c_3} = f_{c_2} \pm f_{LO}$, $f_{LO} - f_{c_2}$. Each should be tested to determine the one that will require $10 \times 10^6 \leq f_{LO} \leq 11 \times 10^6$. First, we test $f_{c_3} = f_{c_2} - f_{LO}$. This case leads to 98.1 MHz $= f_{c_4} = 60 f_{c_3} = 60(f_{c_2} - f_{LO}) = 60(125 f_{c_1} - f_{LO}) = 7.5 \times 10^8 - 60 f_{LO}$. Thus, we have $f_{LO} = 10.865$ MHz, which is in the desired range. We won't test the other cases since this one works. Thus, the final design is $M_1 = 125$, $M_2 = 60$, and $f_{LO} = 10.865$ MHz. This gives $f_{c_2} = 125 f_{c_1} = 12.5$ MHz, $\Delta f_2 = M1 \cdot \Delta f_1 = 1250$ Hz, $f_{c_3} = f_{c_2} - f_{LO} = 12.5 - 10.865 = 1.635$ MHz, $\Delta f_3 = 1250$ Hz. The bandpass filter used will be centered at 1.635 MHz.

(b) Given the multiplication factors already used, the tunable range given by

$$60(f_{c_2} - f_{LO}) = 60(f_{c_2} - (10 \text{ MHz to } 11 \text{ MHz})) = 90 \text{ MHz to } 150 \text{ MHz}$$

5.3-3 The design is shown in Fig. S5.3-3. In this case, the NBFM generator generates $f_{c_1} = 150$ kHz, and $\Delta f_1 = 10$ Hz. The final WBFM should have $f_{c_4} = 96.3$ MHz, and $\Delta f_4 = 20.48$ kHz. The total factor of frequency multiplication needed is $M_1 \cdot M_2 = \frac{\Delta f_4}{\Delta f_1} = 2048$. Because only frequency doublers are available, we find that $M_1 \cdot M_2 = 2^{11} = 2048$. Now, $M_1 = 2^{n_1}$, $M_2 = 2^{n_2}$, $n_1 + n_2 = 11$, $f_{c_2} = 2^{n_1} f_{c_1}$, and $f_{c_4} = 2^{n_2} f_{c_3}$. In order to find f_{LO}, there are three possible relationships: $f_{c_3} = f_{c_2} \pm f_{LO}$, $f_{LO} - f_{c_2}$. Each should be tested to determine the one that will require $13 \times 10^6 \leq f_{LO} \leq 14 \times 10^6$.

First, we test $f_{c_3} = f_{c_2} - f_{LO}$. This case leads to 96.3 MHz $= f_{c_4} = 2^{n_2} f_{c_3} = 2^{n_2}(f_{c_2} - f_{LO}) = 2^{n_2}(2^{n_1} f_{c_1} - f_{LO}) = 2^{n_1+n_2} f_{c_1} - 2^{n_2} f_{LO} = 2^{11}(150 \times 10^3) - 2^{n_2} f_{LO}$. Thus, we have $f_{LO} = 2^{-n_2}(3.072 \times 10^8 - 9.63 \times 10^7) = 2^{-n_2}(2.109 \times 10^8)$.

In this case, if $n_2 = 4$, then $f_{LO} = 13.1813$ MHz, which is in the desired range. We won't test the other cases since this one works. Thus, the final design is $M_1 = 128$, $M_2 = 16$, and $f_{LO} = 13.1813$ MHz. This gives $f_{c_2} = 2^{n_1} f_{c_1} = 19.2$ MHz, $\Delta f_2 = M1 \cdot \Delta f_1 = 1280$ Hz, $f_{c_3} = f_{c_2} - f_{LO} = 19.2 - 13.1813 = 6.0187$ MHz, $\Delta f_3 = 1280$ Hz. The bandpass filter used will be centered at 6.0187 MHz.

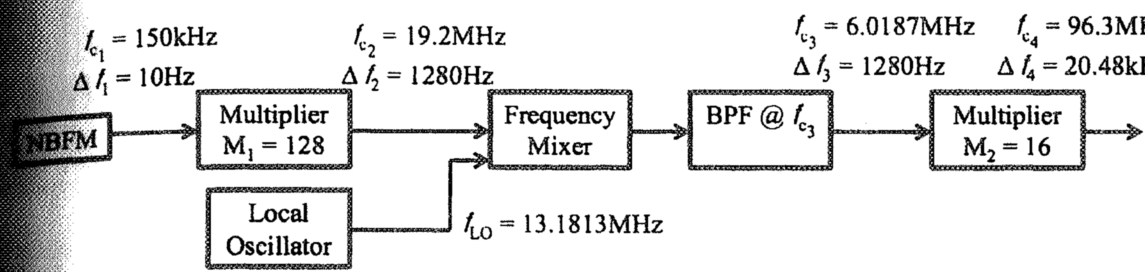

Fig. S5.3-3

5.4-1

(a) When $\varphi_{\text{PM}}(t) = A\cos\left[\omega_c t + k_p m(t)\right]$ is passed through an ideal FM demodulator, the output is $k_p\dot{m}(t)$. This signal, when passed through an ideal integrator, yields $k_p m(t)$. Hence, an FM demodulator followed by an ideal integrator acts as a PM demodulator. If $m(t)$ has a discontinuity, $\dot{m}(t) = \infty$ at the point(s) of discontinuity, and the system will fail. If a dc offset is present, the integrator will generate an increasing dc offset to $k_p m(t)$ that will distort the output.

(b) When $\varphi_{\text{FM}}(t) = A\cos\left[\omega_c t + \int^t m(\alpha)\,d\alpha\right]$ is passed through an ideal PM demodulator, the output is $k_f \int^t m(\alpha)\,d\alpha$. This signal, when passed through an ideal differentiator, yields $k_f m(t)$. Hence, a PM demodulator followed by an ideal differentiator, acts as an FM demodulator regardless of whether $m(t)$ has jump discontinuities or not. If a dc offset is present after the PM demodulator, the ideal differentiator will eliminate it anyway.

5.4-2 Given that $f_c = 10$ kHz, $\Delta f = 1$ kHz, and the message is periodic square wave of period T_0, the resulting FM signal simply switches instantaneous frequency from 11 kHz to 9 kHz and back over one period. Thus,

$$\varphi_{\text{FM}}(t) = A\cos\left[20000\pi t \pm 2000\pi t\right]$$

over any given half period. Now, after the ideal differentiator,

$$\dot{\varphi}_{\text{FM}}(t) = -(20000\pi \pm 2,000\pi)A\sin\left[2,000\pi t \pm 2000\pi t\right]$$

Next, after the envelope detector, the output will be a periodic square wave proportional to $(20000\pi \pm 2,000\pi)A$, with a dc offset. After dc blocking, the result is a periodic square wave proportional to $m(t)$. This is illustrated in Fig. S5.4-2.

5.4-3 $s(t) = 2\cos\left[10^7\pi t + 2\sin(2000\pi t + 0.3\pi) - 3\pi\cos(100t)\right]$.

(a) The baseband bandwidth of this angle-modulated signal is $B = 2000\pi/2\pi = 1$ kHz. Also,

$$\omega_i(t) = 10^7\pi + 4000\pi\cos(2000\pi t + 0.3\pi) + 300\pi\sin(100t)$$

Therefore $\Delta\omega = 4300\pi$ and $\Delta f = 2.15$ kHz. Thus, $B_{\text{FM}} = 2(\Delta f + B) = 2(2.15 + 1) = 6.3$ kHz.

(b) Since $s(t)$ is an angle-modulated signal with a constant amplitude of 2, the output of an ideal envelope detector would be just a constant.

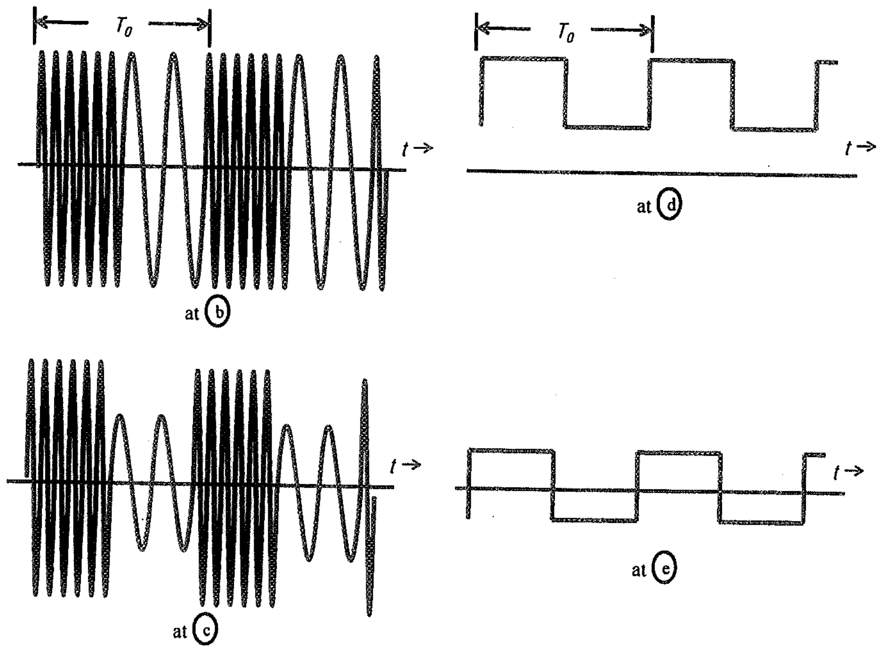

Fig. S5.4-2

(c) If $s(t)$ is first differentiated,

$$\dot{s}(t) = (2)(-1)\left[10^7\pi + 4000\pi\cos(2000\pi t + 0.3\pi) + 300\pi\sin(100t)\right]$$
$$\times \sin\left[10^7\pi t + 2\sin(2000\pi t + 0.3\pi) - 3\pi\cos(100t)\right]$$

Thus, the output of the ideal envelope detector would be

$$2\left[10^7\pi + 4000\pi\cos(2000\pi t + 0.3\pi) + 300\pi\sin(100t)\right]$$

(d) Clearly, it is necessary to first differentiate the signal to obtain the message. If $k_f = 200\pi$, then

$$m(t) = (4000\pi\cos(2000\pi t + 0.3\pi) + 300\pi\sin(100t))/k_f = 20\cos(2000\pi t + 0.3\pi) + 1.5\sin(100t)$$

5.4-4 We can use small error analysis to find the Laplace transform of the phase error $\theta_e(t)$ as follows

$$\Theta_e(s) = \frac{s}{s + AKH(s)}\Theta_i(s)$$

For $\theta_i = kt^2$, $\Theta_i(s) = 2k/s^3$ and

$$\Theta_e(s) = \frac{2k}{s^2[s + AKH(s)]}$$

If $H(s) = 1$, the steady-state phase error is

$$\lim_{t\to\infty}\theta_e(t) = \lim_{s\to 0} s\Theta_e(s) = \lim_{s\to 0}\frac{2k}{s(s + AK)} = \infty$$

Hence, the incoming signal cannot be tracked. If $H(s) = \frac{s+a}{s}$, then

$$\Theta_e(s) = \frac{2k}{s^2[s + \frac{AK(s+a)}{s}]} = \frac{2k}{s[s^2 + AK(s+a)]}$$

$$\lim_{t\to\infty} \theta_e(t) = \lim_{s\to 0} s\Theta_e(s) = \lim_{s\to 0} \frac{2k}{s^2 + AK(s+a)} = \frac{2k}{AKa}$$

Hence, the incoming signal can be tracked within a constant phase $2k/AKa$ radians. Now if

$$H(s) = \frac{s^2 + as + b}{s^2}$$

then

$$\Theta_e(s) = \frac{2k}{s^2[s + \frac{AK(s^2+as+b)}{s^2}]} = \frac{2k}{s^3 + AK(s^2+as+b)}$$

and

$$\lim_{t\to\infty} \theta_e(t) = \lim_{s\to 0} s\Theta_e(s) = \lim_{s\to 0} \frac{2ks}{s^3 + AK(s^2+as+b)} = 0$$

In this case, the incoming signal can be tracked with zero phase error.

-1 Given a preemphasis filter design of $H_p(f) = j2\pi f$, it is clear that to obtain $H_p(f)H_d(f) = 1$, we must have $H_d = 1/(j2\pi f)$. Clearly, the preemphasis filter is a differentiator and the deemphasis filter is an integrator. With frequency modulation and demodulation in between, the system is essentially a PM system, where the input of the preemphasis filter can be considered to be the message $m(t)$.

-1 Given that the IF frequency is $f_{\rm IF} = 455$ kHz and $f_c = 1530$ kHz, it is clear that an image station exists at a distance of $2f_{\rm IF} = 910$ kHz. This means that at another carrier frequency of $f_c' = 1530 - 910 = 620$ kHz, $f_{\rm LO} = f_c' + f_{\rm IF} = 1075$ kHz, and $f_c - f_{\rm LO}' = f_{\rm IF}$. Thus, at f_c' the station at f_c will be heard owing to a poor RF-stage bandpass filter.

-2 With $f_{\rm IF} = 10.7$ MHz and a desired range of 88 to 108 MHz, $[f_{\rm LO}]_{\rm min} = 88 + 10.7 = 98.7$ MHz and $[f_{\rm LO}]_{\rm max} = 108 + 10.7 = 118.7$ MHz. Image stations exist at frequencies $2f_{\rm IF} = 21.4$ MHz apart. Since $88 + 21.4 = 109.4$ MHz and $108 - 21.4 = 86.6$ MHz, all of the image stations will be out of the frequency band used for this FM system, so it is not possible to pick up a signal from a second one of these FM stations.

-3

(a) With $f_{\rm IF} = 455$ kHz and a desired range of 9.4 to 9.9 MHz, $[f_{\rm LO}]_{\rm min} = 9.4 + 0.455 = 9.855$ MHz and $[f_{\rm LO}]_{\rm max} = 9.9 + 0.455 = 10.355$ MHz.

(b) Since the range of the 31-meter band is only 500 kHz, it is not possible to receive an image station and a desired station from that band. The image stations exist at a distance of $2f_{\rm IF} = 910$ kHz, which means that for any given station within the desired range, the image station is outside the same band.