Chapter 7

7.2 -1
7.2 -4
7.3 -1

7.6 -1

Chapter 7

7.2-1

(a) The Fourier spectrum of triangular pulse $p_1(t) = \Delta(t/T_b)$ is

$$\frac{T_b}{2}\Delta\left(t/T_b\right) = \Pi\left(\frac{t}{0.5T_b}\right) * \Pi\left(\frac{t}{0.5T_b}\right)$$

$$\begin{aligned}\frac{T_b}{2}\mathcal{F}\left\{\Delta\left(t/T_b\right)\right\} &= \mathcal{F}\left\{\Pi\left(\frac{t}{0.5T_b}\right) * \Pi\left(\frac{t}{0.5T_b}\right)\right\} \\ &= \mathcal{F}\left\{\Pi\left(\frac{t}{0.5T_b}\right)\right\}\mathcal{F}\left\{\Pi\left(\frac{t}{0.5T_b}\right)\right\} \\ &= \frac{T_b^2}{4}\operatorname{sinc}^2\left(\frac{\pi f T_b}{2}\right)\end{aligned}$$

$$P(f) = \frac{T_b}{2}\operatorname{sinc}^2\left(\frac{\pi f T_b}{2}\right)$$

For polar signaling [Eq. (7.13)],

$$S_y(f) = \frac{|P(f)|^2}{T_b}R_0$$

and $R_0 = 1$

Therefore,

$$S_y(f) = \frac{\frac{T_b^2}{4}\operatorname{sinc}^4\left(\frac{\pi f T_b}{2}\right)}{T_b} = \frac{T_b}{4}\operatorname{sinc}^4\left(\frac{\pi f T_b}{2}\right)$$

For on-off signaling [Eq. (7.20)],

$$S_y(f) = \frac{|P(f)|^2}{4T_b}\left[1 + \frac{1}{T_b}\sum_{n=-\infty}^{n=\infty}\delta\left(f - \frac{n}{Tb}\right)\right]$$

$$\begin{aligned}S_y(f) &= \frac{\frac{T_b^2}{4}\operatorname{sinc}^4\left(\frac{\pi f T_b}{2}\right)}{4T_b}\left[1 + \frac{1}{T_b}\sum_{n=-\infty}^{n=\infty}\delta(f - \frac{n}{Tb})\right] \\ &= \frac{T_b\operatorname{sinc}^4\left(\frac{\pi f T_b}{2}\right)}{16}\left[1 + \frac{1}{T_b}\sum_{n=-\infty}^{n=\infty}\delta(f - \frac{n}{Tb})\right]\end{aligned}$$

For bipolar signaling [Eq. (7.21a)],

$$S_y(f) = \frac{|P(f)|^2}{T_b}\sin^2\left(\pi f T_b\right)$$

$$\begin{aligned}S_y(f) &= \frac{\frac{T_b^2}{4}\operatorname{sinc}^4\left(\frac{\pi f T_b}{2}\right)}{T_b}\sin^2(\pi f T_b) \\ &= \frac{T_b\operatorname{sinc}^4\left(\frac{\pi f T_b}{2}\right)}{4}\sin^2(\pi f T_b)\end{aligned}$$

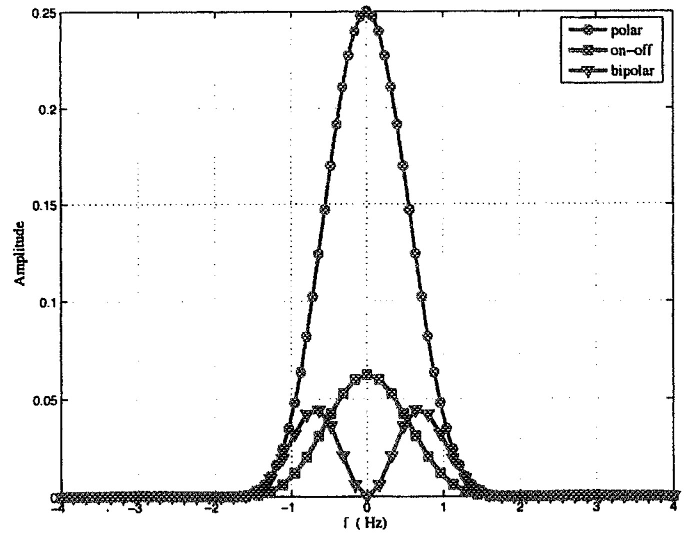

Fig. S7.2-1

(b) Bandwidth: For the three cases, bandwidth is about $1.5/T_b$ Hz. See Fig. S7.2-1.

7.3-2

(a) The Fourier spectrum of triangular pulse $p_2(t) = \Pi\left(t/0.5T_b\right)$ is

$$\mathcal{F}(p_2(t)) = \frac{T_b}{2}\operatorname{sinc}\left(\frac{\pi f T_b}{2}\right)$$

$$P(f) = \frac{T_b}{2}\operatorname{sinc}\left(\frac{\pi f T_b}{2}\right)$$

$$|P(f)|^2 = \frac{T_b^2}{4}\operatorname{sinc}^2\left(\frac{\pi f T_b}{2}\right)$$

For polar signaling [Eq. (7.13)],

$$S_y(f) = \frac{|P(f)|^2}{Tb}\mathcal{R}_0$$

and $R_0 = 1$

Therefore,

$$S_y(f) = \frac{\frac{T_b^2}{4}\operatorname{sinc}^2\left(\frac{\pi f T_b}{2}\right)}{T_b} = \frac{T_b}{4}\operatorname{sinc}^2\left(\frac{\pi f T_b}{2}\right)$$

For on-off signaling [Eq. (7.20)],

$$S_y(f) = \frac{|P(f)|^2}{4T_b}\left[1 + \frac{1}{T_b}\sum_{n=-\infty}^{n=\infty}\delta\left(f - \frac{n}{Tb}\right)\right]$$

$$
\begin{aligned}
P(f) &= \frac{T_b}{2}\left[\text{sinc}\left(\frac{2\pi f T_b + \pi}{2}\right) + \text{sinc}\left(\frac{2\pi f T_b - \pi}{2}\right)\right] \\
&= \frac{T_b}{2}\left[\frac{\sin\left(\pi f T_b + \frac{\pi}{2}\right)}{\left(\pi f T_b + \frac{\pi}{2}\right)} + \frac{\sin\left(\pi f T_b - \frac{\pi}{2}\right)}{\left(\pi f T_b - \frac{\pi}{2}\right)}\right] \\
&= \frac{T_b}{2}\left[\frac{\cos(\pi f T_b)}{\left(\pi f T_b + \frac{\pi}{2}\right)} - \frac{\cos(\pi f T_b)}{\left(\pi f T_b - \frac{\pi}{2}\right)}\right] \\
&= \frac{T_b}{2}\left[\frac{\pi f T_b \cos(\pi f T_b) - \frac{\pi}{2}\cos(\pi f T_b) - \pi f T_b \cos(\pi f T_b) - \frac{\pi}{2}\cos(\pi f T_b)}{(\pi f T_b)^2 - \left(\frac{\pi}{2}\right)^2}\right] \\
&= \frac{T_b}{2}\left[\frac{-\pi\cos(\pi f T_b)}{(\pi f T_b)^2 - \left(\frac{\pi}{2}\right)^2}\right] \\
&= \frac{T_b}{2\pi}\left[\frac{\cos(\pi f T_b)}{(1/4) - (f T_b)^2}\right]
\end{aligned}
$$

(a) For polar signaling [Eq. (7.13)],

$$S_y(f) = \frac{|P(f)|^2}{T_b} R_0 \qquad R_0 = 1$$

Therefore,

$$S_y(f) = \frac{\frac{T_b^2}{4}\left[\text{sinc}(\pi f T_b + \pi/2) + \text{sinc}(\pi f T_b - \pi/2)\right]^2}{T_b} = \frac{T_b\left[\text{sinc}(\pi f T_b + \pi/2) + \text{sinc}(\pi f T_b - \pi/2)\right]^2}{4}$$

For on-off signaling [Eq. (7.20)],

$$S_y(f) = \frac{|P(f)|^2}{4T_b}\left[1 + \frac{1}{T_b}\sum_{n=-\infty}^{n=\infty}\delta\left(f - \frac{n}{Tb}\right)\right]$$

$$S_y(f) = \frac{T_b\left[\text{sinc}(\pi f T_b + \pi/2) + \text{sinc}(\pi f T_b - \pi/2)\right]^2}{16}\left[1 + \frac{1}{T_b}\sum_{n=-\infty}^{n=\infty}\delta\left(f - \frac{n}{Tb}\right)\right]$$

For bipolar signaling Eq. (7.21a), $S_y(f) = \frac{|P(f)|^2}{T_b}\sin^2(\pi f T_b)$

$$S_y(f) = \frac{T_b\left[\text{sinc}(\pi f T_b + \pi/2) + \text{sinc}(\pi f T_b - \pi/2)\right]^2}{4}\sin^2(\pi f T_b)$$

(b) Bandwidth: For three cases, bandwidth is about $1.5/T_b$ Hz or 1.5 Hz for $T_b = 1$. See Fig. S7.2-3b

7.2-4

(a) The pulse waveform is $p(t) = \Pi\left(\frac{t + \frac{T_b}{4}}{\frac{T_b}{2}}\right) - \Pi\left(\frac{t - \frac{T_b}{4}}{\frac{T_b}{2}}\right)$.

See Figure S7.2-4a for the modulated data

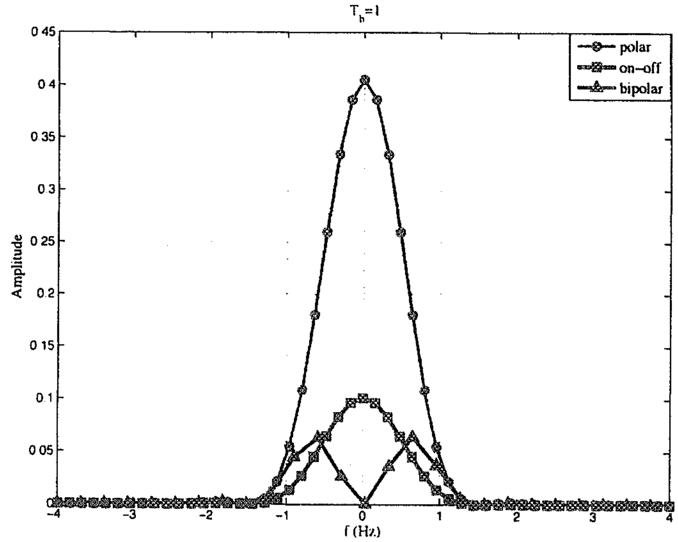

Fig. S7.2-3b

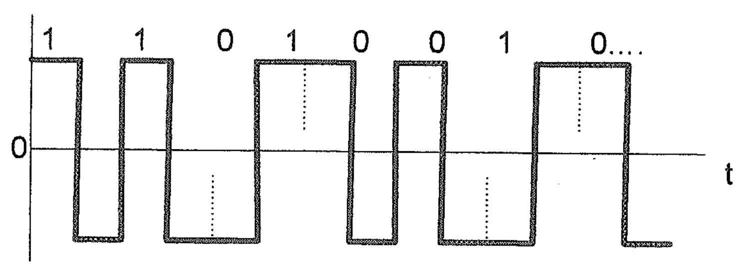

Fig. S7.2-4a

$$P(f) = \frac{T_b}{2}\text{sinc}\left(\frac{\pi f T_b}{2}\right)\exp\left(\frac{j2\pi f T_b}{4}\right) - \frac{T_b}{2}\text{sinc}\left(\frac{\pi f T_b}{2}\right)\exp\left(\frac{-j2\pi f T_b}{4}\right)$$
$$= jT_b\,\text{sinc}\left(\frac{\pi f T_b}{2}\right)\sin\left(\frac{\pi f T_b}{2}\right)$$

1 and 0 are assumed to be equally likely, we have $S_y(f) = \dfrac{|P(f)|^2}{T_b}$

$$S_y(f) = \frac{T_b^2\text{sinc}^2\left(\frac{\pi f T_b}{2}\right)\sin^2\left(\frac{\pi f T_b}{2}\right)}{T_b}$$
$$= T_b\text{sinc}^2\left(\frac{\pi f T_b}{2}\right)\sin^2\left(\frac{\pi f T_b}{2}\right)$$

(first null) bandwidth is $\dfrac{4\pi}{T_b}$ rad/s or $2/T_b = R_b$Hz.

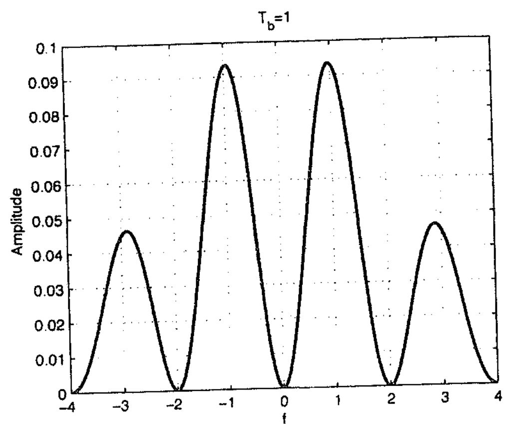

Fig. S7.2-4b

7.2-5

(a) The Fourier transform of the pulse is $P(f)$.

For the binary signal using differential signaling, all the bits are equally likely. Thus, we have

$$R_0 = \lim_{N\to\infty}\frac{1}{N}\left[\frac{N}{2}(1)^2 + \frac{N}{2}(-1)^2\right] = 0$$

To compute R_1 the four possible sequences 00, 11, 01, 10 are equally likely, so

$$\text{R}_1 = \lim_{N\to\infty}\frac{1}{N}\left[\frac{N}{2}(1) + \frac{N}{2}(-1)\right] = 0$$

Similarly, $R_n = 0$, $n > 0$. Therefore

$$S_y(f) = \frac{|P(f)|^2}{T_b}\text{R}_0 = \frac{T_b\left[\text{sinc}\left(\frac{2\pi f T_b + \pi}{2}\right) + \text{sinc}\left(\frac{2\pi f T_b - \pi}{2}\right)\right]^2}{4}$$

7.2-6

(a) Random binary sequence is: 010100110111010... See Figure 7.2-6a.

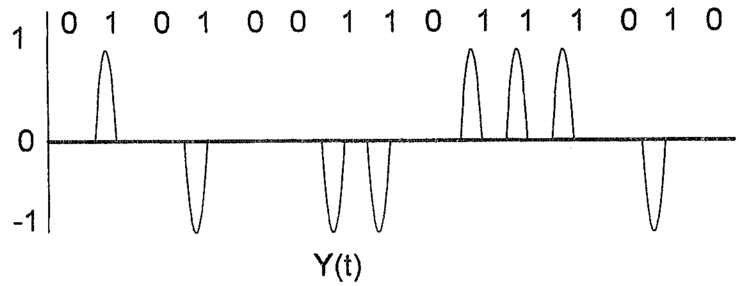

Fig. S7.2-6a

(b) To compute R_0, we observe that, on the average, half of the pulses have $a_k = 0$ and remaining half have $a_k = 1$or -1. Hence

$$R_0 = \lim_{N\to\infty} \frac{1}{N}\left[\frac{N}{2}(\pm 1)^2 + \frac{N}{2}(0)^2\right] = \frac{1}{2}$$

To determine R_1, we need to find different cases of $a_k a_{k+1}$. There are four possible equally likely sequences with two bits: 00, 01, 10, 11. This means on average $3N/4$ combinations have $a_k a_{k+1} = 0$ and the remaining $N/4$ combinations have nonzero $a_k a_{k+1}$. Because of the duobinary rule, the bit sequence 11 can only be encoded by 2 consecutive pulses of the same polarity (either both positive or both negative). This means $a_k a_{k+1} = 1$ for 11. Therefore

$$R_1 = \lim_{N\to\infty} \frac{1}{N}\left[\frac{N}{4}(1) + \frac{3N}{4}(0)\right] = \frac{1}{4}$$

To compute R_2 in a similar way, we need to observe $a_k a_{k+2}$. We need to observe all possible combinations of three bits. There are 8 combinations, 111, 101, 000, 001, 010, 011, 100, 110,. The last six combinations have either first bit or last bit 0. Using the duobinary rule, we encode the the first combination by three pulses of the same polarity, leading to $a_k a_{k+2} = 1$. For the second combination, we encode with the first and the last bits of opposite polarities, leading to $a_k a_{k+2} = -1$.

Therefore, given N combinations, on the average, $a_k a_{k+2}$ equals -1, 1 and 0 for $N/8$, $N/8$, and $3N/4$ combinations, respective.

Hence

$$R_2 = \lim_{N\to\infty} \frac{1}{N}\left[\frac{N}{8}(1) + \frac{N}{8}(-1) + \frac{3N}{4}(0)\right] = 0$$

In a similar way, we find $R_n = 0$, $n > 2$.

7.3-1 Bit rate equals to the pulse rate $R_b = 5 \times 10^6$ bit/s and $r = 0.25$ are given. Minimum required bandwidth with rolloff r is

$$B_T = \frac{(1+r)\,R_b}{2}$$

$$B_T = \frac{(1+0.25)\,5\times 10^6}{2} = 3.125\times 10^6 \text{ Hz}$$

7.3-2 The pulse rate $R_b = 13.0248 \times 10^6$ bit/s and $r = 0.2$ are given. Minimum required bandwidth with roll-off r is $B_T = \dfrac{(1+r)\,R_b}{2}$:

$$B_T = \frac{(1+0.2)\,13.0248\times 10^6}{2} = 7.8149\times 10^6 \text{ Hz}$$

$$\mathbf{P}_r = \begin{pmatrix} 0.17 & 0 & 0 & 0 & 0 \\ 0.6 & 0.17 & 0 & 0 & 0 \\ 1 & 0.6 & 0.17 & 0 & 0 \\ -0.1 & 1 & 0.6 & 0.17 & 0 \\ -0.2 & -0.1 & 1 & 0.6 & 0.17 \\ 0 & -0.2 & -0.1 & 1 & 0.6 \\ 0 & 0 & -0.2 & -0.1 & 1 \\ 0 & 0 & 0 & -0.2 & -0.1 \\ 0 & 0 & 0 & 0 & -0.2 \end{pmatrix}, \ \mathbf{p}_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

MMSE, $\mathbf{c} = \mathbf{P}_r^{\dagger}\mathbf{p}_0 = [0.1632 \quad -0.5689 \quad 1.0150 \quad -0.1212 \quad 0.1837]$

MSE of 5-tap MMSE=0.0023

MSE of 5-tap ZF=0.0024

5-tap MMSE and ZF equalizers in this problem are better than better than the 3-tap equalizers.

6-1 $p(t) = \Pi\left(\dfrac{t}{3T_b/4}\right)$ See Figures S7.6-1a, S7.6-1b S7.6-1c, and S7.6-1d for the eye diagrams.

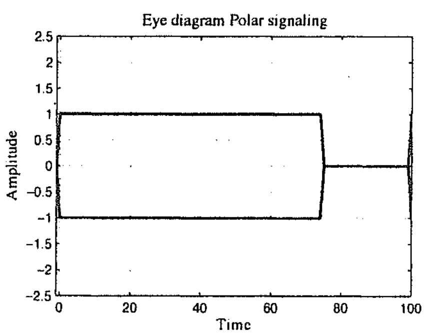

Fig. S7.6-1a

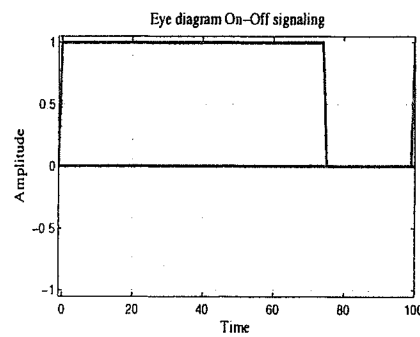

Fig. S7.6-1b

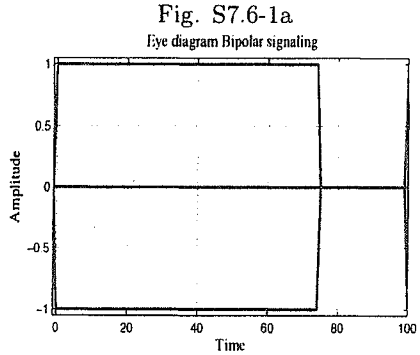

Fig. S7.6-1c

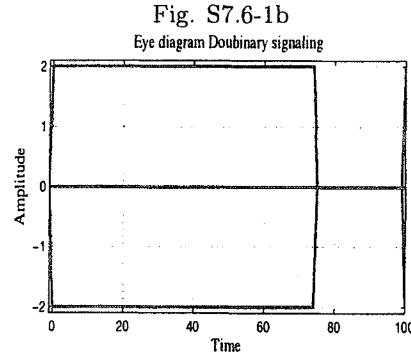

Fig. S7.6-1d