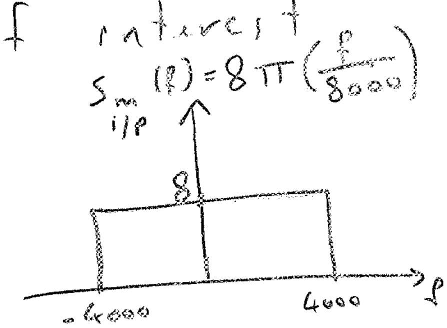
Chapter 10

$\boxed{10.1\text{-}1}$ If the channel transfer function is $H_c(f)$, the PSD of the output signal is equal to the PSD of the input signal multiplied by $|H_c(f)|^2$

$$\therefore \; S_{m_{o/p}}(f) = S_{m_{i/p}}(f) \cdot |H_c(f)|^2$$

SNR is the integration of the SNR density over the Bandwidth of interest.

In this problem, the bandwidth of interest is the input signal BW.



Required SNR = 40 dB = 10^4

Hence,

$$10^4 = \int_{-w}^{w} \frac{S_{m_{i/p}}(f) \cdot |H_c(f)|^2}{S_n(f)}\, df$$

$$10^4 = \int_{-w}^{w} \frac{8 \cdot 10^{-4}}{10^{-8}\left[(2\pi f)^2 + (3000\pi)^2\right]}\, df$$

$$\cancel{10^4} = 8 * \cancel{10^4} \int_{-w}^{w} \frac{1}{(2\pi f)^2 + (3000\pi)^2}\, df$$

Note that $\int \frac{1}{1+x^2}\,dx = \arctan(x)$

Hence

$$1 = 8*2\int_0^W \frac{1/(3000\pi)^2}{1+\left(\frac{2\pi f}{3000\pi}\right)^2}\,df$$

I have used here $\int_{-w}^{w} \text{even } f_2 = 2\int_0^w \text{even } f_2$

$$1 = \frac{8*2}{(3000\pi)^2}\int_0^W \frac{1}{1+\left(\frac{f}{1500}\right)^2}\,d\frac{f}{1500}\cdot 1500$$

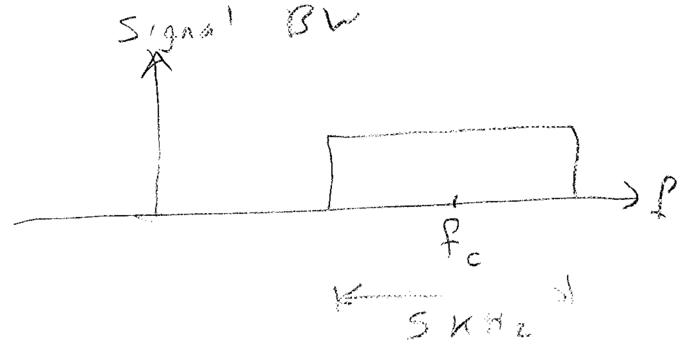
$$1 = \frac{8*1500*2}{(3000\pi)^2}\left[\arctan\left(\frac{f}{1500}\right)\right]_0^W$$

$$\arctan\left(\frac{W}{1500}\right) = \frac{(3000\pi)^2}{2*8*1500} = 3.7*10^3$$

$$\frac{W}{1500} = \tan(3.7*10^3) = 0.3154$$

$$\boxed{\therefore W = 473 \text{ Hz}}$$

Hence, W is the BW required for the SNR to be 40 dB



10.2.1 DSB-SC

$S_n(f) = 10^{-12}$, $BW = 5$ KHz

SNR = 47 dB

(a) $SNR = \dfrac{\text{received signal power}}{\text{received noise power}} = \dfrac{S_o}{N_o}$

where N_o is filtered by the receiver filter which is limited to the signal BW

$\therefore N_o = S_n(f) * BW = 10^{-12} * 5000 = 5 * 10^{-9}$

$\therefore SNR = 47 \text{ dB} = 10^{4.7} = \dfrac{S_o}{N_o} = \dfrac{S_o}{5 * 10^{-9}}$

$\therefore S_o = 2.5 * 10^{-4}$

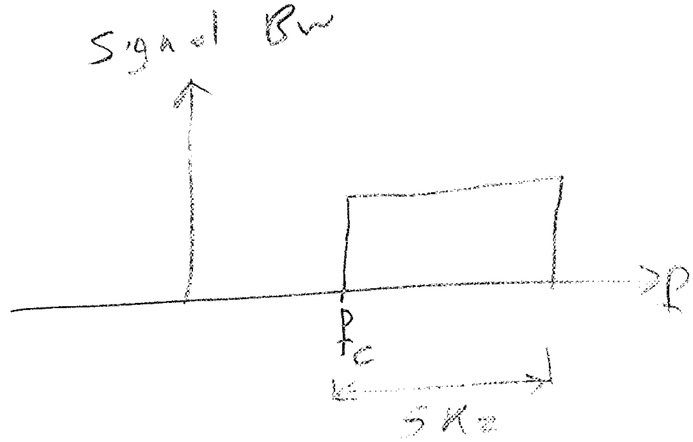
(b) received output noise $= N_o = 5 * 10^{-9}$

(c) the signal S_T is filtered by the channel filter $H_c(f) = 10^{-3}$

$\therefore S_o = |H_c(f)|^2 \, S_T = 10^{-6} \, S_T$

$\therefore S_T = 10^{6} * S_o = 250$

$\boxed{10-2-2}$ SSB-SC



$S_n(f) = 10^{-12}$; $BW = 5\ KHz$

$SNR = 45\ dB$

(a) $SNR = \dfrac{\text{received signal power}}{\text{received noise power}} = \dfrac{S_o}{N_o}$

$N_o = S_n(f) * BW = 5 * 10^{-9}$

$SNR = 47\,dB = 10^{4.7} = \dfrac{S_o}{N_o} = \dfrac{S_o}{5 * 10^{-9}}$

$\therefore S_o = 2.5 * 10^{-4}$

(b) $N_o = 5 * 10^{-9}$

(c) $S_o = |H_c(f)|^2 S_T = 10^{-6} S_T$

$S_T = 10^{6} * S_o = 250$

$\boxed{10.3\_1}$ $SNR = \frac{S_o}{N_o} = 28 \text{ dB} = 631$

$$\frac{S_o}{N_o} = 631 = 3\beta^2\gamma \frac{\overline{m^2(t)}}{m_p^2}$$

$$631 = 3(5)^2 \gamma \frac{\sigma_m^2}{(3\sigma_m)^2}$$

$$\gamma = \frac{3 * 631}{25} = 75.7$$

(a) $\gamma = \frac{S_i}{NB}$

$$S_i = \gamma NB = 75.7 * 2*10^{-10} * 15000$$

$$= 2.27 * 10^{-4}$$

(b) $\beta = \frac{\Delta w}{2\pi B} = \frac{K_p m_p}{2\pi B}$

$$5 = \frac{K_p(3\sigma_m)}{30000\pi} \longrightarrow K_p \sigma_m = 50\,000\,\pi$$

$$S_o = \alpha^2 K_p^2 \overline{m^2(t)} = \alpha^2 K_p^2 \sigma_m^2$$

$$= (10^{-2})^2 * (50\,000\pi)^2 = 250000\,\pi^2$$

(c) $N_o = \frac{S_o}{SNR} = \frac{250000\,\pi^2}{631} = 3.9 * 10^3$

$\boxed{10.3.6}$ $\quad S_m(f) = \dfrac{|2\pi f|}{\sigma^2} e^{-\frac{(2\pi f)^2}{\sigma^2}}$

total power of $m(t) = P_{tot} = \int_{-\infty}^{\infty} S_m(f)\, df$

$$P_{tot} = \int_{-\infty}^{\infty} \underbrace{\frac{|2\pi f|}{\sigma^2} e^{-\frac{(2\pi f)^2}{\sigma^2}}}_{\text{even function in } f} df$$

$$= 2 \int_0^{\infty} \frac{2\pi f}{\sigma^2} e^{-\frac{(2\pi f)^2}{\sigma^2}} df$$

$$= \cancel{2} \int_0^{\infty} \frac{2\pi}{\sigma^2} e^{-\frac{(2\pi f)^2}{\sigma^2}} d\, \frac{f^2}{\cancel{2}}$$

$$= \frac{1}{2\pi} \int_0^{\infty} e^{-\frac{(2\pi f)^2}{\sigma^2}} d\, \frac{(2\pi f)^2}{\sigma^2}$$

substituting for $x = \dfrac{(2\pi f)^2}{\sigma^2}$

$$P_{tot} = \frac{1}{2\pi} \int_0^{\infty} e^{-x} dx = \frac{1}{2\pi}$$

$p(w)$ is the power with in the band $-w$ to w

$$p(w) = \int_{-w}^{w} S_m(f)\, df = \frac{1}{2\pi} \int_0^{w} e^{-x} dx$$

$$= \frac{1}{2\pi} \left[1 - e^{-\frac{(2\pi w)^2}{\sigma^2}} \right]$$

(a) $x = 99 \rightarrow \frac{P(w)}{P_{tot}} = 0.99$

$$= 1 - e^{-\frac{(2\pi w)^2}{\sigma^2}}$$

$$\therefore -\frac{(2\pi w)^2}{\sigma^2} = \log_e(0.01)$$

$$w^2 = \frac{\sigma^2}{(2\pi)^2} \log_e(100)$$

$$w^2 = 0.1167\,\sigma^2$$

to check the concentration of frequency components of $S_m(f)$, we calculate $\overline{f^2}$

$$\overline{f^2} = \int_{-\infty}^{\infty} f^2 \cdot 2\pi S_m(f)\, df$$

$$= 4\pi \int_0^{\infty} \frac{2\pi f^3}{\sigma^2} e^{-\frac{(2\pi f)^2}{\sigma^2}}\, df$$

$$= \frac{4\pi^2}{\sigma^2} \int_0^{\infty} f^2 e^{-\frac{(2\pi f)^2}{\sigma^2}}\, df^2$$

substituting for $f^2 = x$

$$\overline{f^2} = \frac{4\pi^2}{\sigma^2} \int_0^{\infty} x\, e^{-\frac{(2\pi)^2 x}{\sigma^2}}\, dx$$

$$\begin{array}{ll} x & e^{-\frac{(2\pi)^2 x}{\sigma^2}} \\ 1 & -\frac{\sigma^2}{(2\pi)^2} e^{-\frac{(2\pi)^2 x}{\sigma^2}} \\ 0 & \frac{\sigma^4}{(2\pi)^4} e^{-\frac{(2\pi)^2 x}{\sigma^2}} \end{array}$$

$$= \frac{4\pi^2}{\sigma^2} \left[-\frac{\sigma^2}{(2\pi)^2} x e^{-\frac{(2\pi)^2 x}{\sigma^2}} - \frac{\sigma^4}{(2\pi)^4} e^{-\frac{(2\pi)^2 x}{\sigma^2}} \right]_0^{\infty}$$

$$= \frac{4\pi^2}{\sigma^2} \left[\frac{\sigma^4}{(2\pi)^4} \right] = \frac{\sigma^2}{(2\pi)^2} = 0.0253\,\sigma^2$$

Hence, $\frac{W^2}{3} = \frac{0.1167\sigma^2}{3} = 0.039\sigma^2$

for $\chi = .99$, $\frac{W^2}{3} = 0.039\sigma^2 > \overline{f^2}$

Hence, PM is superior

(b) $\chi = 0.9 \longrightarrow \frac{P(W)}{P_{tot}} = 0.9$

$= 1 - e^{-\frac{(2\pi W)^2}{\sigma^2}}$

$-\frac{(2\pi W)^2}{\sigma^2} = \log_e(0.1)$

$\frac{W^2}{3} = 0.019\sigma^2 < \overline{f^2}$

Hence, FM is superior

(c) $\chi = 0.7 \longrightarrow \frac{P(W)}{P_{tot}} = 0.7$

$= 1 - e^{-\frac{(2\pi W)^2}{\sigma^2}}$

$-\frac{(2\pi W)^2}{\sigma^2} = \log_e(0.3)$

$\frac{W^2}{3} = 0.01\sigma^2 < \overline{f^2}$

Hence, FM is superior

$\boxed{10.4-1}$ $BW_{m(t)} = 4.5\ MHz$, $SNR_{quantized} = 50\ dB$

(a) For uniform distribution

$$\overline{m^2} = \frac{1}{2m_p} \int_{-m_p}^{m_p} m^2\, dm = \frac{1}{3} m_p^2$$

$$SNR = 50dB = 10^5 = 3 * (2)^{2n} \left(\frac{\overline{m^2}}{m_p^2} \right)$$

$$= \cancel{3} (2)^{2n} \left(\frac{1}{\cancel{3}} \right)$$

$$2n = 16.61$$

$$\therefore n = \left\lceil \frac{16.61}{2} \right\rceil = 9$$

Hence, # quantization levels $= L = 2^n = 512$

(b) Hence, for $n = 9$

$$SNR = 3 * (2)^{2n} \left(\frac{\overline{m^2}}{m_p^2} \right)$$

$$= 3 * (2)^{18} * \frac{1}{3}$$

$$= 262144 = 54.18\ dB$$

(c) $BW_{PCM} = 2n\, BW_{m(t)}$

$= 2 * 9 * 4.5$

$= 81$ MHz

(d) If SNR $= 50 + 6 = 56$ dB

SNR $= 56$ dB $= 10^{5.6} = 3 * (2)^{2n} \left(\frac{\overline{m^2}}{m_p^2}\right)$

$= 3 (2)^{2n} \left(\frac{1}{3}\right)$

$2n = 18.6$

$n = \left\lceil \frac{18.6}{2} \right\rceil = 10$

$L = 2^n = 1024$

$BW_{PCM} = 2n\, BW_{m(t)}$

$= 2 * 10 * 4.5$

$= 90$ MHz