Chapter 2 solutions

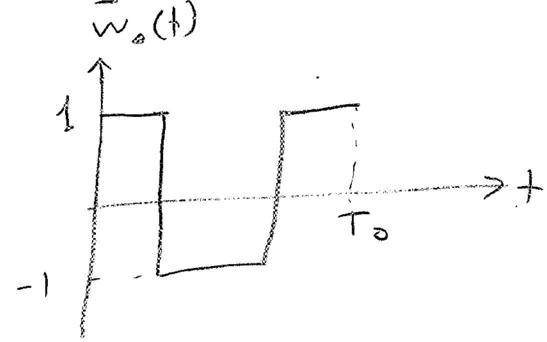
$\boxed{2.1\text{-}1}$ If $g(t)$ is a periodic signal with period T,

$$\text{average power} = \frac{1}{T}\int_0^T |g(t)|^2\,dt$$

for $\varphi(t)$, $T = \pi$

$$P_{av} = \frac{1}{\pi}\int_0^{\pi} \left(e^{-t/2}\right)^2 dt = \frac{1}{\pi}\int_0^{\pi} e^{-t}\,dt$$

$$= \frac{1}{\pi}\left(-e^{-t}\right)\Big|_0^{\pi} = \frac{1}{\pi}\left[1 - e^{-\pi}\right]$$



for $w_0(t)$, $T = T_0$

$$P_{av} = \frac{1}{T_0}\int_0^{T_0} \left(w_0(t)\right)^2 dt$$

you can notice that

$\left(w_0(t)\right)^2$ is always equal to 1 because

$(1)^2$ or $(-1)^2 = 1$

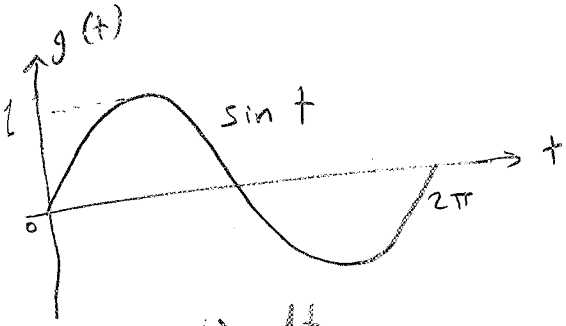
$$\therefore\ P_{av} = \frac{1}{T_0}\int_0^{T_0} 1\,dt = 1$$

$\boxed{2.1-6}$ The energy of a signal $g(t)$ is E, where

$$E = \int_{-\infty}^{\infty} |g(t)|^2 \, dt$$

(a) The signal $g(t)$ is defined as

$$g(t) = \begin{cases} \sin t & , \; 0 \leqslant t \leqslant 2\pi \\ 0 & , \; \text{otherwise} \end{cases}$$



$$\therefore E = \int_0^{2\pi} (\sin t)^2 \, dt = \int_0^{2\pi} \frac{1}{2}(1 - \cos 2t) \, dt$$

$$= \int_0^{2\pi} \frac{1}{2} \, dt - \frac{1}{2} \int_0^{2\pi} \cos 2t \, dt = \pi$$

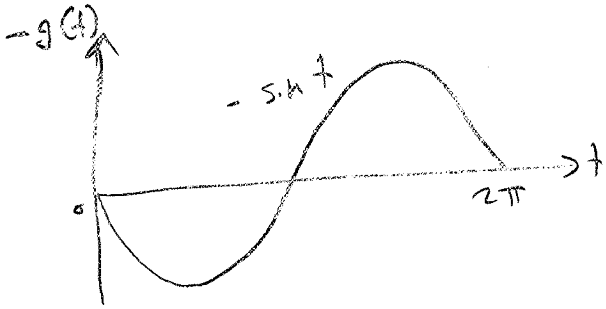
0 because $\cos 2t = \cos \frac{2\pi}{T} t$

i.e. the periodic time $= T = \pi$

Any integration of a sinusoidal signal over any multiple integer of its periodic time $=$ zero

i.e., $\int_0^{k\pi} \cos 2t \, dt = 0$ where $k = 1, 2, \ldots$

(b) The negative of $g(t)$



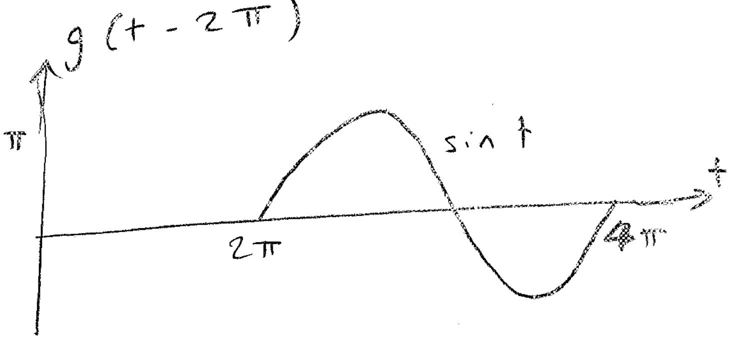
$$E = \int_0^{2\pi} (-g(t))^2 \, dt$$

$$= \int_0^{2\pi} (-\sin t)^2 \, dt = \int_0^{2\pi} (\sin t)^2 = \pi$$

Hence, sign change does <u>not</u> affect the energy of a signal

(c) Time shifting of $g(t)$

$$h(t) = g(t-2\pi) = \begin{cases} \sin t, & 2\pi \leq t \leq 4\pi \\ 0 & \text{o.w.} \end{cases}$$

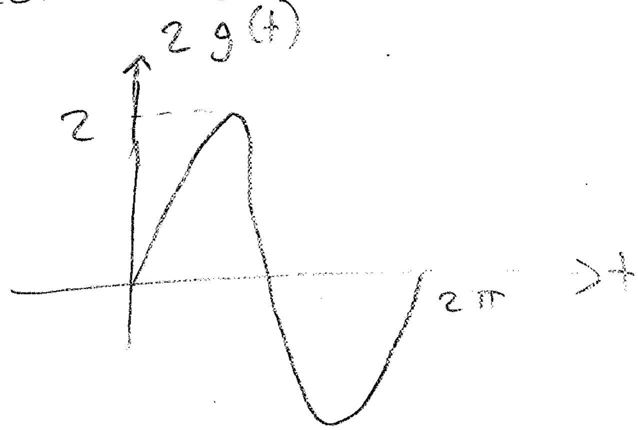


$$E = \int_{2\pi}^{4\pi} (\sin t)^2 \, dt = \int_{2\pi}^{4\pi} \frac{1}{2}(1 - \cos 2t) \, dt$$

$$= \frac{1}{2} \int_{2\pi}^{4\pi} 1 \, dt = \frac{1}{2}[4\pi - 2\pi] = \pi$$

Time shifting does <u>not</u> affect a signal energy

(d) signal doubling



$$E = \int_{0}^{2\pi} (2 \sin t)^2 \, dt$$

$$= 4 \int_{0}^{2\pi} (\sin t)^2 dt = 4\pi$$

signal doubling scales the signal energy by a factor of 4

If a signal is multiplied by K, the energy will be scaled by a factor K^2

$\boxed{2.1\text{-}7}$ If we have a power signal $g(t)$, the power of this signal can be defined as follows

$$P_g = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |g(t)|^2\, dt = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} g(t)\, g^*(t)\, dt$$

$$= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\left(\sum_{k=m}^{n} D_k e^{j\omega_k t}\right) * \left(\sum_{l=m}^{n} D_l^* e^{-j\omega_l t}\right) dt$$

$$= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\sum_{k=m}^{n}\sum_{l=m}^{n} D_k D_l^* e^{j(\omega_k-\omega_l)t}\, dt$$

$$= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\sum_{\substack{k=m\\ k\neq l}}^{n}\sum_{l=m}^{n} D_k D_l^* e^{j(\omega_k-\omega_l)t}\, dt + \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\sum_{k=m}^{n}|D_k|^2\, dt$$

$$= \lim_{T\to\infty}\sum_{\substack{k=m\\ k\neq l}}^{n}\sum_{l=m}^{n} D_k D_l^* \frac{e^{j(\omega_k-\omega_l)T/2} - e^{-j(\omega_k-\omega_l)T/2}}{j(\omega_k-\omega_l)T} + \sum_{k=m}^{n}|D_k|^2$$

$$= \lim_{T\to\infty}\sum_{\substack{k=m\\ k\neq l}}^{n}\sum_{l=m}^{n} 2 D_k D_l^* \frac{\sin\left[(\omega_k-\omega_l)\frac{T}{2}\right]}{(\omega_k-\omega_l)T} + \sum_{k=m}^{n}|D_k|^2$$

$$= 0 + \sum_{k=m}^{n}|D_k|^2 = \sum_{k=m}^{n}|D_k|^2$$

The first term is equal to zero because the numerator $\sin\left[(\omega_k-\omega_l)\frac{T}{2}\right]$ keeps oscillating around -1 and 1 as $T\to\infty$, However, the denominator $(\omega_k-\omega_l)T \to \infty$ as $T\to\infty$

$\boxed{2.2\text{-}1}$ $g(t) = e^{-at}$, $-\infty < t < \infty$

- If a is a real value

$$E = \int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-\infty}^{\infty} (e^{-at})^2 dt$$

$$= \int_{-\infty}^{\infty} e^{-2at} dt = -\frac{1}{2a}\left[e^{-2at}\right]_{-\infty}^{\infty} = \infty$$

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{T/2} |g(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{T/2} e^{-2at} dt = \lim_{T\to\infty} \frac{1}{T}\left(-\frac{1}{2a}\right)\left[e^{-2at}\right]_{-T/2}^{T/2}$$

$$= \lim_{T\to\infty} \frac{1}{2aT}\left[e^{aT} - e^{-aT}\right] = \lim_{T\to\infty} \frac{ae^{aT} + ae^{-aT}}{2a} = \infty$$

Hence, the signal is neither an energy nor a power signal becaus energy signal should have a finite energy and power signal should have a finite power.

- If a is an imaginary value, i.e., $a = j\omega$

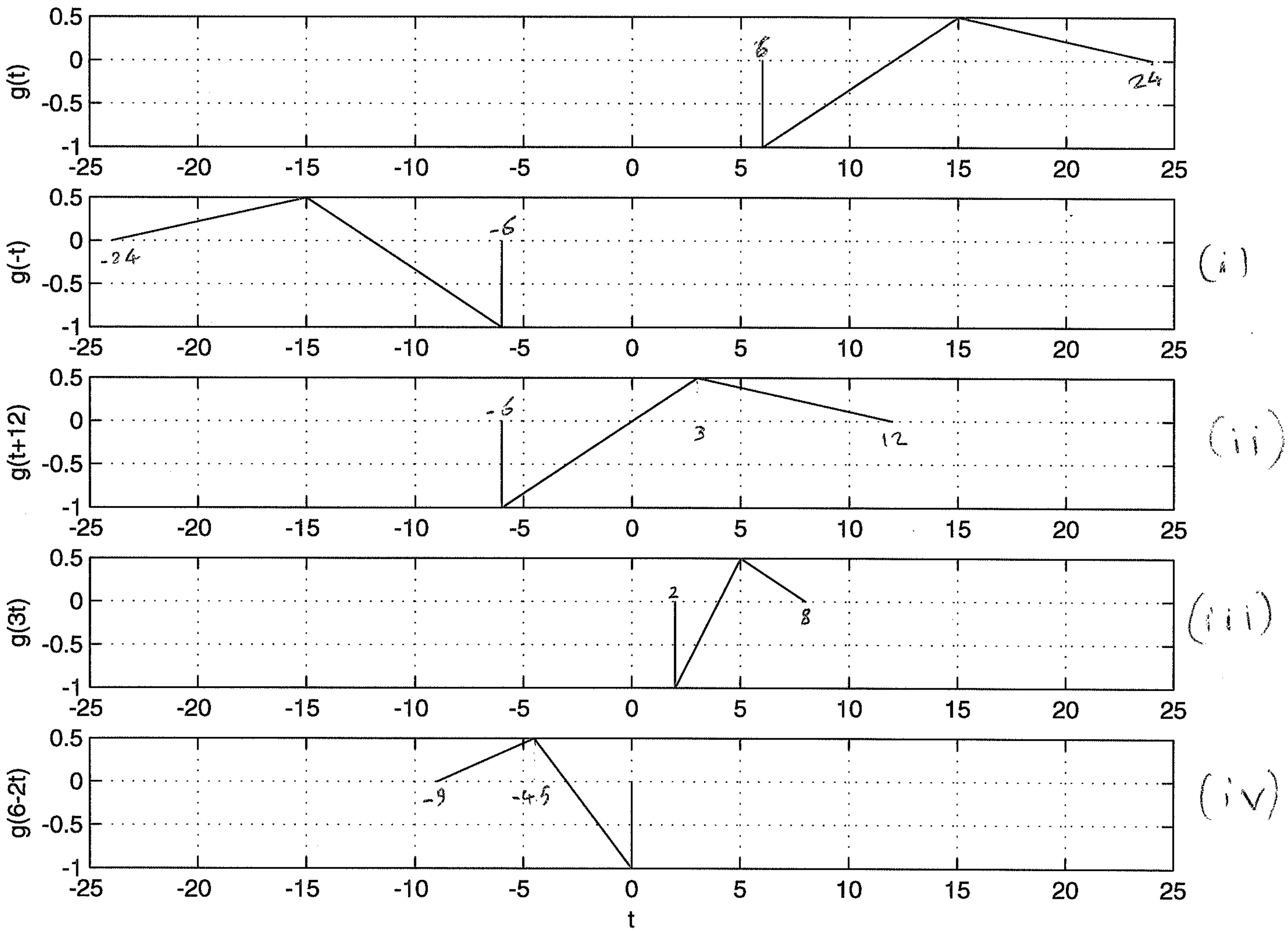
$$h(t) = e^{-j\omega t} \quad , \quad -\infty < t < \infty$$

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{T/2} |e^{-j\omega t}|^2 dt = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{T/2} 1 \, dt$$

$$P = 1$$

Hence, h(t) is a power signal because it has a finite power

2.3-2 (a)



(b) In order to find the energy of a signal, we need to write its equation first

$$g(t) = \begin{cases} \frac{1}{6}t - 2 , & 6 \leqslant t < 15 \\ -\frac{1}{18}t + \frac{4}{3} , & 15 \leqslant t < 24 \\ 0 , & \text{o.w} \end{cases}$$

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 \, dt = \int_6^{15} \left(\frac{1}{6}t - 2\right)^2 dt + \int_{15}^{24} \left(-\frac{1}{18}t + \frac{4}{3}\right)^2 dt$$

$$= 6 \int_6^{15} \left(\frac{1}{6}t - 2\right)^2 d\left(\frac{1}{6}t - 2\right) - 18 \int_{15}^{24} \left(-\frac{1}{18}t + \frac{4}{3}\right)^2 d\left(-\frac{1}{18}t + \frac{4}{3}\right)$$

$$= \frac{6}{3} \left[\left(\frac{1}{6}t - 2\right)^3\right]_6^{15} - \frac{18}{3} \left[\left(-\frac{1}{18}t + \frac{4}{3}\right)^3\right]_{15}^{24}$$

$$= 2\left[\left(\frac{15}{6} - 2\right)^3 - (1 - 2)^3\right] - 6\left[\left(-\frac{24}{18} + \frac{4}{3}\right)^3 - \left(-\frac{15}{18} + \frac{4}{3}\right)^3\right]$$

$$\boxed{E_g = 3}$$

from prob. $\boxed{2.1-6}$, the signal power after sign change or time shifting does not change

$\therefore$ (i) energy of $g(-t) = E_g = 3$

(ii) energy of $g(t+12) = E_g = 3$

(iii) energy of $g(3t) = \int_{-\infty}^{\infty} |g(3t)|^2 \, dt$

$$= \frac{1}{3} \int_{-\infty}^{\infty} |g(3t)|^2 \, d(3t)$$

Substituting $x = 3t$ → integration limits are the same

$\therefore$ energy of $g(3t) = \frac{1}{3} \int_{-\infty}^{\infty} |g(x)|^2 \, dx$

$$= \frac{1}{3} E_g = 1$$

(iv) energy of $g(6-2t) = \int_{-\infty}^{\infty} |g(6-2t)|^2 \, dt$

$$= -\frac{1}{2} \int_{-\infty}^{\infty} |g(6-2t)|^2 \, d(6-2t)$$

Substituting $x = 6 - 2t$

at $t = -\infty \longrightarrow x = \infty$

at $t = \infty \longrightarrow x = -\infty$

$\therefore$ energy of $g(6-2t) = -\frac{1}{2} \int_{x=\infty}^{-\infty} |g(x)|^2 \, dx$

$$= \frac{1}{2} \int_{x=-\infty}^{\infty} |g(x)|^2 \, dx$$

$$= \frac{1}{2} E_g = \frac{1}{2} \cdot 3 = \frac{3}{2}$$

$\boxed{2.4-3}$ (a) $\int_{-\infty}^{\infty} g(\tau+a)\,\delta(t-\tau)\,dt$

$$= g(\tau+a)\Big|_{\tau=t} = g(t+a)$$

(b) $\int_{-\infty}^{\infty} \delta(\tau)\,g(t-\tau)\,d\tau = g(t-\tau)\Big|_{\tau=0} = g(t)$

(c) $\int_{-\infty}^{\infty} \delta(t)\,e^{-j\omega t}\,dt = e^{-j\omega t}\Big|_{t=0} = e^{0} = 1$

(d) $\int_{-\infty}^{1} \delta(t-2)\sin \pi t\,dt = 0$ because the integration region does not include $t = 2$

(e) $\int_{-2}^{\infty} \delta(t+3)\,e^{-t}\,dt = 0$ because $t=-3 \notin [-2, \infty[$

(f) $\int_{-2}^{2} (t^3+4)\,\delta(1-t)\,dt = (t^3+4)\Big|_{t=1} = 1+4 = 5$

(g) $\int_{-\infty}^{\infty} g(2-t)\,\delta(3-t)\,dt = g(2-t)\Big|_{t=3} = g(-1)$

(h) $\int_{-\infty}^{\infty} \cos \frac{\pi}{2}(x-5)\,\delta(2x-3)\,dx = \frac{1}{2}\int_{-\infty}^{\infty} \cos \frac{\pi}{4}(2x-10)\,\delta(2x-3)\,d\,2x$

substituting $y = 2x$

$$= \frac{1}{2}\int_{-\infty}^{\infty} \cos \frac{\pi}{4}(y-10)\,\delta(y-3)\,dy$$

$$= \frac{1}{2} \cos \frac{\pi}{4}(y-10)\Big|_{y=3} = \frac{1}{2}\cos \frac{\pi}{4}(3-10) = \frac{1}{2}\cos\left(-\frac{7\pi}{4}\right)$$

$$= \frac{1}{2}\cos\left(-\frac{7\pi}{4}+\frac{8\pi}{4}\right) = \frac{1}{2}\cos\left(\frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}$$

$\boxed{2-6-1}$ E_x is the energy of $x(t)$

$$x(t) = \begin{cases} \sin(2\pi t) & , \quad 0 \leq t \leq 1 \\ 0 & , \quad o.w \end{cases}$$

$$E_x = \int_0^1 (\sin 2\pi t)^2 dt = \int_0^1 \frac{1}{2}(1 - \cos 4\pi t)\, dt$$

$$= \frac{1}{2}[t]_0^1 = \frac{1}{2}$$

E_{g_1} is the energy of $g_1(t)$

$$g_1(t) = \begin{cases} \sin 4\pi t & , \quad 0 \leq t \leq 1 \\ 0 & , \quad o.w \end{cases}$$

$$E_{g_1} = \int_0^1 (\sin 4\pi t)^2 dt = \int_0^1 \frac{1}{2}(1 - \cos 4\pi t)\, dt$$

$$E_{g_1} = \frac{1}{2}$$

ρ_1 is the correlation coefficient between $x(t)$ & $g_1(t)$

$$\rho_1 = \frac{1}{\sqrt{E_x E_{g_1}}} \int_{-\infty}^{\infty} x(t)\, g_1(t)\, dt$$

$$= \frac{1}{\sqrt{0.5 \times 0.5}} \int_0^1 \sin(2\pi t) \cdot \sin(4\pi t)\, dt$$

$$= \frac{1}{0.5} \int_0^1 \frac{1}{2}[\underbrace{\cos(2\pi t)}_{0} - \underbrace{\cos(6\pi t)}_{0}]\, dt$$

because integration over multiple period of the cosine function

$$\boxed{\rho_1 = 0}$$

E_{g_2} is the energy of $g_2(t)$

$$g_2(t) = \begin{cases} -\sin 2\pi t & , \quad 0 \leq t \leq 1 \\ 0 & \quad o.w \end{cases}$$

$$E_{g_2} = E_{g_1} = \frac{1}{2}$$

$$\therefore \rho_2 = \frac{1}{\sqrt{E_x E_{g_2}}} \int_{-\infty}^{\infty} x(t)\, g_2(t)\, dt$$

$$= \frac{1}{\sqrt{0.5 \times 0.5}} \int_0^1 (\sin 2\pi t)(-\sin 2\pi t)\, dt$$

$$= \frac{-1}{0.5} \int_0^1 \frac{1}{2}(1 - \cos 4\pi t)\, dt = \frac{-1}{0.5} \cdot \frac{1}{2} \cdot 1$$

$$\boxed{\rho_2 = -1}$$

$$g_3(t) = \begin{cases} 0.707 & , \quad 0 \leq t \leq 1 \\ 0 & , \quad o.w \end{cases}$$

$$E_{g_3} = \int_{-\infty}^{\infty} |g_3(t)|^2 = \int_0^1 \left|\frac{1}{\sqrt{2}}\right|^2 dt = \frac{1}{2}$$

$$C_{n_3} = \frac{1}{\sqrt{E_x E_{g_3}}} \int_{-\infty}^{\infty} x(t)\, g_3(t)\, dt$$

$$= \frac{1}{\sqrt{0.5 \times 0.5}} \int_0^1 \frac{1}{\sqrt{2}} \cancel{\sin 2\pi t\, dt}^{\,0}$$

because integration over one period of sine wave

$$\boxed{\rho_3 = 0}$$

$$g_4(t) = \begin{cases} 0.707 & , \quad 0 \leq t < 0.5 \\ -0.707 & , \quad 0.5 \leq t < 1 \\ 0 & \quad \text{o.w} \end{cases}$$

$$E_{g_4} = \int_0^{0.5} \left|\frac{1}{\sqrt{2}}\right|^2 dt + \int_{0.5}^{1} \left|\frac{1}{\sqrt{2}}\right|^2 dt = \frac{1}{2}$$

$$\rho_4 = \frac{1}{\sqrt{E_x E_{g_4}}} \int_{-\infty}^{\infty} x(t)\, g_4(t)\, dt$$

$$= \frac{1}{\sqrt{0.5 \times 0.5}} \left[\int_0^{0.5} \frac{1}{\sqrt{2}} \sin 2\pi t \, dt - \int_{0.5}^{1} \frac{1}{\sqrt{2}} \sin 2\pi t \, dt \right]$$

$$= \sqrt{2} \left[-\frac{1}{2\pi} \Big[\cos 2\pi t\Big]_0^{0.5} + \frac{1}{2\pi} \Big[\cos 2\pi t\Big]_{0.5}^{1} \right]$$

$$= \sqrt{2} \left[-\frac{1}{2\pi} [-1-1] + \frac{1}{2\pi} [1+1] \right] = \frac{2\sqrt{2}}{\pi}$$

$$\boxed{\rho_4 = 0.9}$$

In order to provide maximum protection against noise, the correlaton coefficient (ρ) between the two signals should be minimum

$$-1 \leq \rho \leq 1$$

Hence, the optimal choice of ρ is $\boxed{\rho = -1}$

Thus, $x(t)$ & $g_2(t)$ provide maximum protection against noise where $\boxed{\rho_2 = -1}$

$\boxed{2.8-3}$ (a) any arbitrary function can be written as

$$g(t) = \frac{1}{2}\left[g(t) + g(t)\right] + \frac{1}{2}\left[g(-t) - g(-t)\right]$$

$$= \frac{1}{2}\left[g(t) + g(-t)\right] + \frac{1}{2}\left[g(t) - g(-t)\right]$$

$$= \frac{1}{2}\, g_e(t) + \frac{1}{2}\, g_o(t)$$

where $g_e(t) = g(t) + g(-t)$ is an even function as

$$g_e(-t) = g(t) + g(-t) = g_e(t)$$

and $g_o(t) = g(t) - g(-t)$ is an odd fuction as

$$g_o(-t) = -\left[g(t) - g(-t)\right] = -g_o(t)$$

$$\therefore\ g(t) = \frac{1}{2}\, g_e(t) + \frac{1}{2}\, g_o(t)$$

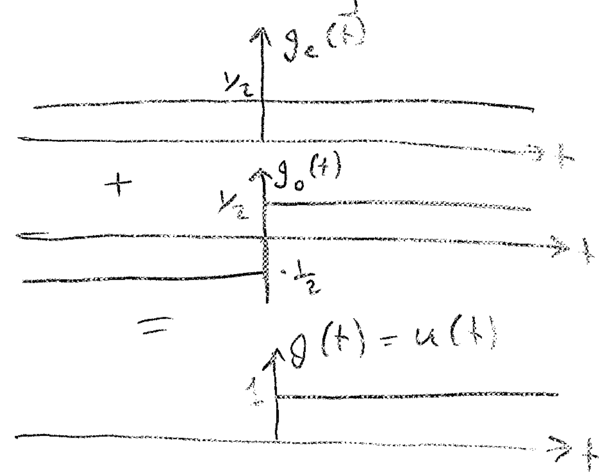
Hence, any function $g(t)$ can be expressed as a sum of an even fuction $g_e(t)$ and an odd function $g_o(t)$

(b) (i) if $g(t) = u(t)$

$$\therefore g(t) = \frac{1}{2}\underbrace{\left[u(t) + u(-t)\right]}_{1} + \frac{1}{2}\left[u(t) - u(-t)\right]$$

$$g(t) = \underbrace{\frac{1}{2}}_{\text{even fn}} + \underbrace{\frac{1}{2}\left[u(t) - u(-t)\right]}_{\text{odd fn}}$$

$$= g_e(t) + g_o(t)$$



(ii) $g(t) = e^{-at}\, u(t)$

$$\therefore g(t) = \frac{1}{2}\left[g(t) + g(-t)\right] + \frac{1}{2}\left[g(t) - g(-t)\right]$$

$$= \underbrace{\frac{1}{2}\left[e^{-at}u(t) + e^{at}u(-t)\right]}_{\text{even fn}} + \underbrace{\frac{1}{2}\left[e^{-at}u(t) - e^{at}u(-t)\right]}_{\text{odd fn}}$$

$$g(t) = g_e(t) + g_o(t)$$

(iii) $g(t) = e^{jt}$

$$g(t) = \frac{1}{2}\left[e^{jt} + e^{-jt}\right] + \frac{1}{2}\left[e^{jt} - e^{-t}\right]$$

$$= \frac{1}{2}\cdot 2\cos t + \frac{1}{2}\cdot 2j\sin t$$

$$= \underbrace{\cos t}_{\text{even fn}} + \underbrace{j\sin t}_{\text{odd fn}}$$

$$g(t) = g_e(t) + g_o(t)$$

$\boxed{2.8\text{-}2}$ $g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$

(a) for even functions: $g(t) = g(-t)$

$$b_n = \frac{2}{T_0} \int_{-\frac{T_0}{2}}^{T_0/2} \underset{\text{even}}{g(t)} \; \underset{\text{odd}}{\sin n\omega_0 t} \, dt$$

$$b_n = \frac{2}{T_0} \int_{-\frac{T_0}{2}}^{T_0/2} \underset{\text{odd}}{h(t)} \, dt = 0$$

$$a_0 = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{T_0/2} \underset{\text{even}}{g(t)} \, dt = \frac{2}{T_0} \int_0^{T_0/2} g(t) \, dt$$

$$a_n = \frac{2}{T_0} \int_{-\frac{T_0}{2}}^{T_0/2} \underbrace{g(t) \cos n\omega_0 t}_{\text{even}} \, dt = \frac{4}{T_0} \int_0^{T_0/2} g(t) \cos n\omega_0 t \, dt$$

(b) for odd functions: $g(t) = -g(-t)$

$$a_0 = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{T_0/2} \underset{\text{odd}}{g(t)} \, dt = 0$$

$$a_n = \frac{2}{T_0} \int_{-\frac{T_0}{2}}^{T_0/2} \underset{\text{odd}}{g(t)} \; \underset{\text{even}}{\cos n\omega_0 t} \, dt$$

$$a_n = \frac{2}{T_0} \int_{-\frac{T_0}{2}}^{T_0/2} \underset{\text{odd}}{h(t)} \, dt = 0$$

$$b_n = \frac{2}{T_0} \int_{-\frac{T_0}{2}}^{T_0/2} \underset{\text{odd}}{g(t)} \; \underset{\text{odd}}{\sin n\omega_0 t} \, dt = \frac{2}{T_0} \int_{-\frac{T_0}{2}}^{T_0/2} \underbrace{g(t) \sin n\omega_0 t}_{\text{even}} \, dt$$

$$b_n = \frac{4}{T_0} \int_0^{T_0/2} g(t) \sin n\omega_0 t \, dt$$

$\boxed{2.8\text{-}2}$ + $\boxed{2.9\text{-}1}$

(a) $T_0 = 4$, $\omega_0 = \frac{2\pi}{T_0} = \frac{\pi}{2}$

$g(t)$ is an even signal

$b_n = 0$

$$a_0 = \frac{2}{T_0} \int_0^{T_0/2} g(t)\, dt = 0$$

$$a_n = \frac{4}{T_0} \int_0^{T_0/2} g(t) \cos n\omega_0 t \, dt$$

$$= \int_0^1 \cos n\tfrac{\pi}{2} t + \int_1^2 -\cos n\tfrac{\pi}{2} t = \frac{\left[\sin n\frac{\pi}{2} t\right]_0^1}{n\frac{\pi}{2}} - \frac{\left[\sin n\frac{\pi}{2} t\right]_1^2}{n\frac{\pi}{2}}$$

$$= \frac{\sin n\frac{\pi}{2}}{n\frac{\pi}{2}} + \frac{\sin n\frac{\pi}{2}}{n\frac{\pi}{2}} = \frac{4}{\pi n} \sin n\frac{\pi}{2}$$

$$g(t) = D_0 + \sum_{n=-\infty\,(n\neq 0)}^{\infty} D_n e^{jn\omega_0 t}$$

$D_0 = a_0 = 0$

$D_n = \frac{1}{2} C_n e^{j\theta_n}$

$$C_n = \sqrt{a_n^2 + b_n^2} = a_n = \frac{4}{\pi n} \sin n\frac{\pi}{2}$$

$$\theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right) = 0$$

$$\boxed{D_n = \frac{2}{\pi n} \sin n\frac{\pi}{2} \qquad g(t) = \sum_{n=-\infty\,(n\neq 0)}^{\infty} D_n e^{jn\frac{\pi}{2} t}}$$

exponential Fourier series

$$\boxed{a_n = \frac{4}{\pi n} \sin n\frac{\pi}{2} \qquad g(t) = \sum_{n=1}^{\infty} a_n \cos\left(n\frac{\pi}{2} t\right)}$$

compact trigonometric Fourier series

(b) $T_0 = 10\pi$, $\omega_0 = \frac{2\pi}{T_0} = \frac{1}{5}$

$$D_0 = C_0 = a_0 = \frac{1}{10\pi}\int_{-\pi}^{\pi} 1 \, dt = \frac{1}{5}$$

$b_n = 0$ (even function)

$$a_n = \frac{4}{10\pi}\int_0^{\pi} \cos n\omega_0 t \, dt = \frac{2}{5\pi}\,\frac{\left[\sin n\frac{1}{5}t\right]_0^{\pi}}{\frac{n}{5}}$$

$$\boxed{\begin{aligned} a_n &= \frac{2}{n\pi}\sin\frac{n\pi}{5} \\ g(t) &= \frac{1}{5} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n}{5}t\right)\end{aligned}}$$ trigonometric form

$C_n = a_n$, $\theta_n = 0$

$$\boxed{\begin{aligned} D_n &= \frac{1}{2}C_n = \frac{1}{n\pi}\sin\frac{n\pi}{5}, \quad D_0 = \frac{1}{5} \\ g(t) &= D_0 + \sum_{n=-\infty,(n\neq 0)}^{\infty} D_n e^{j\frac{n}{5}t}\end{aligned}}$$ exponential form

(c) $T_0 = 2\pi$, $\omega_0 = 1$

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi}\frac{1}{2\pi}t \, dt = \frac{1}{(2\pi)^2}\left[\frac{t^2}{2}\right]_0^{2\pi} = \frac{1}{2}$$

$$a_n = \frac{2}{2\pi}\int_0^{2\pi}\frac{1}{2\pi}t\cdot\cos nt \, dt = \frac{1}{2\pi^2}\frac{1}{n}\int_0^{2\pi} t \, d\sin nt$$

$$= \frac{1}{2\pi^2 n}\left[\left[t\sin nt\right]_0^{2\pi} + \frac{1}{n}\left[\cos nt\right]_0^{2\pi}\right] = 0$$

$$b_n = \frac{2}{2\pi}\int_0^{2\pi}\frac{1}{2\pi}t\cdot\sin nt \, dt = \frac{-1}{2\pi^2}\frac{1}{n}\int_0^{2\pi} t \, d\cos nt$$

$$= \frac{-1}{2\pi^2 n}\left[\left[t\cos nt\right]_0^{2\pi} - \frac{1}{n}\left[\sin nt\right]_0^{2\pi}\right] = \frac{-1}{2\pi^2 n}\, 2\pi\cos 2\pi n$$

$$b_n = -\frac{1}{\pi n}$$

$$C_n = \sqrt{a_n^2 + b_n^2} = \frac{1}{\pi|n|}$$

$$\theta_n = \tan^{-1}\left(-\frac{b_n}{a_n}\right) = \begin{cases} \pi/2 & , n \geq 0 \\ -\pi/2 & , n < 0 \end{cases}$$

$$\boxed{\begin{aligned} D_n &= \frac{1}{2} C_n e^{j\theta_n} = \frac{1}{2\pi|n|} e^{j\theta_n} = \begin{cases} \frac{j}{2\pi n} & ; n \geq 0 \\ \frac{-j}{2\pi|n|} & , n < 0 \end{cases} \\ D_0 &= a_0 = \frac{1}{2} \\ g(t) &= D_0 + \sum_{n=-\infty, (n \neq 0)}^{n=\infty} D_n e^{jnt} = \frac{1}{2} + \sum_{n=1}^{\infty} b_n \sin(nt) \end{aligned}}$$

(d) $T_0 = \pi$, $\omega_0 = \frac{2\pi}{\pi} = 2$

$a_0 = a_n = 0$ (odd function)

$$b_n = \frac{4}{\pi} \int_0^{\pi/4} \frac{4}{\pi} t \sin 2nt \, dt = \frac{-16}{\pi^2} \frac{1}{2n} \int_0^{\pi/4} t \, d \cos 2nt \quad dt$$

$$= \frac{-8}{\pi^2 n} \left[\left[t \cos 2nt \right]_0^{\pi/4} - \frac{1}{2n} \left[\sin 2nt \right]_0^{\pi/4} \right]$$

$$b_n = -\frac{8}{\pi^2 n} \left[\frac{\pi}{4} \cos \frac{\pi n}{2} - \frac{1}{2n} \sin n\frac{\pi}{2} \right]$$

$$\boxed{\begin{aligned} C_n &= |b_n| \\ \theta_n &= \tan^{-1}\left(-\frac{b_n}{a_n}\right) \\ D_n &= \frac{1}{2} C_n e^{j\theta_n} = -\frac{1}{\pi n}\left[\cos\frac{\pi n}{2} - \frac{2}{\pi n}\sin\frac{\pi n}{2}\right] e^{j\theta_n} \\ g(t) &= \sum_{n=-\infty, (n \neq 0)}^{n=\infty} D_n e^{j2nt} = \sum_{n=1}^{\infty} b_n \sin(2nt) \end{aligned}}$$

(e) $T_0 = 3$, $\omega_0 = \frac{2\pi}{3}$

$$D_0 = a_0 = \frac{1}{3}\int_0^1 t\, dt = \frac{1}{6}\left[t^2\right]_0^1 = \frac{1}{6}$$

$$a_n = \frac{2}{3}\int_0^1 t \cos n \frac{2\pi}{3} t\, dt = \frac{2}{3}\frac{3}{2\pi n}\int_0^1 t\, d \sin n \frac{2\pi}{3} t$$

$$= \frac{1}{\pi n}\left[\left[t \sin n \frac{2\pi}{3} t\right]_0^1 + \frac{3}{2\pi n}\left[\cos n \frac{2\pi}{3} t\right]_0^1\right]$$

$$= \frac{1}{\pi n}\left[\sin \frac{2\pi n}{3} + \frac{3}{2\pi n}\left[\cos \frac{2\pi n}{3} - 1\right]\right]$$

$$b_n = \frac{2}{3}\int_0^1 t \sin n \frac{2\pi}{3} t\, dt = -\frac{2}{3}\frac{3}{2\pi n}\int_0^1 t\, d \cos n \frac{2\pi}{3} t$$

$$= -\frac{1}{\pi n}\left[\left[t \cos n \frac{2\pi}{3} t\right]_0^1 - \frac{3}{2\pi n}\left[\sin n \frac{2\pi}{3} t\right]_0^1\right]$$

$$= -\frac{1}{\pi n}\left[\cos \frac{2\pi n}{3} - \frac{3}{2\pi n} \sin \frac{2\pi n}{3}\right]$$

$C_n = \sqrt{a_n^2 + b_n^2}$, $\theta_n = \tan^{-1}\left(-\frac{b_n}{a_n}\right)$

$D_n = \frac{1}{2} C_n e^{j\theta_n}$, $D_0 = \frac{1}{6}$

$$g(t) = D_0 + \sum_{n=-\infty\ (n\neq 0)}^{n=\infty} D_n e^{jn \frac{2\pi}{3} t}$$

$$g(t) = \frac{1}{6} + \sum_{n=1}^{\infty}\left[a_n \cos\left(n \frac{2\pi}{3} t\right) + b_n \sin\left(n \frac{2\pi}{3} t\right)\right]$$

(F) $T_0 = 6$, $\omega_0 = \frac{2\pi}{T_0} = \frac{\pi}{3}$

$$D_0 = a_0 = \frac{2}{6}\left[\int_0^1 1\, dt + \int_1^2 (2-t)\, dt\right]$$

$$= \frac{1}{3}\left[1 + \left[2t - \frac{t^2}{2}\right]_1^2\right] = \frac{1}{3}\left[1 + \cancel{4} - \cancel{2} - 2 + \frac{1}{2}\right]$$

$a_0 = D_0 = \frac{1}{2}$

$b_n = 0$ (even function)

$$a_n = \frac{4}{6}\left[\int_0^1 \cos n\frac{\pi}{3}t\, dt + \int_1^2 (2-t)\cos n\frac{\pi}{3}t\, dt\right]$$

$$= \frac{2}{3}\left[\frac{3}{\pi n}\left[\sin n\frac{\pi}{3}t\right]_0^1 + \frac{6}{\pi n}\left[\sin n\frac{\pi}{3}t\right]_1^2 - \frac{3}{\pi n}\int_1^2 t\, d\sin n\frac{\pi}{3}t\right]$$

$$= \frac{2}{3}\left[\frac{3}{\pi n}\sin\frac{\pi n}{3} + \frac{6}{\pi n}\left[\sin\frac{2\pi n}{3} - \sin\frac{\pi n}{3}\right] - \frac{3}{\pi n}\left[\left[t\sin n\frac{\pi}{3}t\right]_1^2 + \frac{3}{\pi n}\left[\cos n\frac{\pi}{3}t\right]_1^2\right]\right]$$

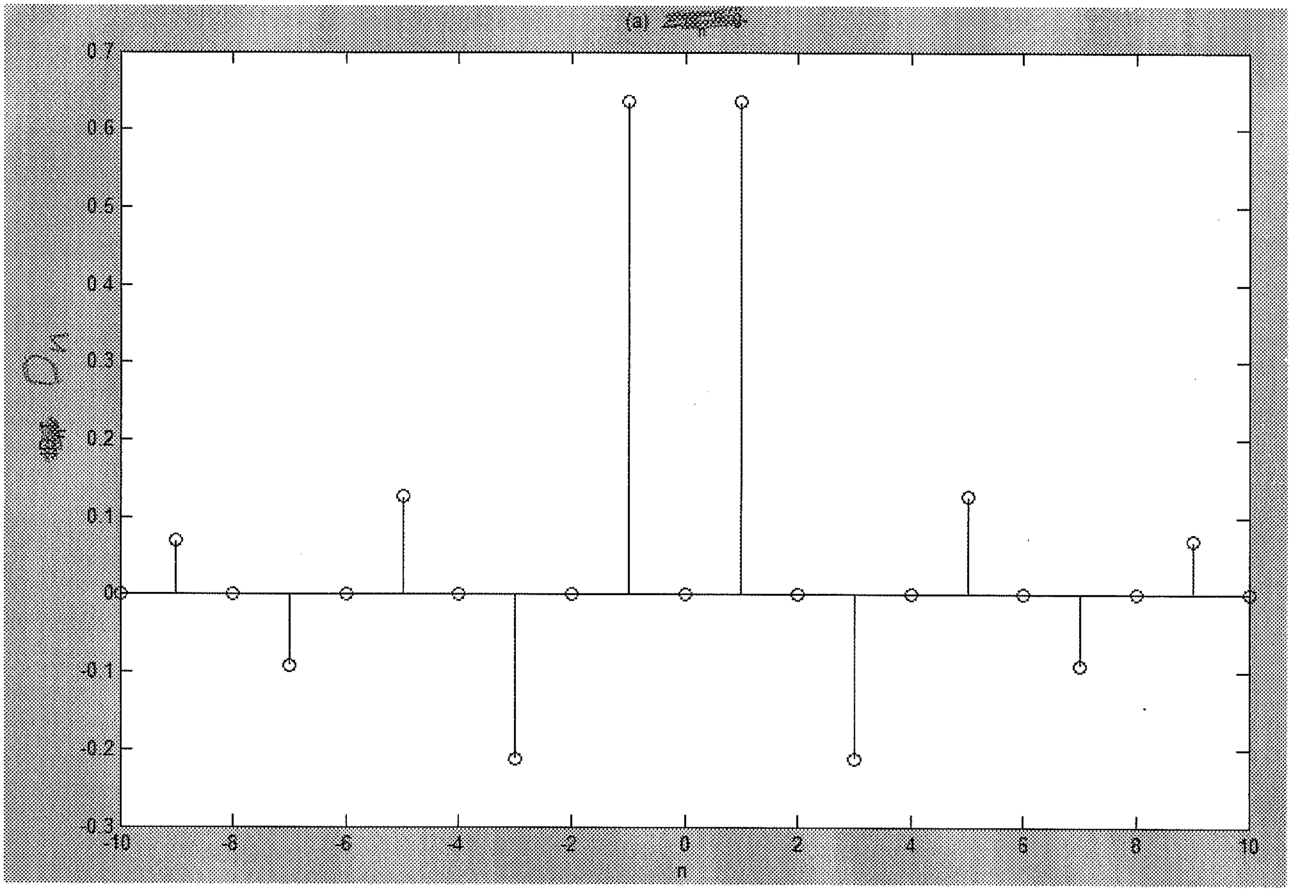
$$= \frac{2}{3}\left[\cancel{\frac{3}{\pi n}\sin\frac{\pi n}{3}} + \cancel{\frac{6}{\pi n}\left[\sin\frac{2\pi n}{3} - \sin\frac{\pi n}{3}\right]} - \frac{3}{\pi n}\left[\cancel{2\sin\frac{2\pi n}{3} - \sin\frac{\pi n}{3}}\right] - \frac{9}{\pi^2 n^2}\left[\cos\frac{2\pi n}{3} - \cos\frac{\pi n}{3}\right]\right]$$

$$a_n = \frac{6}{\pi^2 n^2}\left[\cos\frac{\pi n}{3} - \cos\frac{2\pi n}{3}\right]$$

$C_n = a_n$, $\theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right) = 0$

$D_n = \frac{1}{2}C_n$, $D_0 = \frac{1}{2}$

$$g(t) = D_0 + \sum_{n=-\infty, (n\neq 0)}^{n=\infty} D_n e^{jn\frac{\pi}{3}t} = \frac{1}{2} + \sum_{n=1}^{\infty} a_n \cos\left(n\frac{\pi}{3}t\right)$$



(a)
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0.5
0.4
0.3
0.2
0.1
0
-0.1
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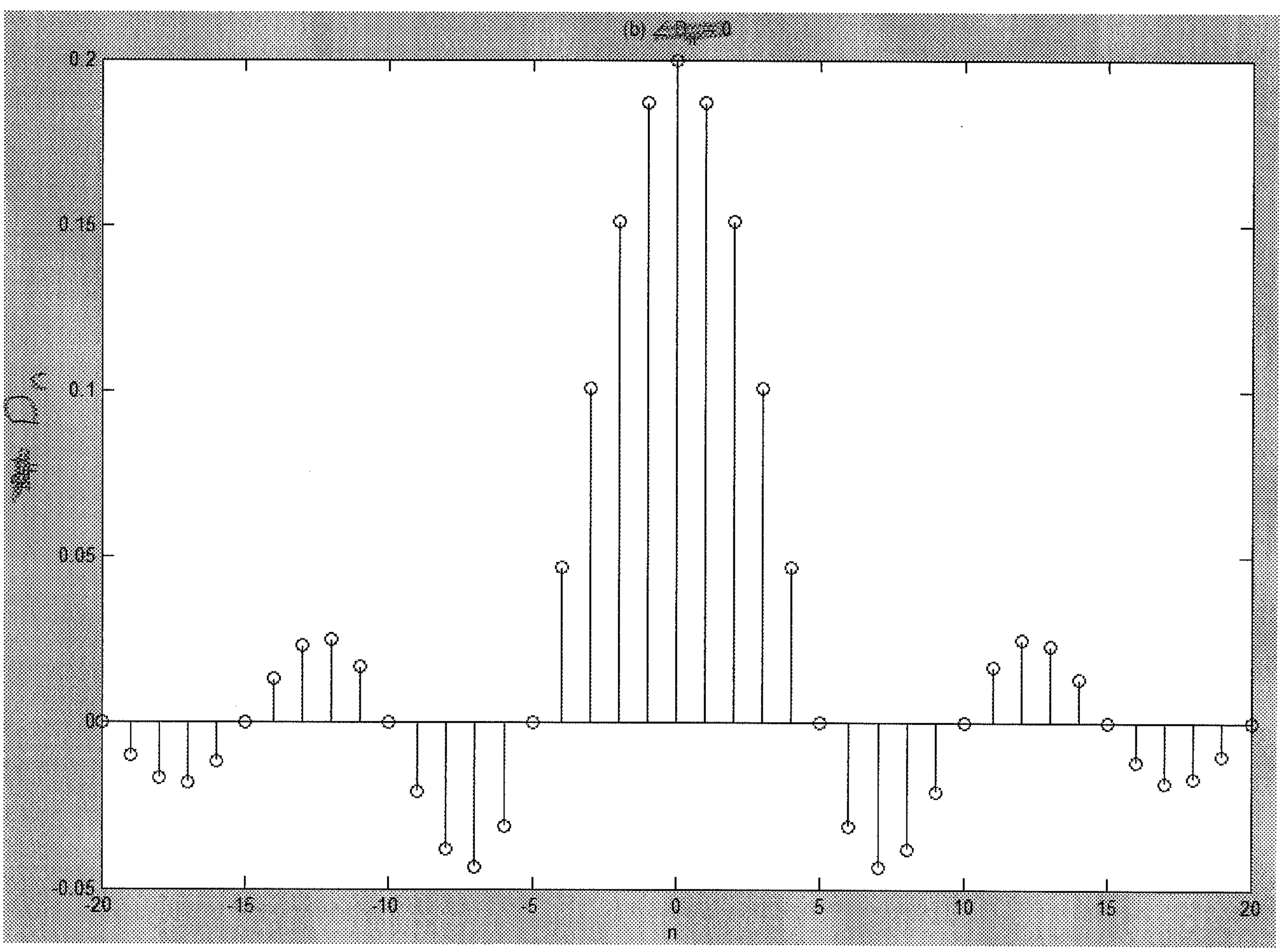
-0.3
-10
-8
-6
-4
-2
0
2
4
6
8
10
n
Dn



(b)
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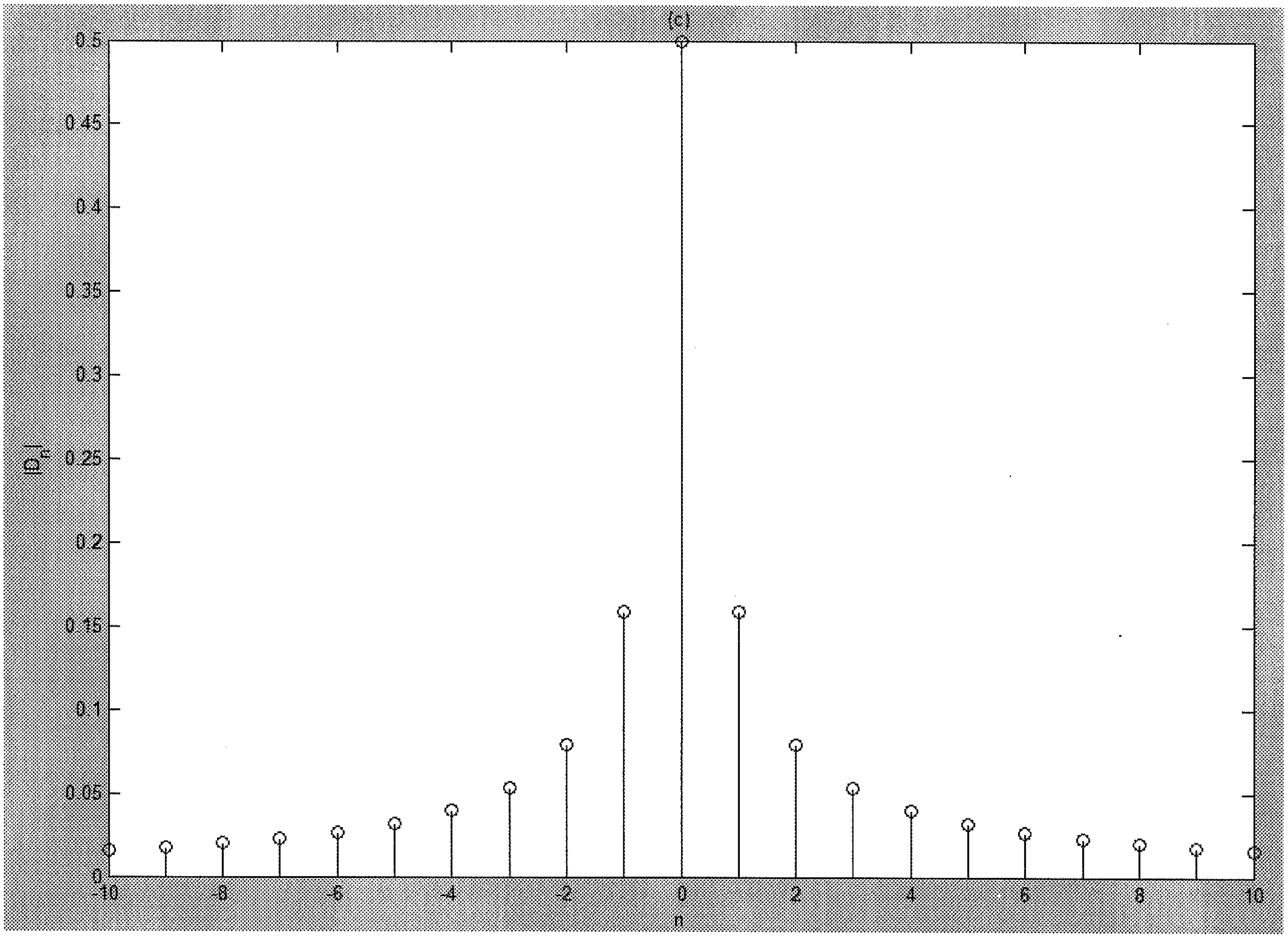
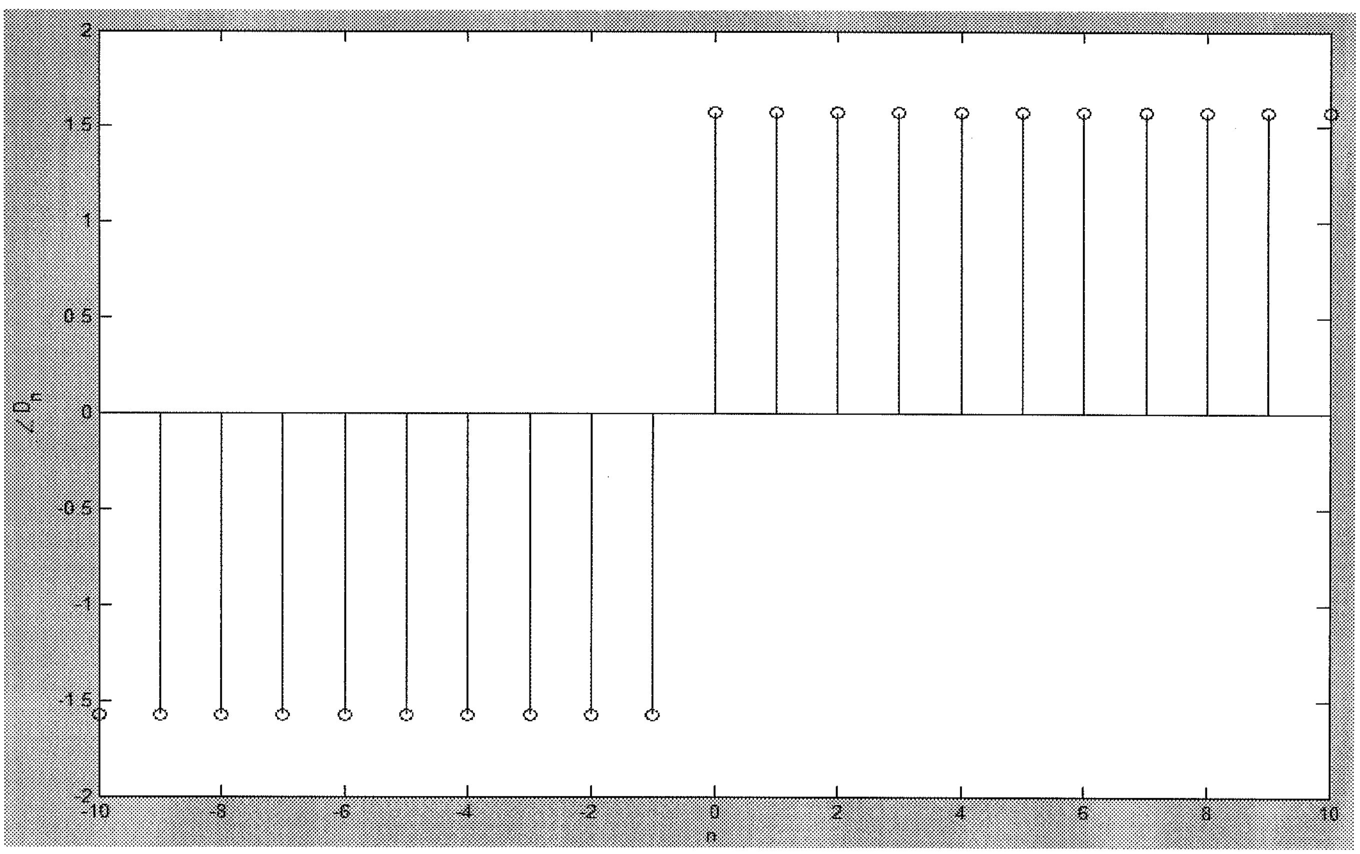
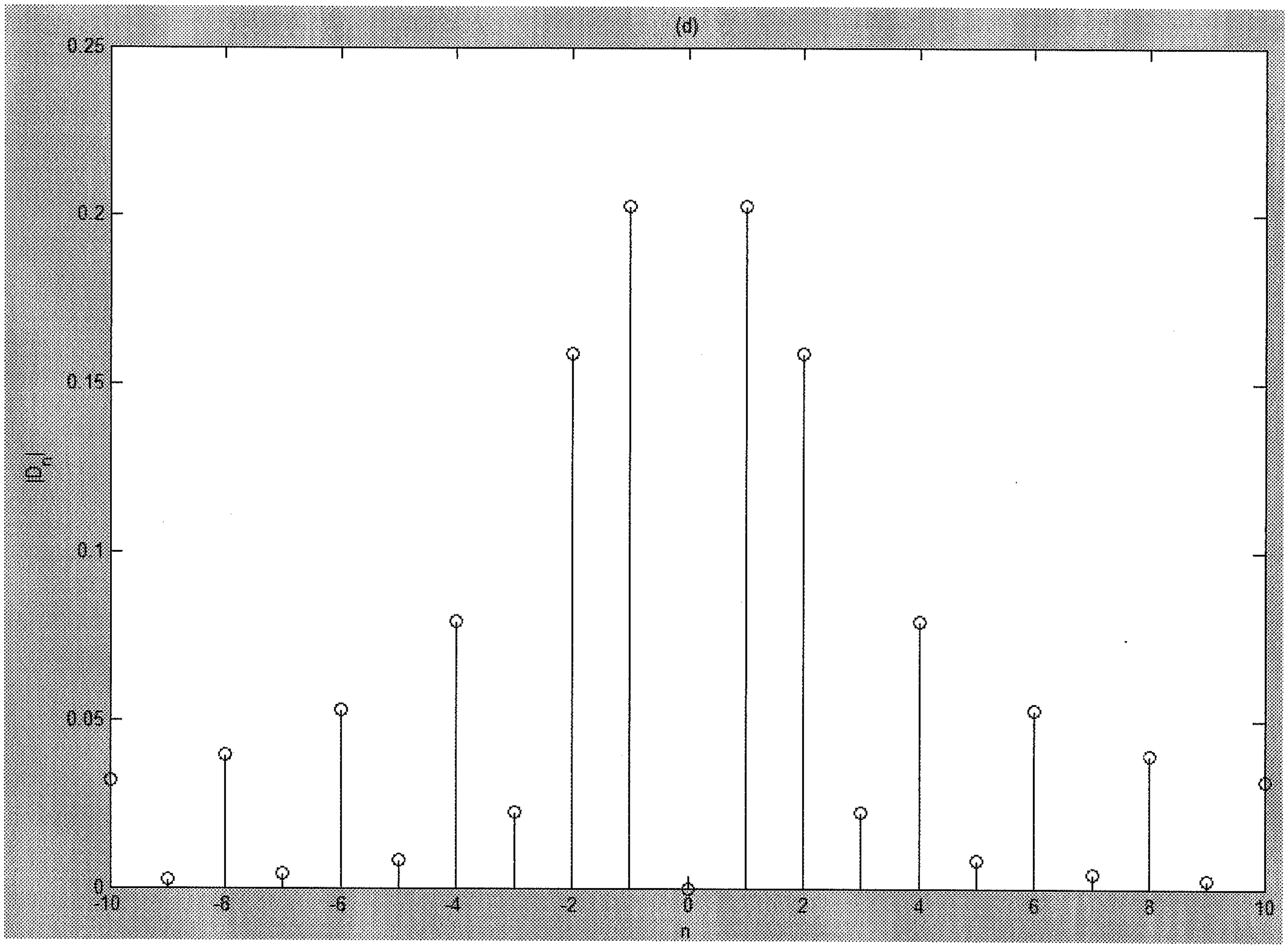
0
-0.05
-20
-15
-10
-5
0
5
10
15
20
n
Dn


(c)
|D_n|
n


∠ D_n
n


(d)
$|D_n|$
n
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0.2
0.15
0.1
0.05
0
-10
-8
-6
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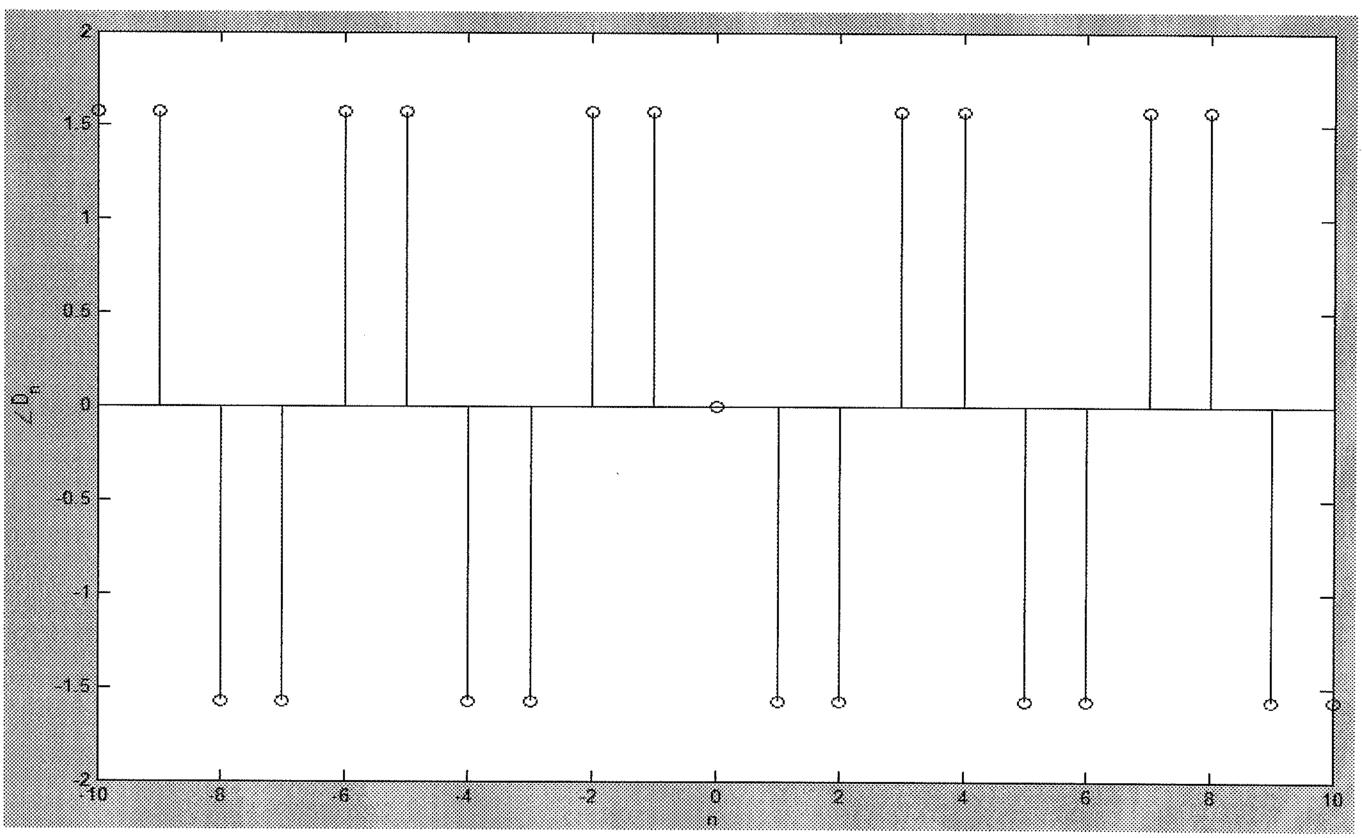
0
2
4
6
8
10


$\angle D_n$
n
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-10
-8
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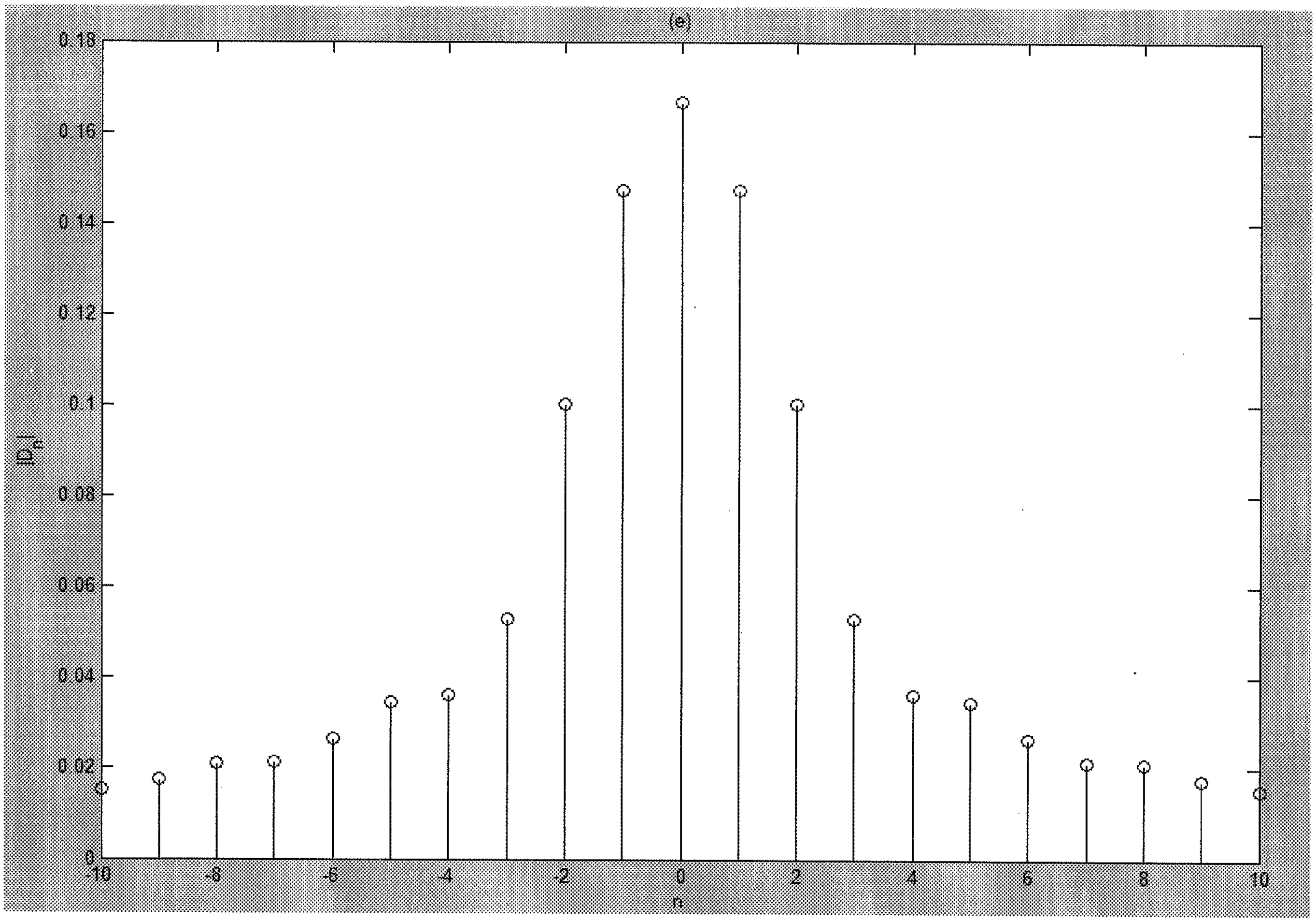
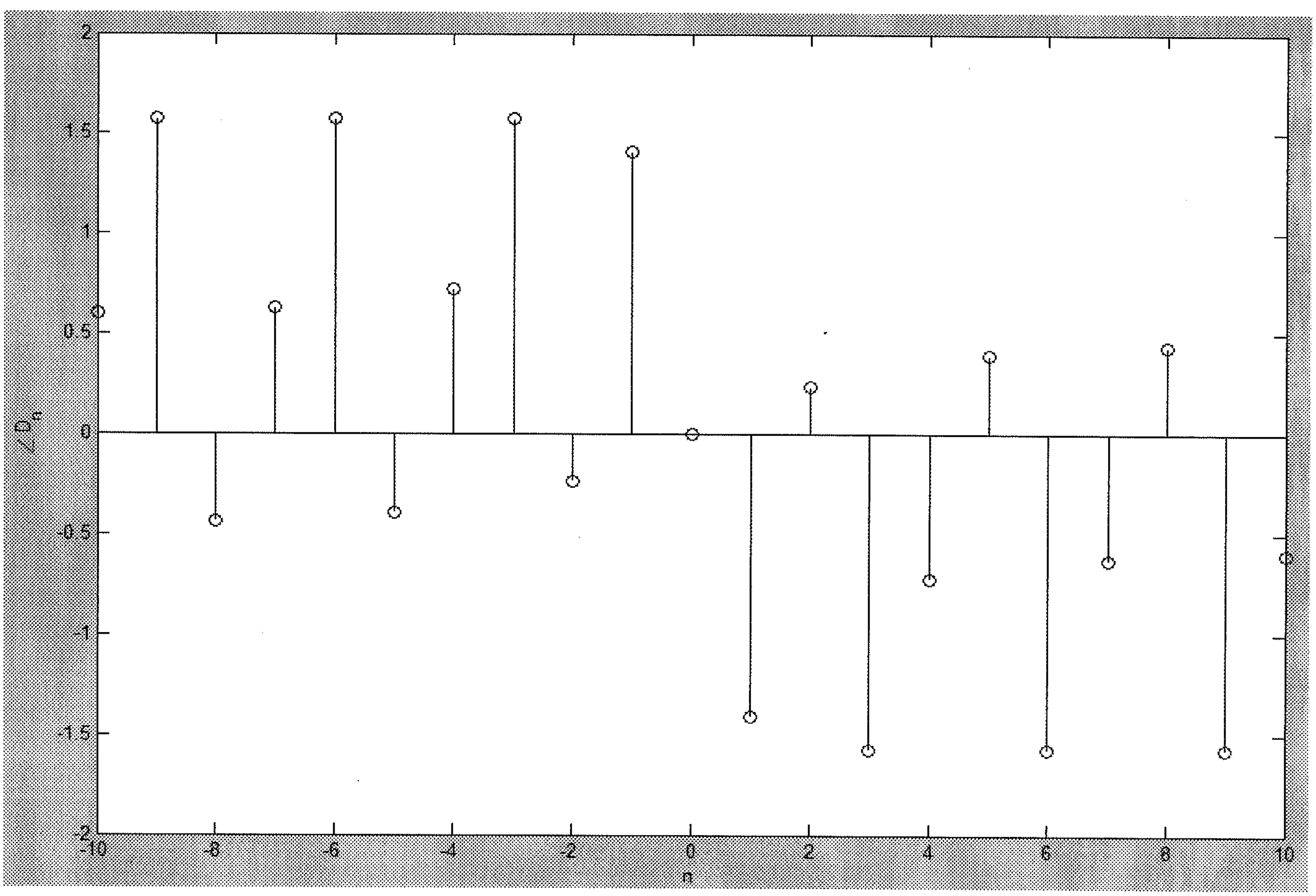
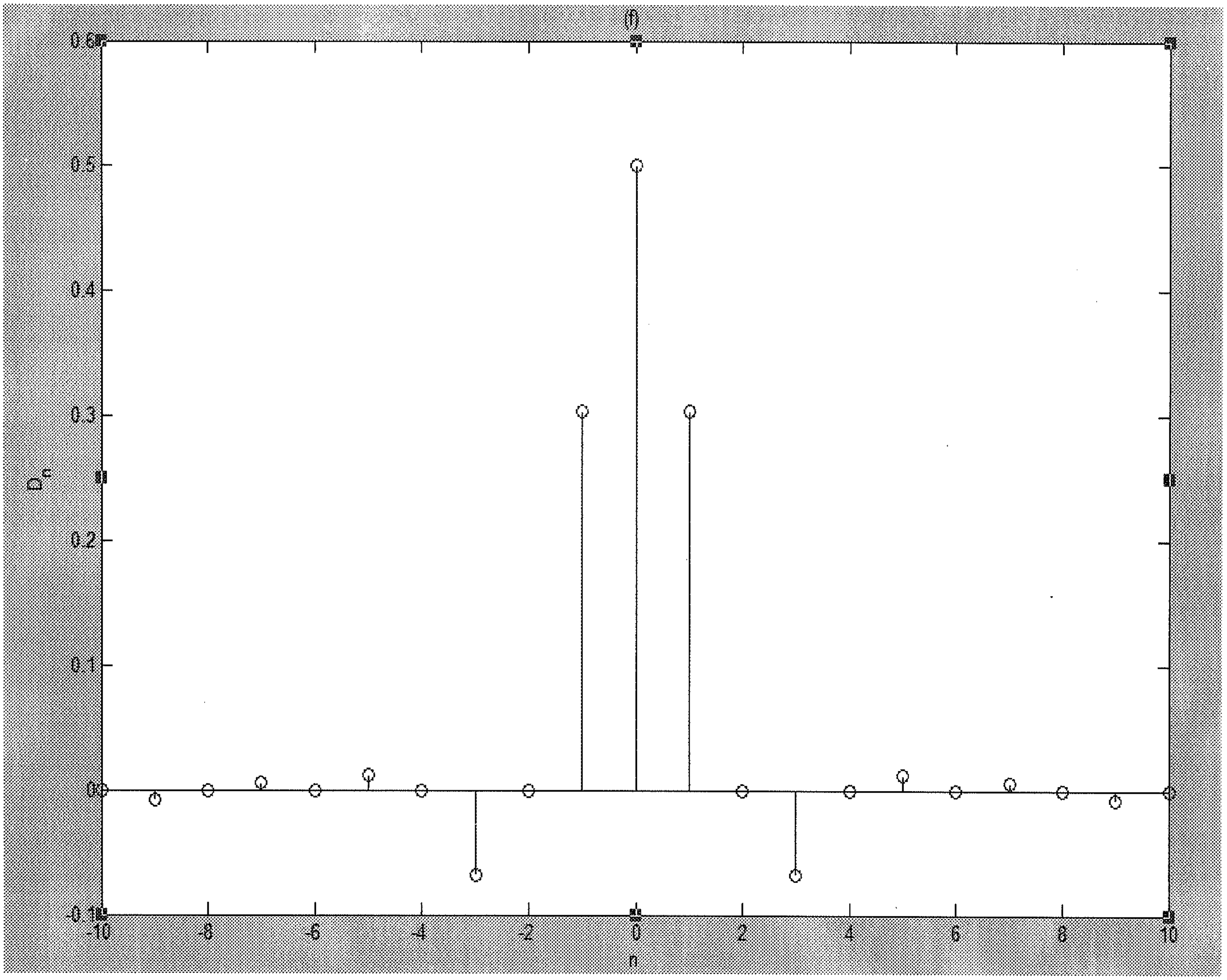
2
4
6
8
10



(e)
$|D_n|$
n



$\angle D_n$
n



(f)
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
D_n
-10
-8
-6
-4
-2
0
2
4
6
8
10
n