Problem 1.2.2 Solution

(a) The sample space of the experiment is

$$S = \{aaa, aaf, afa, faa, ffa, faf, aff, fff\}. \quad (1)$$

(b) The event that the circuit from Z fails is

$$Z_F = \{aaf, aff, faf, fff\}. \quad (2)$$

The event that the circuit from X is acceptable is

$$X_A = \{aaa, aaf, afa, aff\}. \quad (3)$$

(c) Since $Z_F \cap X_A = \{aaf, aff\} \neq \phi$, Z_F and X_A are not mutually exclusive.

(d) Since $Z_F \cup X_A = \{aaa, aaf, afa, aff, faf, fff\} \neq S$, Z_F and X_A are not collectively exhaustive.

(e) The event that more than one circuit is acceptable is

$$C = \{aaa, aaf, afa, faa\}. \quad (4)$$

The event that at least two circuits fail is

$$D = \{ffa, faf, aff, fff\}. \quad (5)$$

(f) Inspection shows that $C \cap D = \phi$ so C and D are mutually exclusive.

(g) Since $C \cup D = S$, C and D are collectively exhaustive.

Problem 1.3.2 Solution

A sample outcome indicates whether the cell phone is handheld (H) or mobile (M) and whether the speed is fast (F) or slow (W). The sample space is

$$S = \{HF, HW, MF, MW\}. \tag{1}$$

The problem statement tells us that $P[HF] = 0.2$, $P[MW] = 0.1$ and $P[F] = 0.5$. We can use these facts to find the probabilities of the other outcomes. In particular,

$$P[F] = P[HF] + P[MF]. \tag{2}$$

This implies

$$P[MF] = P[F] - P[HF] = 0.5 - 0.2 = 0.3. \tag{3}$$

Also, since the probabilities must sum to 1,

$$P[HW] = 1 - P[HF] - P[MF] - P[MW] = 1 - 0.2 - 0.3 - 0.1 = 0.4. \tag{4}$$

Now that we have found the probabilities of the outcomes, finding any other probability is easy.

(a) The probability a cell phone is slow is

$$P[W] = P[HW] + P[MW] = 0.4 + 0.1 = 0.5. \tag{5}$$

(b) The probability that a cell hpone is mobile and fast is $P[MF] = 0.3$.

(c) The probability that a cell phone is handheld is

$$P[H] = P[HF] + P[HW] = 0.2 + 0.4 = 0.6. \tag{6}$$

Problem 1.4.1 Solution

From the table we look to add all the disjoint events that contain H_0 to express the probability that a caller makes no hand-offs as

$$P[H_0] = P[LH_0] + P[BH_0] = 0.1 + 0.4 = 0.5. \tag{1}$$

In a similar fashion we can express the probability that a call is brief by

$$P[B] = P[BH_0] + P[BH_1] + P[BH_2] = 0.4 + 0.1 + 0.1 = 0.6. \tag{2}$$

The probability that a call is long or makes at least two hand-offs is

$$P[L \cup H_2] = P[LH_0] + P[LH_1] + P[LH_2] + P[BH_2] \tag{3}$$

$$= 0.1 + 0.1 + 0.2 + 0.1 = 0.5. \tag{4}$$

Problem 1.4.4 Solution

Each statement is a consequence of part 4 of Theorem 1.4.

(a) Since $A \subset A \cup B$, $P[A] \leq P[A \cup B]$.

(b) Since $B \subset A \cup B$, $P[B] \leq P[A \cup B]$.

(c) Since $A \cap B \subset A$, $P[A \cap B] \leq P[A]$.

(d) Since $A \cap B \subset B$, $P[A \cap B] \leq P[B]$.

Problem 1.4.6 Solution

(a) For convenience, let $p_i = P[FH_i]$ and $q_i = P[VH_i]$. Using this shorthand, the six unknowns $p_0, p_1, p_2, q_0, q_1, q_2$ fill the table as

$$\begin{array}{c|ccc} & H_0 & H_1 & H_2 \\ \hline F & p_0 & p_1 & p_2 \\ V & q_0 & q_1 & q_2 \end{array}. \tag{1}$$

However, we are given a number of facts:

$$p_0 + q_0 = 1/3, \qquad p_1 + q_1 = 1/3, \tag{2}$$

$$p_2 + q_2 = 1/3, \qquad p_0 + p_1 + p_2 = 5/12. \tag{3}$$

Other facts, such as $q_0 + q_1 + q_2 = 7/12$, can be derived from these facts. Thus, we have four equations and six unknowns, choosing p_0 and p_1 will specify the other unknowns. Unfortunately, arbitrary choices for either p_0 or p_1 will lead to negative values for the other probabilities. In terms of p_0 and p_1, the other unknowns are

$$q_0 = 1/3 - p_0, \qquad p_2 = 5/12 - (p_0 + p_1), \tag{4}$$

$$q_1 = 1/3 - p_1, \qquad q_2 = p_0 + p_1 - 1/12. \tag{5}$$

Because the probabilities must be nonnegative, we see that

$$0 \le p_0 \le 1/3, \tag{6}$$

$$0 \le p_1 \le 1/3, \tag{7}$$

$$1/12 \le p_0 + p_1 \le 5/12. \tag{8}$$

Although there are an infinite number of solutions, three possible solutions are:

$$p_0 = 1/3, \qquad p_1 = 1/12, \qquad p_2 = 0, \tag{9}$$

$$q_0 = 0, \qquad q_1 = 1/4, \qquad q_2 = 1/3. \tag{10}$$

and

$$p_0 = 1/4, \qquad p_1 = 1/12, \qquad p_2 = 1/12, \tag{11}$$

$$q_0 = 1/12, \qquad q_1 = 3/12, \qquad q_2 = 3/12. \tag{12}$$

and

$$p_0 = 0, \qquad p_1 = 1/12, \qquad p_2 = 1/3, \tag{13}$$

$$q_0 = 1/3, \qquad q_1 = 3/12, \qquad q_2 = 0. \tag{14}$$

(b) In terms of the p_i, q_i notation, the new facts are $p_0 = 1/4$ and $q_1 = 1/6$. These extra facts uniquely specify the probabilities. In this case,

$$p_0 = 1/4, \qquad p_1 = 1/6, \qquad p_2 = 0, \tag{15}$$

$$q_0 = 1/12, \qquad q_1 = 1/6, \qquad q_2 = 1/3. \tag{16}$$

Problem 1.6.4 Solution

(a) Since $A \cap B = \emptyset$, $P[A \cap B] = 0$. To find $P[B]$, we can write

$$P[A \cup B] = P[A] + P[B] - P[A \cap B] \tag{1}$$
$$5/8 = 3/8 + P[B] - 0. \tag{2}$$

Thus, $P[B] = 1/4$. Since A is a subset of B^c, $P[A \cap B^c] = P[A] = 3/8$. Furthermore, since A is a subset of B^c, $P[A \cup B^c] = P[B^c] = 3/4$.

(b) The events A and B are dependent because

$$P[AB] = 0 \neq 3/32 = P[A]P[B]. \tag{3}$$

(c) Since C and D are independent $P[CD] = P[C]P[D]$. So

$$P[D] = \frac{P[CD]}{P[C]} = \frac{1/3}{1/2} = 2/3. \tag{4}$$

In addition, $P[C \cap D^c] = P[C] - P[C \cap D] = 1/2 - 1/3 = 1/6$. To find $P[C^c \cap D^c]$, we first observe that

$$P[C \cup D] = P[C] + P[D] - P[C \cap D] = 1/2 + 2/3 - 1/3 = 5/6. \tag{5}$$

By De Morgan's Law, $C^c \cap D^c = (C \cup D)^c$. This implies

$$P\left[C^c \cap D^c\right] = P\left[(C \cup D)^c\right] = 1 - P[C \cup D] = 1/6. \tag{6}$$

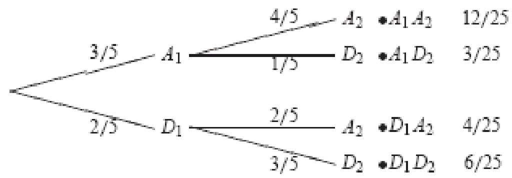
Note that a second way to find $P[C^c \cap D^c]$ is to use the fact that if C and D are independent, then C^c and D^c are independent. Thus

$$P\left[C^c \cap D^c\right] = P\left[C^c\right] P\left[D^c\right] = (1 - P[C])(1 - P[D]) = 1/6. \tag{7}$$

Finally, since C and D are independent events, $P[C|D] = P[C] = 1/2$.

Problem 1.7.6 Solution

Let A_i and D_i indicate whether the ith photodetector is acceptable or defective.



(a) We wish to find the probability $P[E_1]$ that exactly one photodetector is acceptable. From the tree, we have

$$P[E_1] = P[A_1D_2] + P[D_1A_2] = 12/25 + 4/25 = 16/25. \tag{1}$$

(b) The probability that both photodetectors are defective is $P[D_1D_2] = 6/25$.

Problem 1.9.2 Solution

Given that the probability that the Celtics win a single championship in any given year is 0.32, we can find the probability that they win 8 straight NBA championships.

$$P\left[8 \text{ straight championships}\right] = (0.32)^8 = 0.00011. \tag{1}$$

The probability that they win 10 titles in 11 years is

$$P\left[10 \text{ titles in } 11 \text{ years}\right] = \binom{11}{10}(.32)^{10}(.68) = 0.00082. \tag{2}$$

The probability of each of these events is less than 1 in 1000! Given that these events took place in the relatively short fifty year history of the NBA, it should seem that these probabilities should be much higher. What the model overlooks is that the sequence of 10 titles in 11 years started when Bill Russell joined the Celtics. In the years with Russell (and a strong supporting cast) the probability of a championship was much higher.