P11.4. $10 \cos 2\pi \times 10^6 t \cos 6\pi \times 10^6 t$

$$= 5 (\cos 4\pi \times 10^6 t + \cos 8\pi \times 10^6 t)$$

From evaluation of (11.13)-(11.15) for $I_0 = 10$ A, $dl = 1$ m, $\mu = \mu_0$, $\varepsilon = \varepsilon_0$, $r = 10$, and $\phi = \pi/3$,

for $\omega = 4\pi \times 10^6$ term,

$$\bar{E}_r = 3.8824\underline{/-91.272^\circ} \text{ V/m}$$

$$\bar{E}_\theta = 2.8681\underline{/-87.069^\circ} \text{ V/m}$$

$$\bar{H}_\phi = 3.7359 \times 10^{-3}\underline{/-1.272^\circ} \text{ A/m}$$

for $\omega = 8\pi \times 10^6$ term,

$$\bar{E}_r = 2.3358\underline{/-98.045^\circ} \text{ V/m}$$

$$\bar{E}_\theta = 1.3789\underline{/-67.591^\circ} \text{ V/m}$$

$$\bar{H}_\phi = 4.4952 \times 10^{-3}\underline{/-8.045^\circ} \text{ A/m}$$

$\therefore$ The root mean square values are

$$E_r = \sqrt{\left(\frac{3.8824}{\sqrt{2}}\right)^2 + \left(\frac{2.3358}{\sqrt{2}}\right)^2} = 3.2038 \text{ V/m}$$

$$E_\theta = \sqrt{\left(\frac{2.8681}{\sqrt{2}}\right)^2 + \left(\frac{1.3789}{\sqrt{2}}\right)^2} = 2.2503 \text{ V/m}$$

$$H_\phi = \sqrt{\left(\frac{3.7359 \times 10^{-3}}{\sqrt{2}}\right)^2 + \left(\frac{4.4952 \times 10^{-3}}{\sqrt{2}}\right)^2}$$

$$= 4.133 \times 10^{-3} \text{ A/m}$$

P11.7. For $f = 10$ MHz, $\lambda = 30$ m.

Since 1 km >> 30 m, the field is radiation field.

$\therefore$ Amplitude of **E** broadside to the dipole

$$= \frac{\eta\beta I_0\, dl}{4\pi r} = 1 \text{ mV/m}$$

$$I_0 = \frac{10^{-3} \times 4\pi \times 10^3 \times 30}{120\pi \times 2\pi \times 0.5}$$

$$= 0.3183 \text{ A}$$

Time-average power radiated

$$= \frac{1}{2} I_0^2 R_{\text{rad}}$$

$$= \frac{1}{2} \times 0.3183^2 \times 80\pi^2 \left(\frac{0.5}{30}\right)^2$$

$$= 0.0111 \text{ W}$$

P11.8.

$$f(\theta, \phi) = \begin{cases} \operatorname{cosec}^2 \theta & \text{for } \pi/6 \le \theta \le \pi/2 \\ 0 & \text{otherwise} \end{cases}$$

$$D = 4\pi \frac{[f(\theta, \phi)]_{\max}}{\int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} f(\theta, \phi) \sin \theta \, d\theta \, d\phi}$$

$$= 2 \frac{4}{\int_{\pi/6}^{\pi/2} \operatorname{cosec} \theta \, d\theta}$$

$$= \frac{8}{\left[\ln \tan \frac{\theta}{2}\right]_{\pi/6}^{\pi/2}}$$

$$= \frac{8}{0 - (-1.317)}$$

$$= 6.075$$

P11.9. From $D = \frac{[P_r]_{\max}}{[P_r]_{\text{av}}} = \frac{<[P_r]_{\max}>}{<[P_r]_{\text{av}}>}$, $<[P_r]_{\text{av}}> = \frac{<[P_r]_{\max}>}{D}$

$$\therefore \quad \frac{<[P_r]_{\text{av1}}>}{<[P_r]_{\text{av2}}>} = \frac{<[P_r]_{\max 1}>/D_1}{<[P_r]_{\max 2}>/D_2} = \frac{D_2}{D_1}$$

From $<P_{\text{rad}}>=<[P_r]_{\text{av}}> 4\pi r^2 = \frac{1}{2} I_0^2 R_{\text{rad}}$,

$$I_0 = \sqrt{\frac{<[P_r]_{\text{av}}> 8\pi r^2}{R_{\text{rad}}}}$$

$$\therefore \quad \frac{I_{01}}{I_{02}} = \sqrt{\frac{<[P_r]_{\text{av1}}> 8\pi r^2}{R_{\text{rad1}}}} \Bigg/ \sqrt{\frac{<[P_r]_{\text{av2}}> 8\pi r^2}{R_{\text{rad2}}}}$$

$$= \sqrt{\frac{<[P_r]_{\text{av1}}> R_{\text{rad2}}}{<[P_r]_{\text{av2}}> R_{\text{rad1}}}}$$

$$= \sqrt{\frac{D_2}{D_1} \frac{R_{\text{rad2}}}{R_{\text{rad1}}}}$$

P11.10. From P11.4,

$$I = 5\cos 4\pi \times 10^6 t + 5\cos 8\pi \times 10^6 t$$

For $\omega = 4\pi \times 10^6, f = 2 \times 10^6$,

$$\lambda = \frac{3\times 10^8}{2\times 10^6} = 150 \text{ m}$$

$$R_{\text{rad}} = 80\pi^2\left(\frac{dl}{\lambda}\right)^2 = 80\pi^2\left(\frac{1}{150}\right)^2$$

$$= 0.0351\ \Omega$$

$$P_{\text{rad}} = \frac{1}{2}\times 5^2 \times 0.0351$$

$$= 0.4386 \text{ W}$$

For $\omega = 8\pi \times 10^6, f = 4 \times 10^6$,

$$\lambda = \frac{3\times 10^8}{4\times 10^6} = 75 \text{ m}$$

$$R_{\text{rad}} = 80\pi^2\left(\frac{1}{75}\right)^2 = 0.1404\ \Omega$$

$$P_{\text{rad}} = \frac{1}{2}\times 5^2 \times 0.1404$$

$$= 1.7546 \text{ W}$$

$\therefore$ Time-average power radiated

$$= 0.4386 + 1.75746$$

$$= 2.1932 \text{ W}$$