B

VECTOR IDENTITIES AND THEOREMS

$$\overline{A} = \hat{x}\, A_x + \hat{y}\, A_y + \hat{z}\, A_z$$

$$\overline{A} + \overline{B} = \hat{x}\,(A_x + B_x) + \hat{y}\,(A_y + B_y) + \hat{z}\,(A_z + B_z)$$

$$\overline{A} \cdot \overline{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\overline{A} \times \overline{B} = \det \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \hat{x}\,(A_y B_z - A_z B_y) + \hat{y}\,(A_z B_x - A_x B_z) + \hat{z}\,(A_x B_y - A_y B_x)$$

$$\overline{A} \cdot (\overline{B} \times \overline{C}) = \overline{B} \cdot (\overline{C} \times \overline{A}) = \overline{C} \cdot (\overline{A} \times \overline{B})$$

$$\overline{A} \times (\overline{B} \times \overline{C}) = (\overline{A} \cdot \overline{C})\, \overline{B} - (\overline{A} \cdot \overline{B})\, \overline{C}$$

$$(\overline{A} \times \overline{B}) \cdot (\overline{C} \times \overline{D}) = (\overline{A} \cdot \overline{C})\,(\overline{B} \cdot \overline{D}) - (\overline{A} \cdot \overline{D})\,(\overline{B} \cdot \overline{C})$$

$$\nabla \times \nabla \Psi = 0$$

$$\nabla \cdot (\nabla \times \overline{A}) = 0$$

$$\nabla \times (\nabla \times \overline{A}) = \nabla(\nabla \cdot \overline{A}) - \nabla^2 \overline{A}$$

$$(\nabla \times \overline{A}) \times \overline{A} = (\overline{A} \cdot \nabla)\, \overline{A} - \frac{1}{2}\, \nabla(\overline{A} \cdot \overline{A})$$

$$\nabla(\Psi \Phi) = \Psi\, \nabla \Phi + \Phi\, \nabla \Psi$$

$$\nabla \cdot (\Psi \overline{A}) = \overline{A} \cdot \nabla \Psi + \Psi \nabla \cdot \overline{A}$$

$$\nabla \times (\Psi \overline{A}) = \nabla \Psi \times \overline{A} + \Psi \nabla \times \overline{A}$$
$$\nabla(\overline{A} \cdot \overline{B}) = (\overline{A} \cdot \nabla)\,\overline{B} + (\overline{B} \cdot \nabla)\,\overline{A} + \overline{A} \times (\nabla \times \overline{B}) + \overline{B} \times (\nabla \times \overline{A})$$
$$\nabla \cdot (\overline{A} \times \overline{B}) = \overline{B} \cdot (\nabla \times \overline{A}) - \overline{A} \cdot (\nabla \times \overline{B})$$
$$\nabla \times (\overline{A} \times \overline{B}) = \overline{A}\,(\nabla \cdot \overline{B}) - \overline{B}\,(\nabla \cdot \overline{A}) + (\overline{B} \cdot \nabla)\,\overline{A} - (\overline{A} \cdot \nabla)\,\overline{B}$$

Gauss's Divergence Theorem:

$$\int_V \nabla \cdot \overline{G}\, dv = \oint_A \overline{G} \cdot \hat{n}\, da$$

Stokes's Theorem:

$$\int_A (\nabla \times \overline{G}) \cdot \hat{n}\, da = \oint_C \overline{G} \cdot d\overline{\ell}$$

EXPLICIT FORMS OF VECTOR OPERATORS

Cartesian (x, y, z) :

$$\nabla \Psi = \hat{x}\,\frac{\partial \Psi}{\partial x} + \hat{y}\,\frac{\partial \Psi}{\partial y} + \hat{z}\,\frac{\partial \Psi}{\partial z}$$
$$\nabla \cdot \overline{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$
$$\nabla \times \overline{A} = \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$
$$\nabla^2 \Psi = \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2}$$

Cylindrical (ρ, ϕ, z) :

$$\nabla \Psi = \hat{\rho}\,\frac{\partial \Psi}{\partial \rho} + \hat{\phi}\,\frac{1}{\rho}\,\frac{\partial \Psi}{\partial \phi} + \hat{z}\,\frac{\partial \Psi}{\partial z}$$
$$\nabla \cdot \overline{A} = \frac{1}{\rho}\,\frac{\partial (\rho A_\rho)}{\partial \rho} + \frac{1}{\rho}\,\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$
$$\nabla \times \overline{A} = \hat{\rho}\left(\frac{1}{\rho}\,\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right) + \hat{z}\,\frac{1}{\rho}\left(\frac{\partial (\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi}\right)$$
$$\nabla^2 \Psi = \frac{1}{\rho}\,\frac{\partial}{\partial \rho}\left(\rho\,\frac{\partial \Psi}{\partial \rho}\right) + \frac{1}{\rho^2}\,\frac{\partial^2 \Psi}{\partial \phi^2} + \frac{\partial^2 \Psi}{\partial z^2}$$

Spherical (r, θ, ϕ) :

$$\nabla\Psi = \hat{r}\,\frac{\partial\Psi}{\partial r} + \hat{\theta}\,\frac{1}{r}\,\frac{\partial\Psi}{\partial\theta} + \hat{\phi}\,\frac{1}{r\sin\theta}\,\frac{\partial\Psi}{\partial\phi}$$

$$\nabla\cdot\overline{A} = \frac{1}{r^2}\,\frac{\partial(r^2A_r)}{\partial r} + \frac{1}{r\sin\theta}\,\frac{\partial(\sin\theta A_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\,\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla\times\overline{A} = \hat{r}\,\frac{1}{r\sin\theta}\left(\frac{\partial(\sin\theta A_\phi)}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi}\right) + \hat{\theta}\left(\frac{1}{r\sin\theta}\,\frac{\partial A_r}{\partial\phi} - \frac{1}{r}\,\frac{\partial(rA_\phi)}{\partial r}\right) + \hat{\phi}\,\frac{1}{r}\left(\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right)$$

$$\nabla^2\Psi = \frac{1}{r^2}\,\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\,\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\,\frac{\partial^2\Psi}{\partial\phi^2}$$