P1.7.

(a) $$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \begin{vmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \\ A_1 & A_2 & A_3 \\ (B_2C_3 - B_3C_2) & (B_3C_1 - B_1C_3) & (B_1C_2 - B_2C_1) \end{vmatrix}$$

$$= (A_2B_1C_2 - A_2B_2C_1 - A_3B_3C_1 + A_3B_1C_3)\mathbf{a}_1$$

$$+ (A_3B_2C_3 - A_3B_3C_2 - A_1B_1C_2 + A_1B_2C_1)\mathbf{a}_2$$

$$+ (A_1B_3C_1 - A_1B_1C_3 - A_2B_2C_3 + A_2B_3C_2)\mathbf{a}_3$$

$$= (A_1C_1 + A_2C_2 + A_3C_3)B_1\mathbf{a}_1 + (A_1C_1 + A_2C_2 + A_3C_3)B_2\mathbf{a}_2$$

$$+ (A_1C_1 + A_2C_2 + A_3C_3)B_3\mathbf{a}_3 - (A_1B_1 + A_2B_2 + A_3B_3)C_1\mathbf{a}_1$$

$$- (A_1B_1 + A_2B_2 + A_3B_3)C_2\mathbf{a}_2 - (A_1B_1 + A_2B_2 + A_3B_3)C_3\mathbf{a}_3$$

$$= (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C}$$

(b) (i) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) + \mathbf{B} \times (\mathbf{C} \times \mathbf{A}) + \mathbf{C} \times (\mathbf{A} \times \mathbf{B})$

$$= (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C} + (\mathbf{B} \cdot \mathbf{A})\mathbf{C} - (\mathbf{B} \cdot \mathbf{C})\mathbf{A} + (\mathbf{C} \cdot \mathbf{B})\mathbf{A} - (\mathbf{C} \cdot \mathbf{A})\mathbf{B}$$

$$= \mathbf{0}$$

(ii) $(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{B} \times \mathbf{C}) \times (\mathbf{C} \times \mathbf{A})$

$$= (\mathbf{A} \times \mathbf{B}) \cdot [(\mathbf{B} \times \mathbf{C} \cdot \mathbf{A})\mathbf{C} - (\mathbf{B} \times \mathbf{C} \cdot \mathbf{C})\mathbf{A}]$$

= 0

$$= (\mathbf{B} \times \mathbf{C} \cdot \mathbf{A})(\mathbf{A} \times \mathbf{B} \cdot \mathbf{C})$$

$$= (\mathbf{A} \times \mathbf{B} \cdot \mathbf{C})^2$$

P1.29. $f(z, t) = 10 \cos (3\pi \times 10^7 t + 0.1\pi z)$

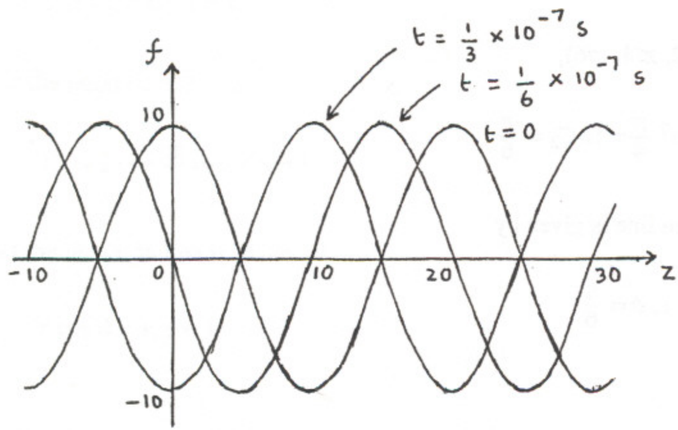
$$f(z, 0) = 10 \cos 0.1\pi z$$

$$f\left(z, \frac{1}{6} \times 10^{-7}\right) = 10 \cos \left(\frac{\pi}{2} + 0.1\pi z\right)$$

$$= -10 \sin 0.1\,\pi z$$

$$f\left(z, \frac{1}{3} \times 10^{-7}\right) = 10 \cos (\pi + 0.1\pi z)$$

$$= -10 \sin 0.1\pi z$$



$f(z, t)$ represents a traveling wave propagating in the $-z$ direction with velocity $\dfrac{5\text{ m}}{\frac{1}{6} \times 10^{-7}\text{ s}} = 3 \times 10^8$ m/s.

P2.3. **(a)** For a solution to exist, a necessary (but not sufficient) condition is that there must be a point of intersection between the straight lines along the field vectors. Thus the two vectors $\mathbf{E}_1 = (2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z)$ V/m at (2, 2, 3) and $\mathbf{E}_2 = (\mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z)$ at (–1, 0, 3) must lie in a plane, or, the determinant

$$\begin{vmatrix} 2 & 2 & 1 \\ 1 & 2 & 2 \\ 3 & 2 & 0 \end{vmatrix}$$

must be zero. Since it is equal to (–8 + 12 – 4) = 0, the two vectors do lie in a plane and hence there is a point of intersection. The equations of the two straight lines are

$$\frac{2-x}{2} = \frac{2-y}{2} = \frac{3-z}{1} \quad \text{or,} \quad x = y, x - 2z = -4$$

and

$$\frac{-1-x}{1} = \frac{0-y}{2} = \frac{3-z}{2} \quad \text{or,} \quad 2x - y = -2, 2x - z = -5$$

and hence the point of intersection is (–2, –2, 1).

Assuming a point charge Q at (–2, –2, 1), its value required to produce $\mathbf{E}_1$ is given by

$$\frac{Q(4\mathbf{a}_x + 4\mathbf{a}_y + 2\mathbf{a}_z)}{4\pi\varepsilon_0(16+16+4)^{3/2}} = 2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z$$

or, $Q = 432\pi\varepsilon_0$. Value of Q required to produce $\mathbf{E}_2$ is given by

$$\frac{Q(\mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z)}{4\pi\varepsilon_0(1+4+4)^{3/2}} = \mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z$$

or, $Q = 108\pi\varepsilon_0$. Since the two values of Q are not the same, there is no solution to the problem.

(b) Following in the same manner as in **(a)**, we first check to see if $\mathbf{E}_1$ and $\mathbf{E}_2$ lie in a plane. Since

P2.3. (continued)

$$\begin{vmatrix} 2 & 2 & 1 \\ 2 & 1 & 2 \\ 0 & 1 & -1 \end{vmatrix} = -6 + 4 + 2 = 0$$

the two vectors do lie in a plane and hence there is a point of intersection. The equations of the two straight lines are

$$\frac{1-x}{2} = \frac{1-y}{2} = \frac{1-z}{1} \quad \text{or,} \quad x = y,\ y - 2z = -1$$

and

$$\frac{1-x}{2} = \frac{2-y}{1} = \frac{0-z}{2} \quad \text{or,} \quad x - 2y = -3,\ 2y - z = 4$$

and the point of intersection is (3, 3, 2). Value of Q required to produce $\mathbf{E}_1$ is given by

$$\frac{Q(-2\mathbf{a}_x - 2\mathbf{a}_y - \mathbf{a}_z)}{4\pi\varepsilon_0(4+4+1)^{3/2}} = 2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z$$

or, $Q = -108\pi\varepsilon_0$. Value of Q required to produce $\mathbf{E}_2$ is given by

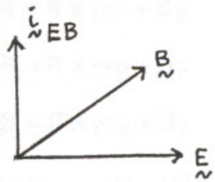
$$\frac{Q(-2\mathbf{a}_x - \mathbf{a}_y - 2\mathbf{a}_z)}{4\pi\varepsilon_0(4+1+4)^{3/2}} = 2\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z$$

or, $Q = -108\pi\varepsilon_0$. Thus the solution is $-108\pi\varepsilon_0$ C at the point (3, 3, 2).

P2.22. $q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$

$$= q\mathbf{E} + q\left(\frac{\mathbf{E} \times \mathbf{B}}{B^2}\right) \times \mathbf{B}$$

$$= q\mathbf{E} + \frac{q}{B^2}(EB\mathbf{a}_{EB} \times \mathbf{B})$$



where $\mathbf{a}_{EB}$ is unit vector perpendicular to both $\mathbf{E}$ and $\mathbf{B}$ and in the right-hand sense.

Noting that $\mathbf{a}_{EB} \times \mathbf{B} = -B\,\frac{\mathbf{E}}{E}$ and proceeding further, we have

$q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$

$$= q\mathbf{E} + \frac{q}{B^2}(EB)\left(-B\,\frac{\mathbf{E}}{E}\right)$$

$$= q\mathbf{E} - q\mathbf{E}$$

$$= \mathbf{0}$$

Hence, the test charge moves with constant velocity equal to the initial value.

For $\mathbf{E} = E_0(2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z)$ and $\mathbf{B} = B_0(\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z)$,

$$\mathbf{v} = \frac{E_0(2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z) \times B_0(\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z)}{\left|B_0(\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z)\right|^2}$$

$$= \frac{E_0 B_0}{9B_0^2}\,(6\mathbf{a}_x - 3\mathbf{a}_y - 6\mathbf{a}_z)$$

$$= \frac{E_0}{3B_0}\,(2\mathbf{a}_x - \mathbf{a}_y - 2\mathbf{a}_z)$$

P3.2. $\mathbf{F} \cdot d\mathbf{l} = (xy\mathbf{a}_x + yz\mathbf{a}_y + zx\mathbf{a}_z) \cdot (dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z)$

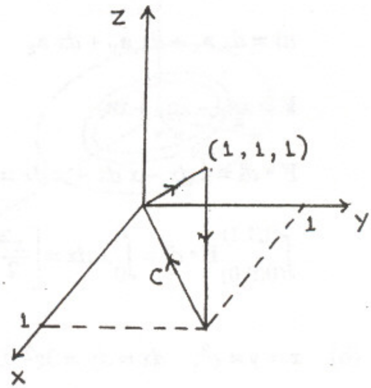
$= xy\,dx + yz\,dy + zx\,dz$

From (0, 0, 0) to (1, 1, 1),

$x = y = z,\ dx = dy = dz$

$\mathbf{F} \cdot d\mathbf{l} = x^2\,dx + x^2\,dx + x^2\,dx = 3x^2\,dx$

$$\int_{(0,0,0)}^{(1,1,1)} \mathbf{F} \cdot d\mathbf{l} = \int_0^1 3x^2\,dx = \left[x^3\right]_0^1 = 1$$



From (1, 1, 1) to (1, 1, 0),

$x = y = 1,\ dx = dy = 0$

$\mathbf{F} \cdot d\mathbf{l} = 0 + 0 + z\,dz = z\,dz$

$$\int_{(1,1,1)}^{(1,1,0)} \mathbf{F} \cdot d\mathbf{l} = \int_1^0 z\,dz = \left[\frac{z^2}{2}\right]_1^0 = -\frac{1}{2}$$

From (1, 1, 0) to (0, 0, 0),

$y = x,\ z = 0;\ dy = dx,\ dz = 0$

$\mathbf{F} \cdot d\mathbf{l} = x^2\,dx + 0 + 0 = x^2\,dx$

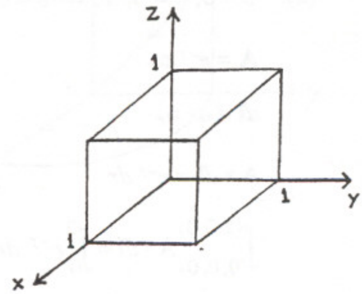
$$\int_{(1,1,0)}^{(0,0,0)} \mathbf{F} \cdot d\mathbf{l} = \int_1^0 x^2\,dx = \left[\frac{x^3}{3}\right]_1^0 = -\frac{1}{3}$$

$$\therefore \oint_C \mathbf{F} \cdot d\mathbf{l} = 1 - \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

P3.6. $\mathbf{A} = x^2yz\mathbf{a}_x + y^2zx\mathbf{a}_y + z^2xy\mathbf{a}_z$

For $x = 0, y = 0, z = 0, \mathbf{A} = 0$

$$\int \mathbf{A} \cdot d\mathbf{S} = 0$$



For $x = 1$,

$$\mathbf{A} = yz\mathbf{a}_x + y^2z\mathbf{a}_y + z^2y\mathbf{a}_z$$

$$d\mathbf{S} = dy\, dz\, \mathbf{a}_x$$

$$\mathbf{A} \cdot d\mathbf{S} = yz\, dy\, dz$$

$$\int \mathbf{A} \cdot d\mathbf{S} = \int_{y=0}^{1}\int_{z=0}^{1} yz\, dy\, dz = \frac{1}{4}$$

For $y = 1$,

$$\mathbf{A} = x^2z\mathbf{a}_x + zx\mathbf{a}_y + z^2x\mathbf{a}_z$$

$$d\mathbf{S} = dz\, dx\, \mathbf{a}_y$$

$$\mathbf{A} \cdot d\mathbf{S} = zx\, dz\, dx$$

$$\int \mathbf{A} \cdot d\mathbf{S} = \int_{z=0}^{1}\int_{x=0}^{1} zx\, dz\, dx = \frac{1}{4}$$

For $z = 1$,

$$\mathbf{A} = x^2y\mathbf{a}_x + y^2x\mathbf{a}_y + xy\mathbf{a}_z$$

$$d\mathbf{S} = dx\, dy\, \mathbf{a}_z$$

$$\mathbf{A} \cdot d\mathbf{S} = xy\, dx\, dy$$

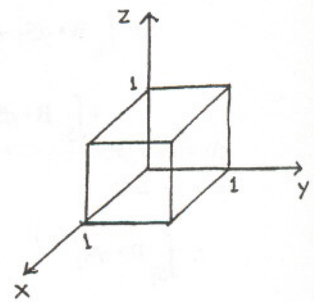
$$\int \mathbf{A} \cdot d\mathbf{S} = \int_{x=0}^{1}\int_{y=0}^{1} xy\, dx\, dy = \frac{1}{4}$$

$$\therefore \oint_S \mathbf{A} \cdot d\mathbf{S} = 0 + 0 + 0 + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

P3.24. $\mathbf{J} = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$

$$\oint_S \mathbf{J} \cdot d\mathbf{S} + \frac{d}{dt}\int_V \rho\, dv = 0$$

$$-\frac{d}{dt}\int_V \rho\, dv = \oint_S \mathbf{J} \cdot d\mathbf{S}$$

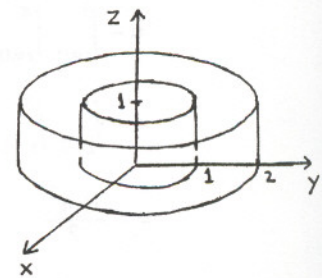


(a) $\oint_S \mathbf{J} \cdot d\mathbf{S} = 0 + 0 + 0 + 1 + 1 + 1 = 3$

$$-\frac{d}{dt}\int_V \rho\, dv = 3 \text{ A}$$

(b) $\mathbf{J} = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$

$= r_c\mathbf{a}_{rc} + z\mathbf{a}_z$



$$\oint_S \mathbf{J} \cdot d\mathbf{S} = 4\pi \times 2 - 2\pi \times 1$$

$$+ 0 + (4\pi - \pi) \times 1$$

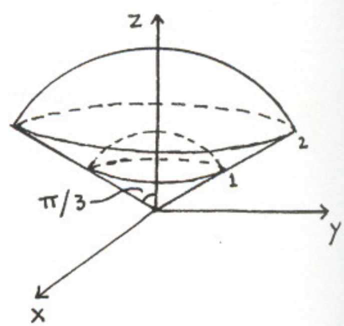
$$= 8\pi - 2\pi + 3\pi$$

$$= 9\pi$$

$$-\frac{d}{dt}\int_V \rho\, dv = 9\pi \text{ A}$$

(c) $\mathbf{J} = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$

$= r_s\mathbf{a}_{rs}$



$$\oint_S \mathbf{J} \cdot d\mathbf{S} = \int_{\theta=0}^{\pi/3}\int_{\phi=0}^{2\pi} 8 \sin\theta\, d\theta\, d\phi$$

$$- \int_{\theta=0}^{\pi/3}\int_{\phi=0}^{2\pi} \sin\theta\, d\theta\, d\phi + 0$$

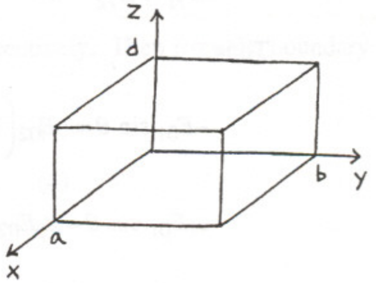
$$= 14\pi\, [-\cos\theta]_0^{\pi/3} = 7\pi$$

$$-\frac{d}{dt}\int_V \rho\, dv = 7\pi \text{ A}$$

P3.37. $\mathbf{E} = E_0 \sin \frac{\pi x}{a} \sin \frac{\pi z}{d} \cos \omega t \, \mathbf{a}_y$

$$\mathbf{H} = H_{01} \sin \frac{\pi x}{a} \cos \frac{\pi z}{d} \sin \omega t \, \mathbf{a}_x$$

$$- H_{02} \cos \frac{\pi x}{a} \sin \frac{\pi z}{d} \sin \omega t \, \mathbf{a}_z$$



Using $\rho_S = \mathbf{a}_n \cdot \mathbf{D} = \mathbf{a}_n \cdot 4\varepsilon_0 \mathbf{E}_0$, we obtain

$$[\rho_S]_{x=0} = \mathbf{a}_x \cdot 4\varepsilon_0 [\mathbf{E}]_{x=0} = 0$$

$$[\rho_S]_{x=a} = -\mathbf{a}_x \cdot 4\varepsilon_0 [\mathbf{E}]_{x=a} = 0$$

$$[\rho_S]_{y=0} = \mathbf{a}_y \cdot 4\varepsilon_0 [\mathbf{E}]_{y=0} = 4\varepsilon_0 E_0 \sin \frac{\pi x}{a} \sin \frac{\pi z}{d} \cos \omega t$$

$$[\rho_S]_{y=b} = -\mathbf{a}_y \cdot 4\varepsilon_0 [\mathbf{E}]_{y=b} = - 4\varepsilon_0 E_0 \sin \frac{\pi x}{a} \sin \frac{\pi z}{d} \cos \omega t$$

$$[\rho_S]_{z=0} = \mathbf{a}_z \cdot 4\varepsilon_0 [\mathbf{E}]_{z=0} = 0$$

$$[\rho_S]_{z=d} = -\mathbf{a}_z \cdot 4\varepsilon_0 [\mathbf{E}]_{z=d} = 0$$

Using $\mathbf{J}_S = \mathbf{a}_n \times \mathbf{H}$, we obtain

$$[\mathbf{J}_S]_{x=0} = \mathbf{a}_x \times [\mathbf{H}]_{x=0} = H_{02} \sin \frac{\pi z}{d} \sin \omega t \, \mathbf{a}_y$$

$$[\mathbf{J}_S]_{x=a} = -\mathbf{a}_x \times [\mathbf{H}]_{x=a} = H_{02} \sin \frac{\pi z}{d} \sin \omega t \, \mathbf{a}_y$$

$$[\mathbf{J}_S]_{y=0} = \mathbf{a}_y \times [\mathbf{H}]_{y=0} = -H_{02} \cos \frac{\pi x}{a} \sin \frac{\pi z}{d} \sin \omega t \, \mathbf{a}_x$$

$$- H_{01} \sin \frac{\pi x}{a} \cos \frac{\pi z}{d} \sin \omega t \, \mathbf{a}_z$$

$$[\mathbf{J}_S]_{y=b} = -\mathbf{a}_y \times [\mathbf{H}]_{y=b} = H_{02} \cos \frac{\pi x}{a} \sin \frac{\pi z}{d} \sin \omega t \, \mathbf{a}_x$$

$$+ H_{01} \sin \frac{\pi x}{a} \cos \frac{\pi z}{d} \sin \omega t \, \mathbf{a}_z$$

$$[\mathbf{J}_S]_{z=0} = \mathbf{a}_z \times [\mathbf{H}]_{z=0} = H_{01} \sin \frac{\pi x}{a} \sin \omega t \, \mathbf{a}_y$$

$$[\mathbf{J}_S]_{z=d} = -\mathbf{a}_z \times [\mathbf{H}]_{z=d} = H_{01} \sin \frac{\pi x}{a} \sin \omega t \, \mathbf{a}_y$$

P4.2. (a) $\mathbf{E} = E_0 \cos 3\pi z \cos 9\pi \times 10^8 t\, \mathbf{a}_x$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} = -\begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 0 & 0 & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} = -\frac{\partial E_x}{\partial z}\mathbf{a}_y$$

$$= 3\pi E_0 \sin 3\pi z \cos 9\pi \times 10^8 t\, \mathbf{a}_y$$

$$\mathbf{B} = \frac{3\pi E_0}{9\pi \times 10^8} \sin 3\pi z \sin 9\pi \times 10^8 t\, \mathbf{a}_y$$

$$= \frac{E_0}{3 \times 10^8} \sin 3\pi z \sin 9\pi \times 10^8 t\, \mathbf{a}_y$$

(b) $\mathbf{E} = E_0 \mathbf{a}_y \cos [3\pi \times 10^8 t + 0.2\pi(4x + 3z)]$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} = -\begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix} = \frac{\partial E_y}{\partial z}\mathbf{a}_x - \frac{\partial E_y}{\partial x}\mathbf{a}_z$$

$$= E_0(-0.6\pi \mathbf{a}_x + 0.8\pi \mathbf{a}_z) \sin [3\pi \times 10^8 t + 0.2\pi(4x + 3z)]$$

$$\mathbf{B} = -\frac{E_0(-0.6\pi \mathbf{a}_x + 0.8\pi \mathbf{a}_z)}{3\pi \times 10^8} \cos [3\pi \times 10^8 t + 0.2\pi(4x + 3z)]$$

$$= \frac{E_0(0.6\mathbf{a}_x - 0.8\mathbf{a}_z)}{3 \times 10^8} \cos [3\pi \times 10^8 t + 0.2\pi(4x + 3z)]$$

P4.4. $\mathbf{E} = E_0 e^{-\alpha z} \cos \omega t \, \mathbf{a}_x$

$$-\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{E} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 0 & 0 & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix}$$

$$= \frac{\partial E_x}{\partial z} \mathbf{a}_y = -\alpha E_0 e^{-\alpha z} \cos \omega t \, \mathbf{a}_y$$

$$\mathbf{B} = \frac{\alpha E_0}{\omega} e^{-\alpha z} \sin \omega t \, \mathbf{a}_y$$

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} = \frac{\alpha E_0}{\omega \mu_0} e^{-\alpha z} \sin \omega t \, \mathbf{a}_y$$

$$\nabla \times \mathbf{H} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 0 & 0 & \frac{\partial}{\partial z} \\ 0 & H_y & 0 \end{vmatrix}$$

$$= -\frac{\partial H_y}{\partial z} \mathbf{a}_x = \frac{\alpha^2 E_0}{\omega \mu_0} e^{-\alpha z} \sin \omega t \, \mathbf{a}_x$$

$$\frac{\partial \mathbf{D}}{\partial t} = \frac{\partial}{\partial t}\left(\varepsilon_0 E_0 e^{-\alpha z} \cos \omega t \, \mathbf{a}_x\right)$$

$$= -\omega \varepsilon_0 E_0 e^{-\alpha z} \sin \omega t \, \mathbf{a}_x$$

For $\nabla \times \mathbf{H}$ to be equal to $\frac{\partial \mathbf{D}}{\partial t}$, $\frac{\alpha^2 E_0}{\omega \mu_0}$ must be equal to $-\omega \varepsilon_0 E_0$, or α^2 must be equal to $-\omega^2 \mu_0 \varepsilon_0$. Since this is not possible for real values of α, the pair of **E** and **B** do not satisfy Ampere's circuital law in differential form.

P4.8. **(a)** $\nabla \cdot (zx\mathbf{a}_x + xy\mathbf{a}_y + yz\mathbf{a}_z)$

$$= \frac{\partial}{\partial x}(zx) + \frac{\partial}{\partial y}(xy) + \frac{\partial}{\partial z}(yz)$$

$$= z + x + y$$

$$= x + y + z$$

(b) $\nabla \cdot [3\mathbf{a}_x + (y-3)\mathbf{a}_y + (2+z)\mathbf{a}_z]$

$$= \frac{\partial}{\partial x}(3) + \frac{\partial}{\partial y}(y-3) + \frac{\partial}{\partial z}(2+z)$$

$$= 0 + 1 + 1$$

$$= 2$$

(c) $\nabla \cdot r \sin\phi\, \mathbf{a}_\phi$

$$= \frac{1}{r}\frac{\partial}{\partial r}(0) + \frac{1}{r}\frac{\partial}{\partial \phi}(r\sin\phi) + \frac{\partial}{\partial z}(0)$$

$$= \cos\phi$$

(d) $\nabla \cdot r\cos\theta\,(\cos\theta\,\mathbf{a}_r - \sin\theta\,\mathbf{a}_\theta)$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}(r^3\cos^2\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(-r\cos\theta\sin^2\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(0)$$

$$= 3\cos^2\theta + \frac{1}{\sin\theta}(\sin^3\theta - 2\sin\theta\cos^2\theta)$$

$$= 3\cos^2\theta + \sin^2\theta - 2\cos^2\theta$$

$$= 1$$

P4.16. (continued)

(b) $\mathbf{A} = \cos y\, \mathbf{a}_x - x \sin y\, \mathbf{a}_y$

$$\oint_C \mathbf{A} \cdot d\mathbf{l} = \oint_C (\cos y\, \mathbf{a}_x - x \sin y\, \mathbf{a}_y) \cdot (dx\, \mathbf{a}_x + dy\, \mathbf{a}_y + dz\, \mathbf{a}_z)$$

$$= \oint_C \cos y\, dx - x \sin y\, dy$$

$$= \oint_C d(x \cos y)$$

$$= \left[x \cos y\right]_{x_1,\, y_1,\, z_1}^{x_1,\, y_1,\, z_1}$$

$$= 0 \text{ for any } C$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ \cos y & -x \sin y & 0 \end{vmatrix}$$

$$= (-\sin y + \sin y)\, \mathbf{a}_z$$

$$= \mathbf{0}$$

$$\therefore \int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{S} = 0 \text{ for any } S$$

Thus Stokes' theorem is verified, without choosing any particular path.

P4.18. (a) $\nabla \cdot \nabla \times \mathbf{A}$

$$= \left(\mathbf{a}_x \frac{\partial}{\partial x} + \mathbf{a}_y \frac{\partial}{\partial y} + \mathbf{a}_z \frac{\partial}{\partial z}\right) \cdot \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \frac{\partial}{\partial y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$= 0$$

(b) $\nabla \times \nabla\Phi$

$$= \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial \Phi}{\partial x} & \frac{\partial \Phi}{\partial y} & \frac{\partial \Phi}{\partial z} \end{vmatrix}$$

$$= \mathbf{0}$$