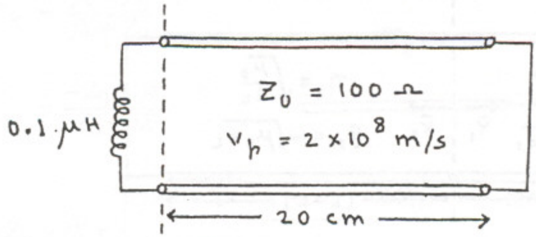
P7.9.



The characteristic equation for resonance is

$$\frac{1}{j\omega L} + \frac{1}{jZ_0 \tan \beta l} = 0$$

$$10^{-7}\omega + 100 \tan\left(\frac{\omega}{2\times 10^8} \times 0.2\right) = 0$$

$$10^{-9}\omega + \tan 10^{-9}\omega = 0$$

$$\tan 10^{-9}\omega = -10^{-9}\omega$$

Letting $x = 10^{-9}\omega$, we have

$$\tan x = -x$$

The three lowest solutions are

$$x = 2.0288,\ 4.9132,\ 7.9787$$

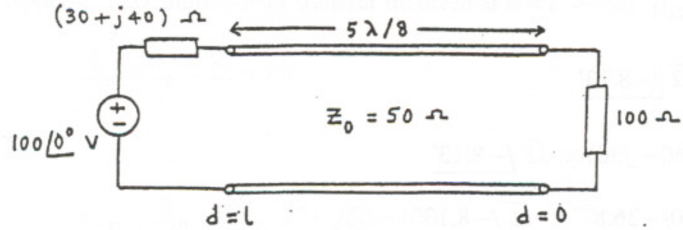
$$\omega = 2.0288 \times 10^9,\ 4.9132 \times 10^9,\ 7.9787 \times 10^9$$

$$f = \frac{\omega}{2\pi}$$

$$= 0.3229 \times 10^9 \text{ Hz},\ 0.7820 \times 10^9 \text{ Hz},\ 1.2698 \times 10^9 \text{ Hz}$$

$$= 0.3229 \text{ GHz},\ 0.7820 \text{ GHz},\ 1.2698 \text{ GHz}$$

P7.17.



$$\bar{\Gamma}_R = \frac{100-50}{100+50} = \frac{50}{150} = \frac{1}{3}$$

$$\bar{\Gamma}(l) = \bar{\Gamma}_R e^{-j2\beta l} = \frac{1}{3}e^{-j(4\pi/\lambda)(5\lambda/8)} = \frac{1}{3}e^{-j5\pi/2}$$

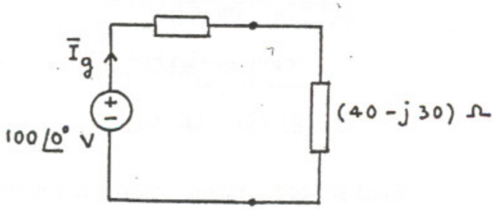
$$= \frac{1}{3}e^{-j\frac{\pi}{2}} = -j\frac{1}{3}$$

$$\bar{Z}_{\text{in}} = Z_0 \frac{1+\bar{\Gamma}(l)}{1-\bar{\Gamma}(l)} = 50\frac{1-j\frac{1}{3}}{1+j\frac{1}{3}}$$

$$= 50\frac{3-j1}{3+j1} = 50\frac{(3-j1)(3-j1)}{9+1}$$

$$= 50\frac{8-j6}{10} = (40-j30)\Omega$$

$$\bar{Z}_{\text{in}} = (40-j30)\Omega$$



$$\bar{I}_g = \frac{100\angle 0^\circ}{70+j10} = \frac{10(7-j1)}{49+1}$$

$$= 1.4 - j0.2$$

$$= \sqrt{2}\,\angle -8.13^\circ$$

P7.17. (continued)

$$\bar{I}(l) = \sqrt{2}\,\angle{-8.13^\circ}$$

$$\bar{V}(l) = (40 - j30) \times \sqrt{2}\,\angle{-8.13^\circ}$$

$$= 50\angle{-36.87^\circ} \times \sqrt{2}\,\angle{-8.13^\circ}$$

$$= 50\sqrt{2}\,\angle{-45^\circ}$$

$$\langle P \rangle = \frac{1}{2}\text{Re}\left[50\sqrt{2}\,\angle{-45^\circ} \times \sqrt{2}\,\angle{8.13^\circ}\right]$$

$$= \frac{1}{2}\text{Re}(100\angle{-36.87^\circ})$$

$$= \frac{1}{2} \times 100 \times \cos 36.87^\circ$$

$$= 40 \text{ W}$$

P7.29. $\bar{Z}_R = (100 + j50)\ \Omega$

$Z_0 = 50\ \Omega$

$\bar{z}_R = (2 + j1)$

(a) $\bar{\Gamma}_R = 0.45\underline{/26.5^\circ}$

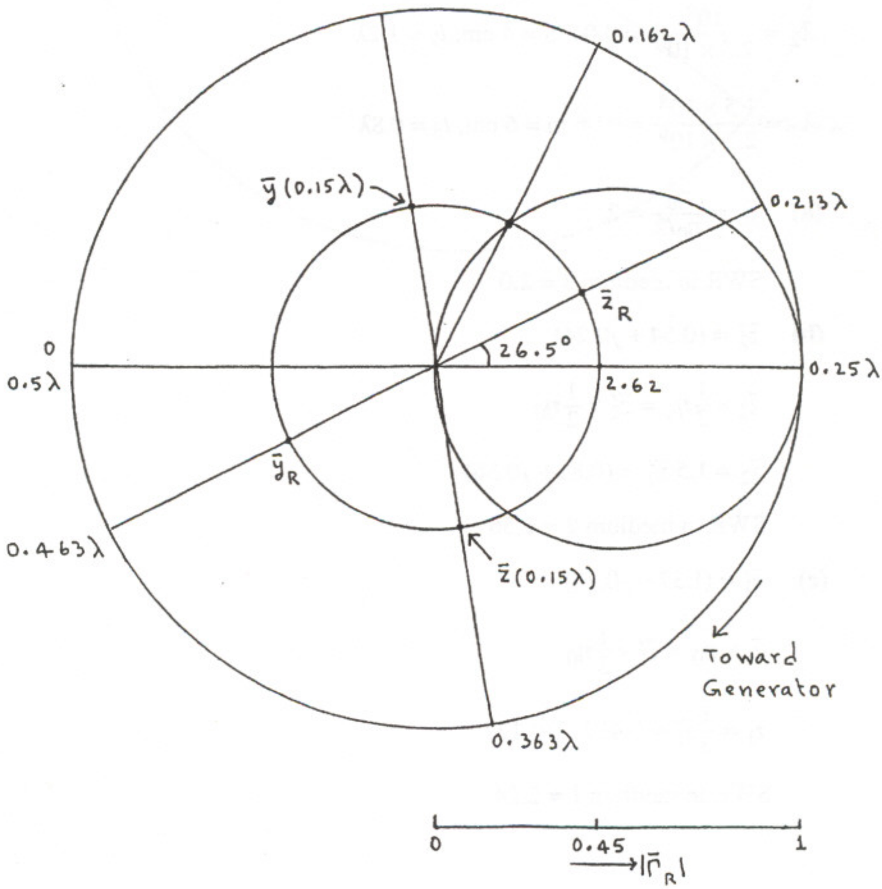
(b) SWR = 2.62

(c) $d_{\min} = (0.5 - 0.213)\lambda = 0.287\lambda$

(d) $\bar{Z}(0.15\lambda) = (0.75 - j0.83)50 = (37.5 - j41.5)\ \Omega$

(e) $\bar{Y}(0.15\lambda) = (0.6 + j0.66)\dfrac{1}{50} = (0.012 + j0.0132)\ \text{S}$

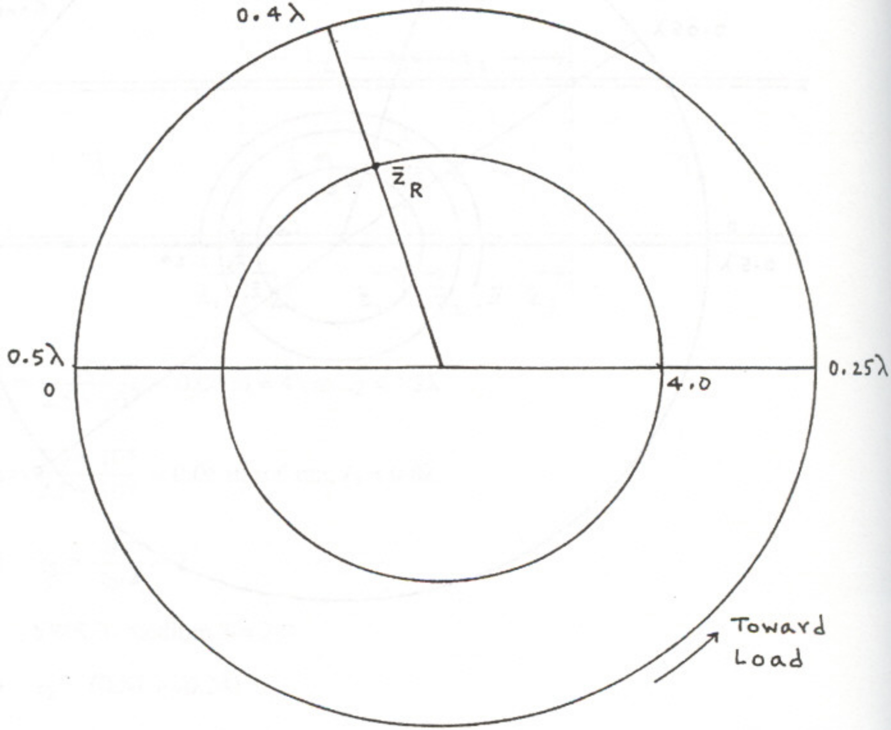
(f) Re $\bar{Y}(d) = Y_0$ at $d = (0.162 + 0.037)\lambda = 0.199\lambda$



P7.31. $d_{min} = 0.4\lambda$

$\bar{z}_R = (0.37 + j0.66)$

$\bar{Z}_R = (0.37 + j0.66)60 = (22.2 + j39.6)\ \Omega$



P7.41. $\mathcal{R} = 0.03\ \Omega/\text{m}, \qquad \mathcal{L} = 1.0\ \mu\text{H/m}$

$\mathcal{G} = 3 \times 10^{-9}\ \text{S/m}, \qquad \mathcal{C} = 50\ \text{pF/m}$

$$\bar{Z}_0 = \sqrt{\frac{\mathcal{R} + j\omega\mathcal{L}}{\mathcal{G} + j\omega\mathcal{C}}}$$

$$= \sqrt{\frac{0.03 + j2\pi \times 10^4 \times 10^{-6}}{3 \times 10^{-9} + j2\pi \times 10^4 \times 50 \times 10^{-12}}}$$

$$= \sqrt{\frac{0.03 + j0.02\pi}{3 \times 10^{-9} + j10^{-6}\pi}} = 100\sqrt{\frac{3 + j2\pi}{3 \times 10^{-3} + j\pi}}$$

$$= 100\sqrt{\frac{6.9326\angle 64.477^\circ}{3.1416\angle 89.945^\circ}} = 147.87\angle -12.734^\circ$$

$$= (144.23 - j32.59)\ \Omega$$

$$\bar{\gamma} = \sqrt{(\mathcal{R} + j\omega\mathcal{L})(\mathcal{G} + j\omega\mathcal{C})}$$

$$= \sqrt{(0.03 + j0.02\pi)(3 \times 10^{-9} + j10^{-6}\pi)}$$

$$= 10^{-4}\sqrt{6.9626\angle 64.477^\circ \times 3.1416\angle 89.945^\circ}$$

$$= 4.677 \times 10^{-4}\angle 77.211^\circ$$

$$= (1.035 + j4.561) \times 10^{-4}\ \text{m}^{-1}$$

P9.1. $\mathbf{E} = 10(2\mathbf{a}_x + \mathbf{a}_y - 2\mathbf{a}_z)\cos[3\pi\times 10^7 t - 0.1\pi(2x - 2y + z)]$

$\varepsilon = 9\varepsilon_0,\ \mu = \mu_0$

(a) $f = \dfrac{\omega}{2\pi} = \dfrac{3\pi\times 10^7}{2\pi} = 1.5\times 10^7\ \text{Hz} = 15\ \text{MHz}$

(b) $\boldsymbol{\beta}\cdot\mathbf{r} = 0.1\pi(2x - 2y + z)$

$= 0.1\pi(2\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z)\cdot\mathbf{r}$

$\therefore\ \boldsymbol{\beta} = 0.1\pi(2\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z)$

$\mathbf{a}_\beta = \dfrac{1}{3}(2\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z)$

(c) $\lambda = \dfrac{2\pi}{\beta} = \dfrac{2\pi}{0.3\pi} = 6\tfrac{2}{3}\ \text{m}$

(d) $\lambda_x = \dfrac{2\pi}{\beta_x} = \dfrac{2\pi}{0.2\pi} = 10\ \text{m}$

$\lambda_y = \dfrac{2\pi}{|\beta_y|} = \dfrac{2\pi}{0.2\pi} = 10\ \text{m}$

$\lambda_z = \dfrac{2\pi}{\beta_z} = \dfrac{2\pi}{0.1\pi} = 20\ \text{m}$

Note that $\dfrac{1}{10^2} + \dfrac{1}{10^2} + \dfrac{1}{20^2} = \dfrac{9}{400} = \dfrac{1}{(20/3)^2}$

(e) $v_{px} = \dfrac{\omega}{\beta_x} = \dfrac{3\pi\times 10^7}{0.2\pi} = 1.5\times 10^8\ \text{m/s}$

$v_{py} = \dfrac{\omega}{|\beta_y|} = \dfrac{3\pi\times 10^7}{0.2\pi} = 1.5\times 10^8\ \text{m/s}$

$v_{pz} = \dfrac{\omega}{\beta_z} = \dfrac{3\pi\times 10^7}{0.1\pi} = 3\times 10^8\ \text{m/s}$

Note that $\dfrac{1}{(1.5\times 10^8)^2} + \dfrac{1}{(1.5\times 10^8)^2} + \dfrac{1}{(3\times 10^8)^2} = \dfrac{1}{(10^8)^2}$

as it should be since $v_p = \dfrac{1}{\sqrt{\mu_0\cdot 9\varepsilon_0}} = \dfrac{c}{3} = 10^8\ \text{m/s}$

P9.16. (continued)

$$\boldsymbol{\beta}_t = 2\pi\sqrt{1.5}\,(\cos 35.264^\circ\ \mathbf{a}_x + \sin 35.264^\circ\ \mathbf{a}_z)$$

$$= 2\pi\left(\mathbf{a}_x + \frac{1}{\sqrt{2}}\mathbf{a}_z\right)$$

$$= \sqrt{2}\pi(\sqrt{2}\mathbf{a}_x + \mathbf{a}_z)$$

Thus we obtain

$$\mathbf{E}_r = 0.0294E_0(\mathbf{a}_x + \mathbf{a}_z)\cos\,[6\pi\times 10^8 t - \sqrt{2}\pi(-\mathbf{a}_x + \mathbf{a}_z)\cdot\mathbf{r}]$$

$$= 0.0294E_0(\mathbf{a}_x + \mathbf{a}_z)\cos\,[6\pi\times 10^8 t + \sqrt{2}\pi(x - z)]$$

$$\mathbf{E}_t = 0.8405\sqrt{2}\,E_0(\sin 35.264^\circ\ \mathbf{a}_x - \cos 35.264^\circ\ \mathbf{a}_z)\cos\,[6\pi\times 10^8 t - \sqrt{2}\pi(\sqrt{2}\mathbf{a}_x + \mathbf{a}_z)\cdot\mathbf{r}]$$

$$= 0.6863E_0(\mathbf{a}_x - \sqrt{2}\mathbf{a}_z)\cos\,[6\pi\times 10^8 t - \sqrt{2}\pi(\sqrt{2}x + z)]$$

Note that at $x = 0$,

$$E_{zi} + E_{zr} = -E_0 + 0.0294E_0 = 0.9706E_0$$

$$= -0.6863\times\sqrt{2}\,E_0 = E_{zt}$$

and

$$E_{xi} + E_{xr} = E_0 + 0.0294E_0 = 1.0294E_0$$

$$= 1.5\times 0.6863E_0 = 1.5E_{xt}$$

thereby satisfying the boundary conditions.