

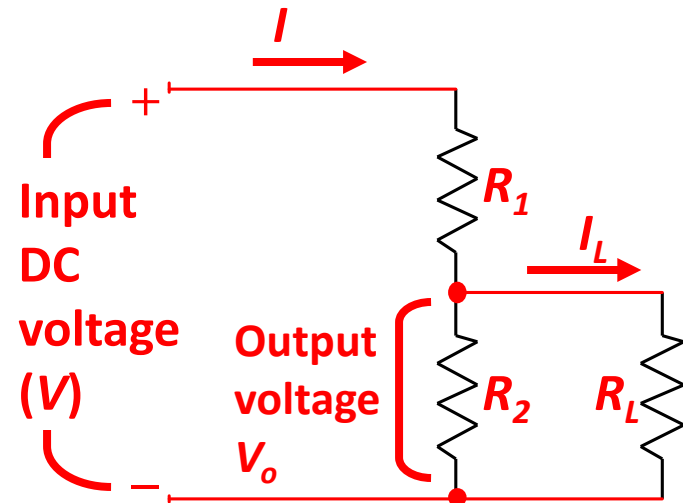


## **Two Circuit problems**

- **Lab 8 presents a pair of circuit design problems which are designed to test the knowledge you have gained in EE 1202 laboratory experiments.**
- **Problem 1 is a simple resistor-based voltage divider, capable of providing a voltage of any value lower than the supply voltage.**
- **Problem 2 deals with reducing the power factor in an AC circuit – a way of reducing the current drawn in an AC circuit that has inductive circuit elements.**

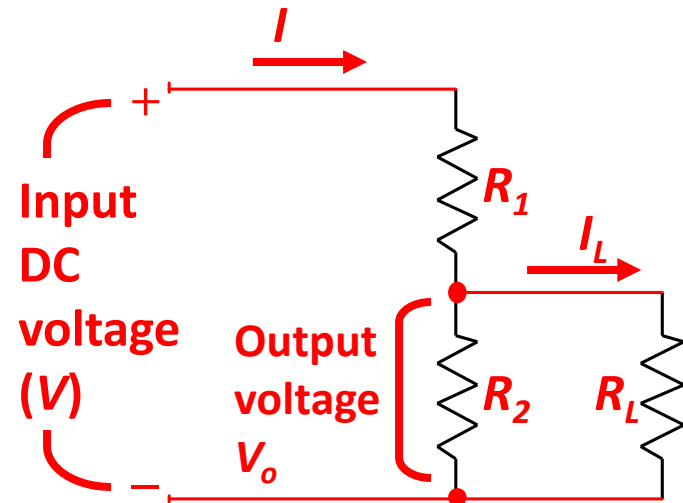
## Resistive Voltage Divider

- A resistive voltage divider produces a DC voltage that is a fraction  $F$  of the input voltage.
- In the diagram, assuming that the load resistor  $R_L$  is not yet connected, then with input voltage  $V$ ,  $I = V/(R_1 + R_2)$ .
- Since the output voltage  $V_o$  (voltage across  $R_2$ ) =  $IR_2$ , then:  
 $V_o = IR_2 = (V \cdot R_2)/(R_1 + R_2)$ .
- If  $R_1$  and  $R_2$  are chosen so that  $R_2/(R_1 + R_2)$  represents the fraction  $F$ , then  $V_o$  is the desired voltage.



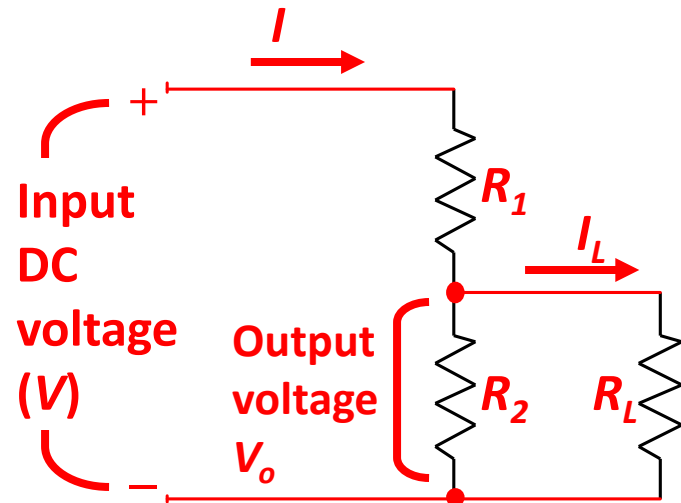
## Voltage Divider Analysis

- With  $R_L$  not connected, then we can choose  $R_1$  and  $R_2$  so that  $(V \cdot R_2)/(R_1 + R_2)$  represents the fractional voltage desired.
- For example, with  $V = 10$  VDC, and the desired voltage  $\approx 3$  V (generally a range is specified), then  $R_2/(R_1 + R_2) = 0.3$ . Then choose  $R_2 = 100 \Omega$  and  $R_1 = 200 \Omega$ . Then  $R_2/(R_1 + R_2) = 100/300 = 0.33$ , which is acceptably close to the desired voltage.
- With some restrictions to be discussed later, the divider as constructed would provide the desired voltage.



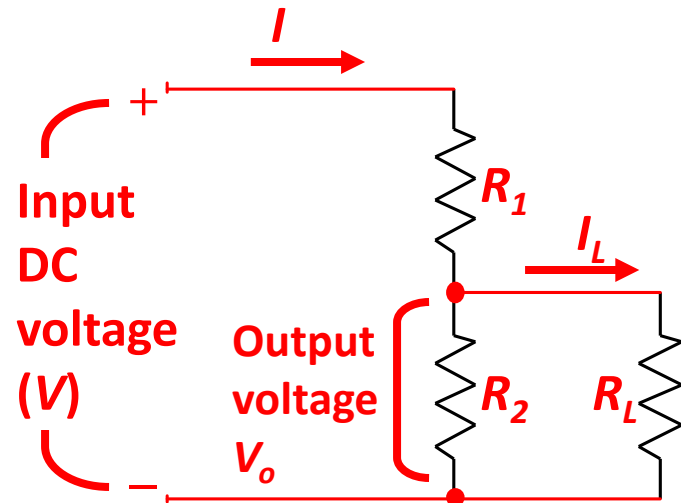
## Adding the Load Resistor

- Now assume that  $R_L$  is connected to the divider circuit (which, after all, is the purpose of the divider). The resistance below  $R_1$  is the parallel resistance of  $R_2$  and  $R_L$ . From previous labs we know that the parallel resistance of  $R_L+R_2$  is  $R_L R_2 / (R_L + R_2)$ .
- This new resistance in the  $R_1$ - $R_2$  circuit will change the value of  $V_o$ .
- We must therefore recalculate the value of the output resistance to determine the effect on  $V_o$ .



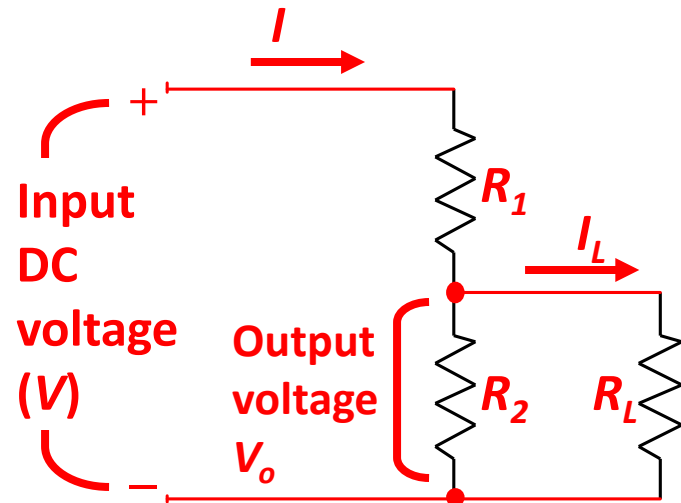
## Adding the Load Resistor (2)

- The new value of  $R_L R_2 / (R_L + R_2)$  can greatly affect the value of  $V_o$ .
- For example, for  $V=10\text{V}$ ,  $R_1 = 200\Omega$ , and  $R_2 = 100\Omega$ , assume that  $R_L$  is  $10\Omega$ , and the desired  $V_o = \text{about } 3\text{V}$ . Then the parallel combination of  $R_L$  and  $R_2$  is:  $10(100)/(10+100) = 1000/110 \approx 9$ , and  $V_o = 10\text{V}(9/[200+9]) \approx 0.4\text{ volt}$ , far outside the desired voltage.
- It is clear that  $V_o$  is greatly influenced by the load,  $R_L$ . What the above equations say is that the voltage divider resistors must be chosen with the value of  $R_L$  in mind.



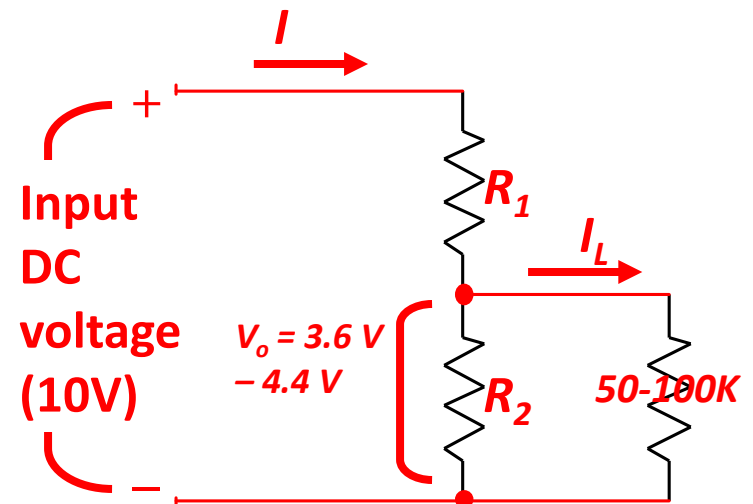
## Adding the Load Resistor (3)

- A voltage divider is usually specified with a range of output voltage  $V_o$ .
- The range of  $R_L$  is also normally specified. For example, say that the range of  $R_L$  is 50K-100K $\Omega$ , and that the desired output voltage is 3V,  $\pm 10\%$ , with an input voltage of 10V.
- This means that  $V_o$  can be 2.7-3.3V and meet the voltage divider “spec.”
- We can now specify the range of resistance values of the divider with respect to the desired performance.
- Note that the value of the lower resistance, with  $R_L$  in place, is  $R_L R_2 / (R_L + R_2)$ .



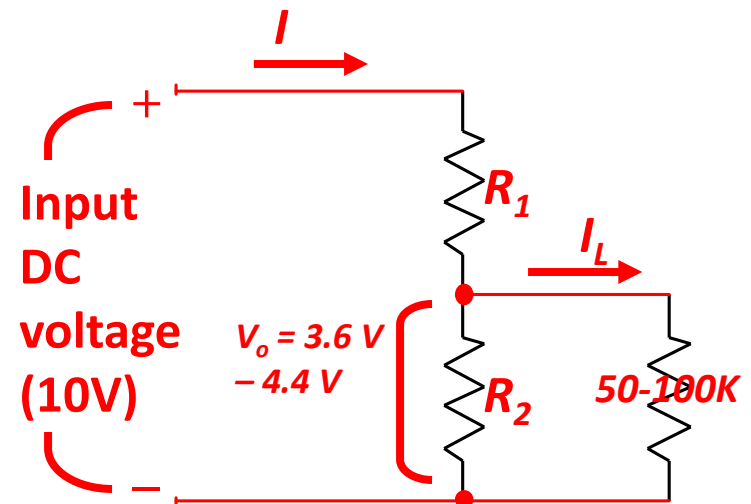
# Resistive Divider Example

- The desired circuit is shown per “spec.”
- The lowest value for  $V_o$  is with the lowest value of  $R_L$  in place.
- Thus,  $V_{oMIN} = I \cdot [R_{LMIN} R_2 / (R_{LMIN} + R_2)]$ , where  $I = V / \{R_1 + [R_{LMIN} R_2 / (R_{LMIN} + R_2)]\}$ .
- We want  $V_{oMIN} \geq 2.7V$ .
- The highest value for  $V_o$  is with the highest value of  $R_L$  in place.
- Then  $V_{oMAX} = I \cdot [R_{LMAX} R_2 / (R_{LMAX} + R_2)]$ , where  $I = V / \{R_1 + [R_{LMAX} R_2 / (R_{LMAX} + R_2)]\}$ .
- We want  $V_{oMAX} \leq 3.3V$ .
- We have only to solve the two inequalities above to determine  $R_1$  and  $R_2$ !



## Finding the Correct Resistors

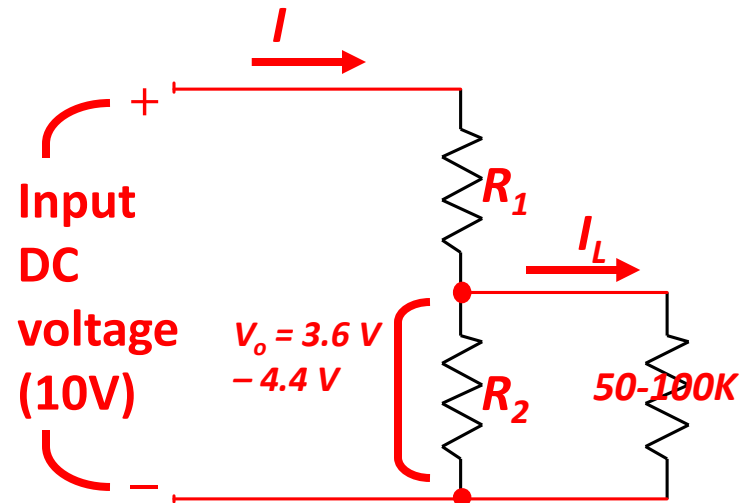
- There is an easier way to find  $R_1$  and  $R_2$ !
- It turns out that a simple “rule of thumb” for a voltage divider is that  $R_2$  should be  $\leq 0.1 R_{LMIN}$ . That is, choose  $R_2$  such that  $R_{LMIN} \geq 10 R_2$ .
- In our example,  $R_{LMIN} = 50 \text{ K}\Omega$ . Then we can choose  $R_2 \approx 5 \text{ K}\Omega$ .
- Now,  $R_2 \approx 0.3 (R_1 + R_2)$ . Solving for  $R_1$ ,  $R_1 \approx (0.7/0.3) R_2 \approx 2.3 R_2$ .
- Or,  $R_1 \approx 11.7 \text{ K}\Omega$ .





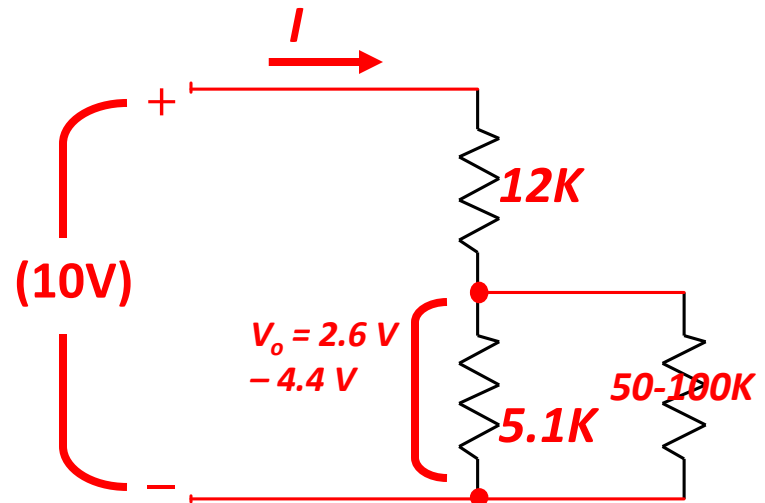
## Finding the Correct Resistors (2)

- We now have specified  $R_2 \approx 5 \text{ K}\Omega$ , and  $R_1 \approx 11.7 \text{ K}\Omega$ .
- We will be using 5% 1/4-W resistors as usual. These only come in standard values. The nearest standard values are 5.1K and 12K. So, choose  $R_2 = 5.1 \text{ K}\Omega$ , and  $R_1 = 12 \text{ K}\Omega$ .
- We now need to check that the chosen values allow us to meet the minimum and maximum output voltages with the minimum and maximum load resistances in place.



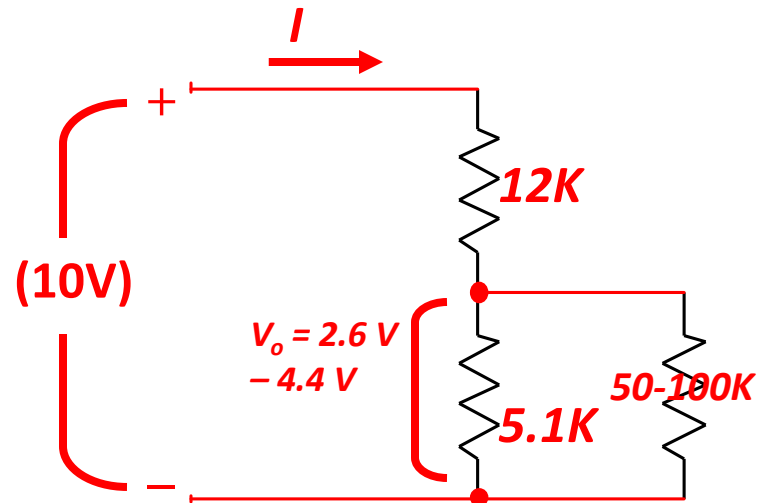
## Finding the Correct Resistors (2)

- According to our “spec,”  $V_{oMIN} = 2.7V$ .
- The parallel resistance of  $R_{LMIN}$  and  $R_2$  is  $(5100 \cdot 50000)/(5100 + 50000) = 4628\Omega$ .
- Then  $V_{oMIN} = 10[4628/(4628 + 12000)] = 2.78V$ . Check!
- The “spec,” states that  $V_{oMAX} = 3.3V$ .
- The parallel resistance of  $R_{LMAX}$  and  $R_2$  is  $(5100 \cdot 100000)/(5100 + 100000) = 4852\Omega$ .
- Then  $V_{oMAX} = 10[4852/(4852 + 12000)] = 2.88V$ . Check!
- Our voltage divider, with the resistor values as shown, satisfies the “spec.”



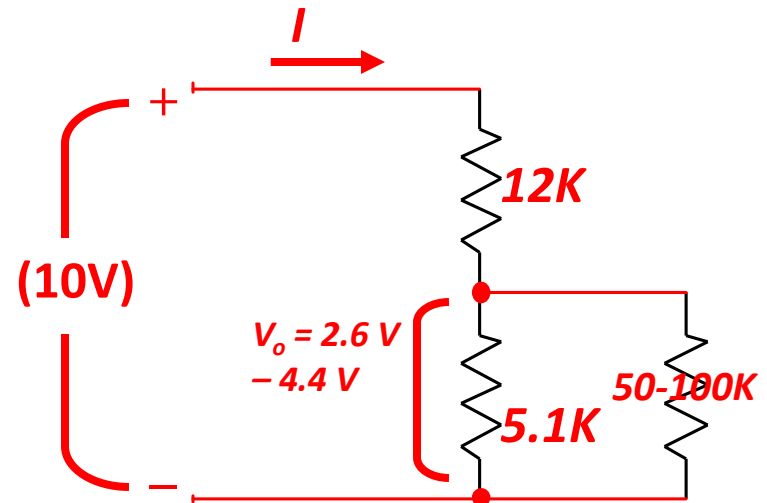
# Limiting Voltage Divider Power Dissipation

- A further concern with our voltage divider: power dissipated in the two resistors.
- One might be tempted to make  $R_1$  and  $R_2$  very small to allow for a wider range of  $R_L$ .
- Consider the absurd situation where we choose  $R_1 = 12\Omega$  and  $R_2 = 5.1\Omega$ . Then the 50-100K range of  $R_L$  is inconsequential. However, consider the power in the divider resistors:
- $I \approx 10/(12+5.1) \approx 0.6$  A. Since the DC power in a resistor =  $I \cdot V = I^2 R$ , then  $R_1$  power =  $(0.6)^2 \cdot 12 \approx 4$  W. A  $\frac{1}{4}$  resistor would probably explode!



## Choosing Resistors to Eliminate Power Problems

- In order to assure that the voltage divider resistors do not use too much power, they should be chosen with an eye to the current in  $R_1$ .
- Clearly  $R_1$  will dissipate more power than  $R_2$  for a given current  $I$ .
- In general, the way to structure the divider is to choose  $R_2 \leq 0.1 R_{LMIN}$ .
- Then calculate  $I$  for the case of  $R_{LMIN}$  (as shown on slide 7). Now,  $I^2 \cdot R_1$  gives the power dissipation in  $R_1$ . It should be  $< 1/4$  watt. If so, the voltage divider resistors are protected.
- Then, assuming the divider meets the voltage specs, it is complete.



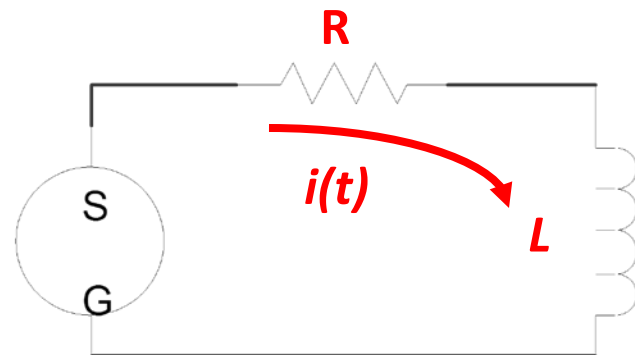


## Reducing the $V$ - $I$ Phase Angle

- All industries use AC motors (air conditioners, assembly line motors, etc.).
- Electric motors work due to inductive coils.
- **Thus large industries present an inductive load to the power company, and the resulting inductive current increases the current drawn from the power company.**
- **Inductive current is “imaginary” mathematically, but the power company still charges for it.**
- For a major power user, reducing inductive load (making AC voltage and current as close to in-phase as possible) reduces the power bill.
- **The common name for reducing the phase angle is “making the power factor one.” The “power factor” is the cosine of the phase angle between the voltage and current in an AC circuit.**
- **If the power factor is 1, then the phase angle is 0 ( $\cos\theta = 1$ ), and there is no reactive current.**

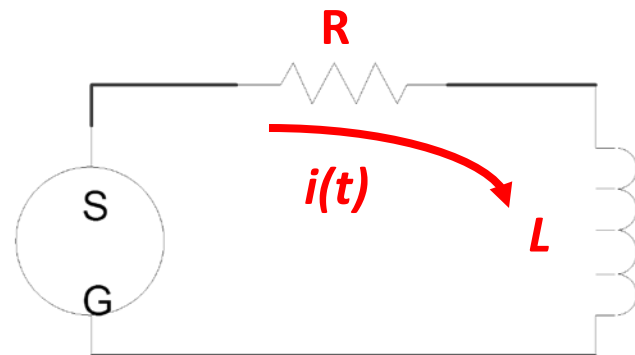
## Reminder of RL AC Circuit Analysis

- In an  $RL$  circuit with a sinusoidal AC voltage applied, current lags the voltage by phase angle  $\theta$ .
- From Lab 5, the  $\omega$ -domain current is  $I = V_p / (R + j\omega L)$ , where  $V_p$  is the maximum AC voltage amplitude.
- $I$  is a complex number in the  $\omega$ -domain, with the imaginary part due to the inductive impedance,  $j\omega L$ . **If the inductive impedance in the circuit was reduced to 0, all inductive current would cease and the overall current magnitude would decrease.**



# Inductive Current is NOT “Imaginary”

- **Inductive current is mathematically imaginary, but physically real; it can be observed and measured.** It requires additional current be generated by the power company, and it is carried by a transmission line into the circuit.
- Removing inductive current from the circuit would reduce the amount of current that is supplied to the circuit.





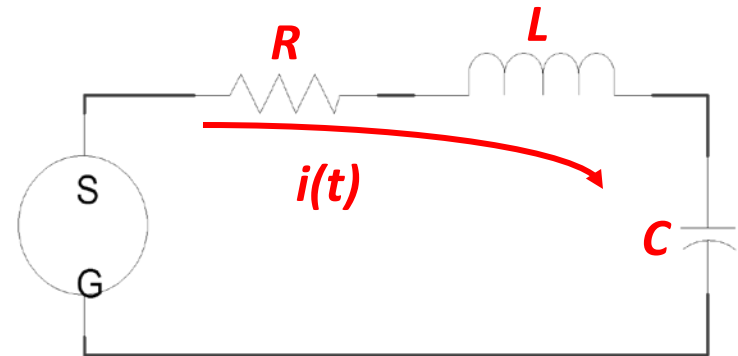
## Making the “Power Factor” 1

- Inductive current is of the form  $I_{MAX} \cos\theta$  in the time domain. The angle  $\theta$  is defined as  $\arctan(X/R)$ , where  $R$  is the circuit resistance and  $X$  is the total circuit reactance.
- In AC circuits, the smaller  $\theta$ , the more efficiently power is utilized.
- Reduce the phase angle to 0 and the circuit impedance is resistive; voltage and current are “in phase” and no reactive current exists.
- Since  $\theta$  is 0, the so-called “power factor,”  $\cos\theta$ , is 1. This “power factor” is a measure of efficient power usage in AC circuits.



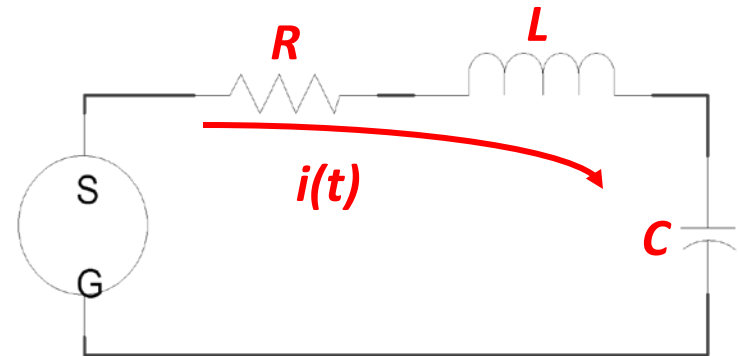
## Adding Capacitance

- In a sinusoidal AC circuit, inductors and capacitors produce impedances of opposite signs.
- It stands to reason that if a circuit is highly inductive, drawing a significant of reactive current, then adding capacitance to the circuit will reduce the total reactance and hence the reactive current.
- In the circuit at right, the inductive impedance is  $j\omega L$ , and the capacitive impedance is  $1/j\omega C = -j/\omega C$ .



## Adding Capacitance (2)

- If one wishes to make the AC current solely resistive, then adding negative capacitive impedance equal to the positive inductive impedance should make the circuit impedance purely resistive.
- Then we want  $|j\omega L| = |-j/\omega C|$ , or  $\omega L = 1/\omega C$ .
- You can solve this equation with respect  $C$  in terms of  $L$ , given the AC frequency of the circuit.
- Do so as a part of your pre-lab assignment.





## Your Lab Assignment

- **Design a voltage divider that meets the specification in your lab text. When it is completed, demonstrate its operation to the TA on duty, and have the TA sign your data sheet to validate that it works.**
- **Calculate the capacitance required to reduce the phase angle to 0 (or make the power factor 1) in the circuit specified in the lab text. Again, have the TA sign and validate that your phase reduction circuit works properly.**
- **There is no lab report for Experiment #8. Turn your data sheet into the instructor in the next EE 1202 class that shows your calculations and has the TA signature validations. At that point your EE 1202 labs are completed!**