EE 1202 Experiment #4 – Capacitors, Inductors, and Transient Circuits

1. **Introduction and Goal: Exploring transient behavior due to inductors and capacitors in DC circuits; gaining experience with lab instruments.**
2. **Equipment List: The following are required for this experimental procedure:**

• **Multimeter, HP Model # 34401A.** • **Signal Generator HP Model 33220A.** • **Oscilloscope, Agilent Model # 54622D.** • **RCL Meter, Fluke Model PM6303A (Appendix A, Section 10).** • **Electronic prototyping board.**	• **Resistors, 5%, ¼ Watt: 1000 Ω (1), and 51 Ω (1), plus others determined in experiment.** • **Inductor, 10 milli-Henry (mH).** • **Capacitors, 0.05 micro-Farad (μF), and 0.01 μF, others determined in experiment.** • **Oscilloscope probes, set to 1X, and connecting leads for DMM and signal generator.**

3. **Experimental Theory: The three common <u>passive circuit elements</u> are resistor, capacitor and inductor. <u>We study DC capacitor and inductor circuits today</u>.**
   - **3.1. Capacitor: A capacitor collects electrical charge. It is made of two or more conductors separated by insulators.**
     - **3.1.1. Applying DC voltage causes current (charge flow) to enter a capacitor. Charge accumulates on its surfaces like water in a reservoir (Fig. 1).**
     - **3.1.2. In Fig. 2, when voltage V is applied, the capacitor develops an <u>equal and opposite</u> voltage (V_C) to the DC source V. When $|V_C| = |V|$, current ceases. Thus <u>a capacitor blocks DC current flow</u>.**

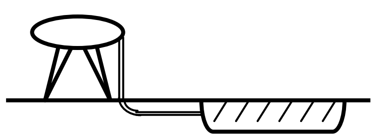


Fig. 1. As a reservoir collects water, the capacitor collects electronic charge.

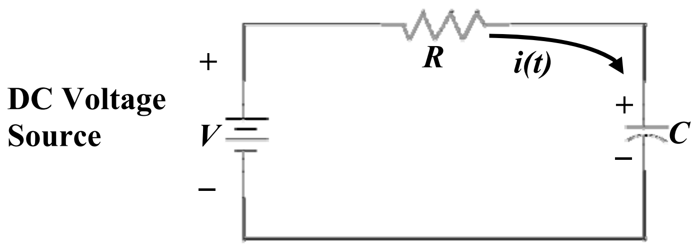


Fig. 2. Capacitor "Collecting Charge"
(Symbol for Capacitor)

   - **3.1.3. With no initial charge, the capacitor has a large capacity to absorb current initially. Thus <u>voltage across a capacitor cannot change instantaneously</u>. It builds from 0 as charge collects on the capacitor plates. When $V_C = -V$, current flow ceases ($+V-V=0$).**
   - **3.1.4. Capacitors are used in all modern electrical circuits (TV's, iPhones, etc.). The unit of capacitance is the <u>Farad</u>, (after Faraday, another early**

experimenter). The Farad is a very large measure of capacitance, so capacitors usually have values of micro- (10^{-6}), and pico- (10^{-12})Farads.

3.2. Inductor: An inductor is a coil of wire with the property of electrical inertia. An analogy is the inertia of mass. A large truck accelerates slowly due to its mass. At high speed, it is hard to stop for the same reason. Similarly, inductors resist increase or decrease in current (Figs. 3 and 4).

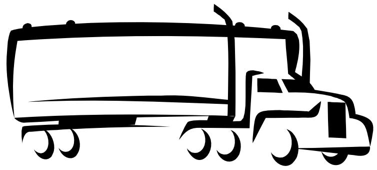


Figure 3. A large, heavy truck would take a great deal of effort or energy to start or stop.

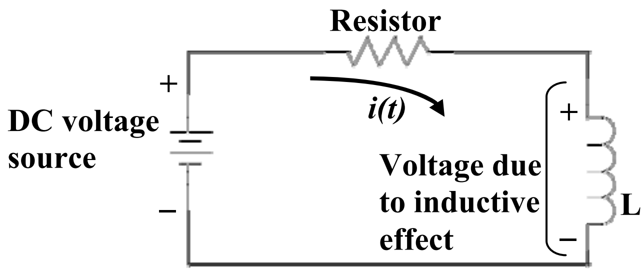


Fig. 4. Inductor "Resisting Current"

(Symbol for Inductor)

3.2.1. Inductor characteristics are due to its magnetic properties. The inductive effect in a coil of wire occurs due to changes of the current. Constant inductor current (or no current) causes no inductive effect.

3.2.2. As voltage cannot change instantaneously on a capacitor, current cannot change instantaneously in an inductor.

3.2.3. Inductance is measured in Henry's (for another pioneer). As one Henry is a large inductor, practical inductors are in milli-Henry's.

3.3. Capacitors and Inductors in a DC Circuit: Capacitors and inductors cause very brief non-linear effects when a DC voltage is applied or changed. Shortly after a DC voltage change, capacitor and inductor circuits reach "steady state." These extremely brief effects are called transient behavior.

3.3.1. Exponential functions review: If $y = 3^x$, y is an exponential function of x. In many exponential functions, e (≈ 2.71828, the "base of natural logarithms") appears. In Fig. 5, $y = 1 - e^{-x}$. At $x = 0$, $y = 0$ ($e^0 = 1$). As $x \to \infty$, $e^{-x} \to 0$, so $y \to 1$. Mathematically, y never reaches 1, although by $x = 10$, ($e^{-10} \approx 0.000045$), the difference is negligible. Approaching a value ("asymptote") but never reaching it is typical of exponential functions. Such functions describe capacitor and inductor behavior in DC circuits.

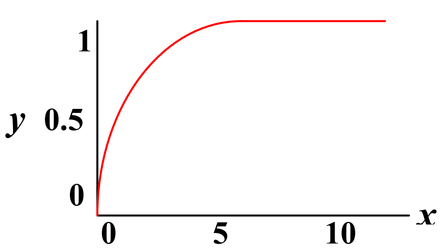


Fig. 5. Plot of $y = 1 - e^{-x}$

3.3.2. The equation $v_C(t) = V(1 - e^{-(t/\tau)})$ describes behavior of current in the RC circuit of Fig. 6, where $v_C(t)$ is the capacitor voltage after the switch is closed ($t = 0$), and V is the DC voltage. Since v_C cannot change instantaneously, $v_C(t=0) = 0$ (assuming no charge on the capacitor at $t = 0$). τ is the "time constant," the time it takes for the voltage to change to $(1-1/e)$ of its former value. Thus, <u>τ is a measure of the duration of transient behavior</u>. The unit of a <u>time constant</u> is seconds, and the smaller it is, the quicker transient behavior is over. Although an exponential function never <u>mathematically</u> reaches its asymptote, <u>transient behavior is over in about ten time constants</u>. For a series *RC* circuit, $\tau = RC$ (*R* in Ohms, *C* in Farads). Thus, $v_C(t) = V(1 - e^{-(t/RC)})$.

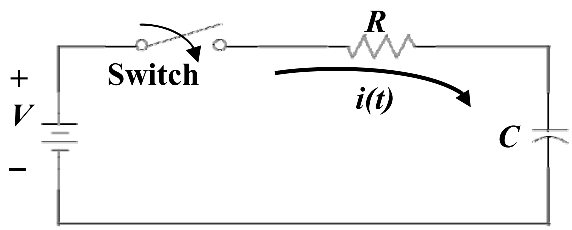


Fig. 6. Simple RC Circuit

3.3.3. In Fig. 7, since current cannot start instantaneously in an inductor, the inductor voltage $v_L = V$ <u>when the switch is closed</u> ($i = 0$, thus $i \cdot R = 0$). As current increases, v_L falls. At steady state, $v_L = 0$, and current equals V/R. An equation for inductor voltage is: $v_L(t) = Ve^{-(t/[L/R])} = Ve^{-(R/L)t}$. At $t = 0$, $v_L = V$. τ (= L/R) is the *RL* circuit time constant (inductance in Henrys, resistance in Ω).

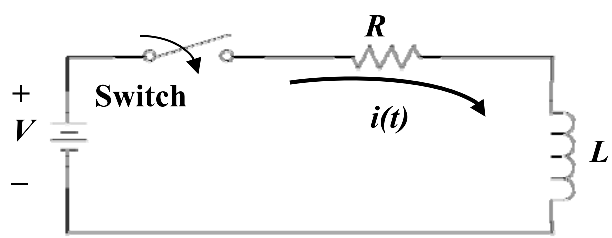


Fig. 7. Simple RL Circuit

3.4. *RLC* Circuit: A capacitor, inductor, and resistor circuit can <u>oscillate</u>.

3.4.1. In Fig. 8, at $t = 0$, V causes current flow in the circuit. Current gradually increases, due to inductor effects, and charge collects on the capacitor. When the capacitor charges to $-V$, (reverse polarity), current flow ceases.

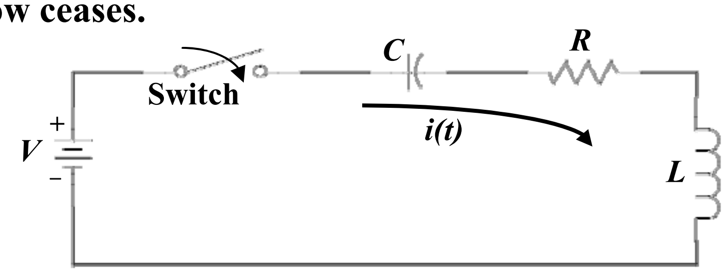


Fig. 8. Series RLC Circuit

3.4.2. The *RLC* circuit will resonate just as a bell that is rung, with proper choice of *R*, *L*, and *C*. The oscillation is also transient.

3.4.3. Skipping the mathematical derivation, for a resonant series *RLC* circuit, capacitor voltage can be expressed as: $v_c(t) = V(1-[\cos\omega_d t]e^{-\alpha t})$, if the components *R*, *L*, and *C*, are chosen properly (for many component values, no oscillation occurs), where *V* is the applied voltage.

3.4.4. The cosine function above describes the voltage oscillation, and the *e*-term clearly makes the behavior transient. ω_d is the radian frequency of oscillation ($\omega_d = 2\pi f_d$, f_d the resonant frequency of the circuit in Hz), defined as $\omega_d = \sqrt{(1/LC)-(R/2L)^2}$. α is the damping factor, defined as $\alpha = R/2L$. Like τ, it determines how fast the oscillation dies away.

4. Pre-work: We will use the equations above as we examine transient behavior. Make sure you understand the concepts of transient behavior discussed above.

5. Experimental Procedure – *RL* and *RC* circuits:

5.1. Voltage Across a Capacitor in a Series RC Circuit: The capacitor voltage equation is: $v_C(t) = V(1-e^{-(t/RC)})$, where the time constant $\tau = RC$.

5.1.1. Select 1KΩ resistor and 0.05 µF capacitor. Measure R and C values (use LC meter for capacitor; see Appendix A for LC meter instructions).

5.1.2. In our RC circuit, $\tau = RC \approx 1000 \times 0.05\ 10^{-6}$ sec. or 0.00005 sec. That is, $\tau \approx 50$ µsec. Since transient circuit behavior lasts ~ 10τ, the behavior of the circuit lasts about 500 microseconds, or ½ millisecond.

5.1.3. Half-millisecond events are hard to see, so we will use an oscilloscope to observe our transients and a signal generator for "DC voltage."

5.1.4. Connect capacitor and resistor as shown in Fig. 9, with signal generator and oscilloscope across the resistor and capacitor as shown.

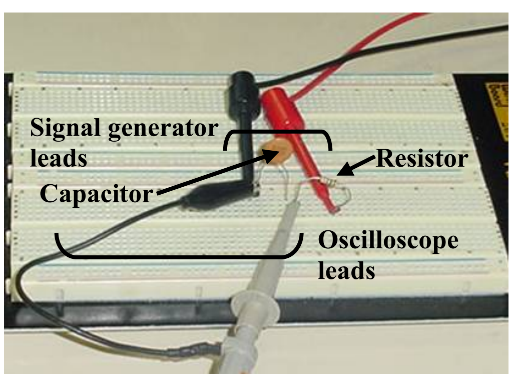


Fig. 9. RC Transient Test Circuit

5.1.5. The signal generator can generate a "DC voltage" for our circuit. A 0-5V, 500 Hz square wave generated by the signal generator will provide an "off" (0 V) and an "on" (5 V) over a 2 millisecond (msec) period, so that the voltage is in each state for 1 msec. Since 1 msec is 20 time constants for the circuit, the square wave signal will simply be a DC voltage being turned on and off rapidly for the circuit.

5.1.6. Turn on signal generator and oscilloscope. Use oscilloscope to set the signal generator to a 500 Hz square wave at 5 V_{pp} (peak-to-peak).

5.1.7. We want a square wave signal of +5V for 1 msec, then 0V for 1 msec. However, the generator currently outputs a 500 Hz signal that is ±2.5 V.

5.1.8. To change the square wave, select the oscilloscope Channel 1 menu and change coupling to "DC," which will make DC voltages visible on the display (depress the "1" button, and the menu shows DC or AC coupling as options). Press the DC offset button on the function generator and using the oscilloscope, add +2.5 V of DC offset to the AC signal, using the adjustment wheel. The signal generator has very precise controls, so you can easily "dial in" DC voltage as needed. Note that the algebraic sum of DC offset and AC signal is output to the circuit. Verify this 0-5V square wave on the oscilloscope. Note: The signal generator is sensitive to "load" (components connected to it), so output voltage may vary as you change components. Check and reset V_{pp} for each exercise.

5.1.9. You may not see transient behavior at first, as it is very brief.

5.1.10. Increase "sweep rate" (or use "Autoscale") until sweep is around 20-50 μsec/div, and you should see the very rounded leading edges of the square wave, as in Fig. 10.

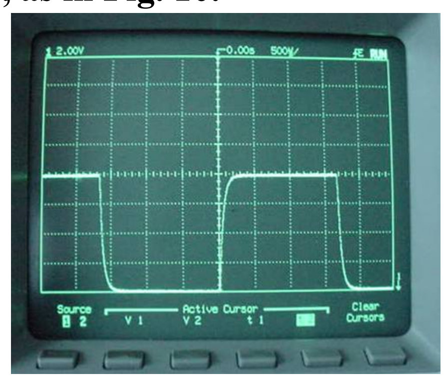


Fig. 10: Capacitor Voltage in Series RC Circuit.

5.1.11. Note that transients occur on DC voltage turn-on and turn-off.

5.1.12. Activate vertical (time) cursors. Set left cursor exactly where the signal starts to rise. Using τ (=RC) calculated in your data sheet, place right cursor to intersect signal trace one time constant after it starts to rise. (Note: your τ should be ~ 50 μsec, but use your calculated value).

5.1.13. Activate horizontal (voltage) cursors, place the bottom cursor at the bottom of the trace and use the upper cursor to measure the voltage level where the second time cursor intersects the rising voltage line on the signal (at $t = \tau$). Record this value.

5.1.14. Move the time cursor horizontally until it intersects the signal at 2τ and take another reading. Continue measurements at 3τ, 4τ, 5τ, 6τ, 8τ, and 10τ, or until changes are indistinguishable. Increase the oscilloscope sweep rate if necessary.

5.2. Inductor Voltage in a Series RL Circuit:

5.2.1. Select a 10 mH inductor and carefully measure its inductance and resistance on the LC meter. Replace the capacitor with the inductor, leaving signal generator and oscilloscope connected as before (Fig. 11). Check that the signal is still a 5 V pp square wave, with a 2.5 V. DC offset, and frequency of 500 Hz.

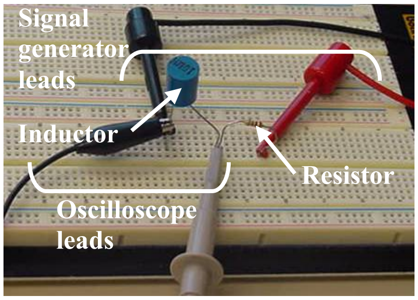


Fig. 11. RL Transient Test Circuit

5.2.2. For an inductor circuit, $\tau = L / R$. This should be ~ 10 μsec, but calculate it using your R and L (remember $R = R_{resistor} + R_{inductor}$).

5.2.3. You may not see an oscilloscope signal at first. You may see "spikes" (vertical lines) every millisecond. Increase sweep rate (~ 10-25 μsec per division should work). The display (Fig. 12) should depict inductor voltage jumping to about 5 V., then rapidly decreasing to 0.

5.2.4. Note that inductor voltage falls off exponentially. Also, the inductor voltage spikes <u>negatively</u> on voltage turn-off. <u>This means that the inductor opposes any change in current</u>. Record the time until transient behavior ends. Convert this value to a time constant on your data sheet.

5.3. RLC Circuit: Use a 51Ω resistor and 0.01 μF capacitor in addition to the inductor and capacitor used above. Measure their <u>exact</u> value and set aside.

5.3.1. RLC Circuit: Connect the 1K resistor, inductor, and 0.05 μF capacitor as shown below (Fig. 13). <u>Remember inductor resistance</u>!

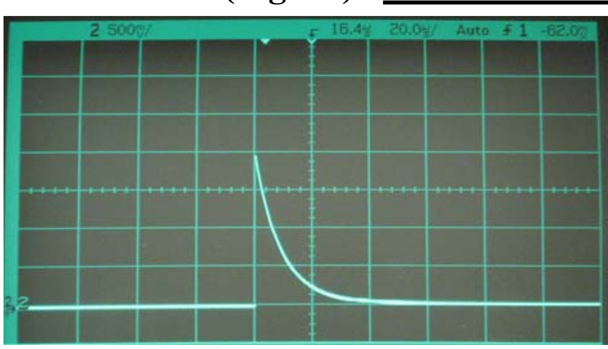


Fig. 12. Inductor Voltage in RL Test Circuit.

5.3.1. Connect oscilloscope probe to check that the square wave signal is still 5V pp, 500-Hz, offset of +2.5V, across the three series components.

5.3.2. Connect oscilloscope across the capacitor (Fig. 13). Press "Autoscale" to see a waveform that is close to that in Figure 10.

5.3.3. For oscillation to occur, there must be the right combination of *R*, *L*, and *C*. As set up initially, the circuit will not oscillate.

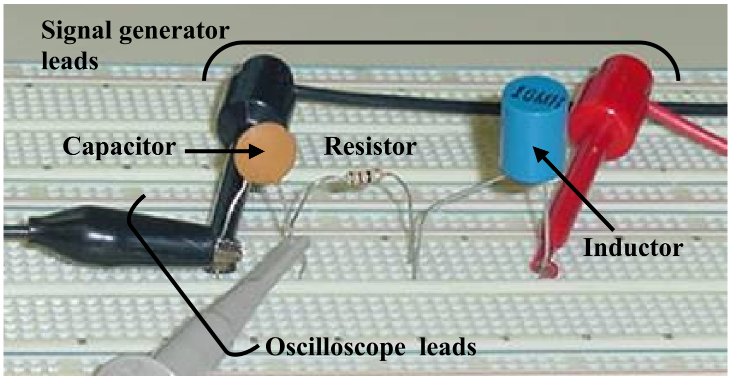


Fig. 13. Series RLC Circuit

5.3.4. Replace 1000 Ω resistor with 51 Ω resistor. You should see a trace as in Fig. 14 (change sweep rate if necessary; ~ 50 μsec/division should work).

Fig. 14: Capacitor Voltage in Series RLC Circuit.

5.3.5. The capacitor voltage "overshoots" the 5V level (up to ~ 7-8 volts), oscillates several times, then settles to 5 V after 5 or 6 cycles.

5.3.6. Use vertical cursors to measure the period of the waveform (the distance between identical points on the wave, e.g., two maxima) and record. Also record length of oscillation to die out point.

5.3.7. Substitute 0.01 μF capacitor for 0.05 μF capacitor. On applying the square-wave, you should see a different frequency. Using vertical cursors, measure and record the period of the new waveform, and elapsed time to end. This should be the same as that measured above.

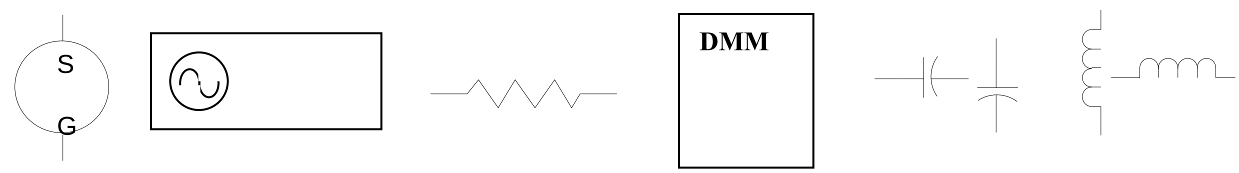
5.3.8. In the Lab. 4 worksheet, you derived a relation for the resistor at which oscillation in an *RLC* circuit would cease, when L = 10 mH, C = 0.01 μF. Pick out resistors from your parts kit that are roughly half and twice this value.

5.3.9. First, substitute the lower-value resistor. Does the circuit oscillate? If so, record the frequency. Now substitute the higher-value resistor. Does oscillation occur? If so, record the frequency; if not, simply note that no oscillation occurred.

5.3.10. Also in worksheet 4, you calculated a resistor value to double α in the *RLC* circuit. Insert that resistor value in the circuit (you may have to

combine some resistors to get a value that is close). Does the oscillation end in half the time? Does this resistor eliminate oscillation?

6. **Laboratory Cleanup: Return parts to storage; <u>make sure work area is clean</u>.**
7. **Writing Laboratory Report: Include the following:**
   - **7.1.** **Draw <u>labeled</u> diagrams for your three circuits (see Fig. 15 for symbols).**
   - **7.2.** **Using data collected in 5.1, plot capacitor voltage (amplitude versus time in µsec). Using the theoretical equation for $v_C(t)$, calculate amplitude at τ = 1, 2, 3, 4, 5, 6, 8, and 10. Plot on the same chart for comparison and discuss the match, advancing an explanation for any discrepancies. (Excel makes it easy to plot these graphs together, or use some other tool if you wish.)**
   - **7.3.** **For the RLC circuit in 5.3.5, calculate the frequency of oscillation from the period that you measured, for both capacitors.**
   - **7.4.** **For the 51 Ω and 10 mH inductor used in 5.3.5, and using measured values for R, L, and C, calculate f_d (= $\omega_d/2\pi$) for that circuit using the formula in 3.4.5, for both capacitors. How do these compare to those you measured?**
   - **7.5.** **For the resistor values on either side of the calculated value of resistance at which resonance ends, did oscillation/non-oscillation occur as predicted?**
   - **7.6.** **Did changing α by using a different resistor reduce the transient behavior? If the transience was shortened, was the reduction about as calculated?**



Signal Generator **Oscilloscope** **Resistor** **Multimeter (DMM)** **Capacitor** **Inductor**

Fig. 15. Special Symbols Required for Drawing Circuits

Experiment #4 Data Sheet

1. **Measured value: 1KΩ resistor ______ 0.05 µF capacitor ______**
2. **Calculate RC time constant (τ), using values above: __________**
3. **Measured voltage amplitude for eight time constants in RC circuit:**

Time	Voltage	Time	Voltage
τ		5τ	
2τ		6τ	
3τ		8τ	
4τ		10τ	

4. **Calculated voltage amplitude for eight time constants in RC circuit:**

Time	Voltage	Time	Voltage
τ		5τ	
2τ		6τ	
3τ		8τ	
4τ		10τ	

5. **Value of 10 mH inductor inductance: __________**
6. **Calculated RL time constant (still using 1K resistor): __________**
7. **Measured time of RL transient behavior (µsec): __________**
8. **Measured time of RL transient behavior (τ's): __________**

Experiment #4 Data Sheet (Continued)

9. Measured values: 51Ω resistor: _______ 0.01 µF capacitor: _______
10. Calculated f_d (Hz) (= $\omega_d/2\pi$) with 0.05 µF capacitor: _______
11. Period of RLC circuit oscillation (0.05 µF capacitor): _______
12. Measured f_d (Hz) of RLC circuit oscillation (0.05 µF cap.): _______
13. Duration of transient behavior (0.05 µF capacitor): _______
14. Calculated f_d (Hz) (= $\omega_d/2\pi$) with 0.01 µF capacitor: _______
15. Period of RLC circuit oscillation (0.01 µF capacitor): _______
16. Measured f_d (Hz) of RLC circuit oscillation (0.01 µF cap.): _______
17. Duration of transient behavior (0.01 µF capacitor): _______

Discovery Exercise #1:

18. Calculated R above which oscillation ends (worksheet): _______
19. Measured value of chosen resistors above and below this value:

    Smaller resistor: ______ Oscillation (if yes, show freq.): _______

    Larger resistor: _______ Oscillation (if yes, show freq.): _______

Discovery Exercise #2:

20. Calculated value of resistor to reduce transient behavior 2X:

    ____

21. Measured value of resistor in kit closest to this value: ________
22. Duration of transience at this resistance: ________
23. Does oscillation still occur: ________
24. Frequency of oscillation at this resistance (if occurring): ________

Experiment #4 Worksheet

Note: Refer to Experiment #4 outline to answer questions below.

1. **Circle the exponential functions:** a. $y = 3x + 7$ b. $r = -\sqrt{a^2 + b^2}$

   c. $z = e^y - 8$ d. $x = 2^n$

2. **The expression for capacitor voltage in RC circuit (assuming no initial charge) when DC voltage is applied is $v_C(t) = V(1 - e^{-(t/RC)})$. Calculate capacitor voltage (in terms of V) when t = :**

   RC __________ **5 *RC*** __________ **10 *RC*** __________

3. **The voltage across an inductor when a DC voltage is switched on is $v_L(t) = Ve^{-(t/[L/R])} = Ve^{-(R/L)t}$. Assume V = 10 V, L = 1 mH, and R = 1 Ω. Find the voltage across the capacitor at t = :**

   0 msec _____________ **1msec** _____________ **10 msec** _____________

4. **Derivation #1: Consider the equation for ω_d, shown in Section 3.4.5 of Experiment 4. Clearly, oscillation will occur so long as $(1/LC) > (R/2L)^2$, else ω_d is imaginary (which implies that the behavior reverts to a straight exponential, as $j\omega_d$ would be real). Using the relation above, develop an equation for the value of R where this occurs. Given the 0.01µF capacitance and 10 mH inductance in the circuit of section 5.3, determine R for these values. You will use these results in the lab.**

5. **Derivation #2: If one wished the transient behavior of an oscillation to be more brief, clearly the value of α must be increased. Assuming L = 10 mH, from the definition of α, what value of R must be chosen to reduce the transient behavior twice as quickly? You will use this result in your lab exercise.**

Note: The formulas for ω_d and α are: $\omega_d = \sqrt{(1/LC) - (R/2L)^2}$, and $\alpha = R/2L$.