EE 1202 Experiment #5 – Inductors and Capacitors in AC Circuits and Phase Relationships

1. **Introduction and Goal**: Capacitors and inductors in AC circuits are studied. Reactance, impedance, and phase relationships of AC voltage and current are defined. Frequency-dependence of inductor and capacitor impedance is introduced. Phase relationships of AC voltage and current are defined.
2. **Equipment List**: The following instruments and components are required:

• Multimeter, HP 34401A. • Signal Generator, HP 33220A. • Oscilloscope, Agilent 54622D. • RCL Meter, Fluke PM6303A. • Electronic prototyping board.	• Resistor, 5%, ¼ Watt: 16 Ω (1). • Capacitor, 10 μF (2). • Inductor, 10 mH (2). • Oscilloscope, banana plug, and coaxial leads.

3. **Experimental Theory**: Capacitors and inductors change the voltage-current relationship in AC circuits. Since most single-frequency AC circuits have a sinusoidal voltage and current, exercises in Experiment 5 use sinusoidal AC voltages. Note that in an RLC AC, current frequency will be identical to the voltage, although the current waveform will be different.

   3.1. "Imaginary" Numbers, the Complex Plane, and Transforms:

   3.1.1. Definition of *j*: As $\sqrt{-1}$ is not a real number, EE's normally define $j = +\sqrt{-1}$. Physicists and mathematicians use *i* $(= -\sqrt{-1})$ for this same purpose, so $j = (-i)$, but that will not affect our theory.

   3.1.2. EE problem solutions often include imaginary numbers. It is useful to consider real and imaginary numbers as existing in a two dimensional space, one axis of which is a real-number axis, and the other of which is the "imaginary" axis.

   3.1.3. In the complex plane (Fig. 1), real numbers (–7 , 10) lie on the x-axis, imaginary numbers (–3*j*, 42*j*) on the y-axis. Complex numbers (2 + 6*j*, –43 – 17*j*) lie off-axis.

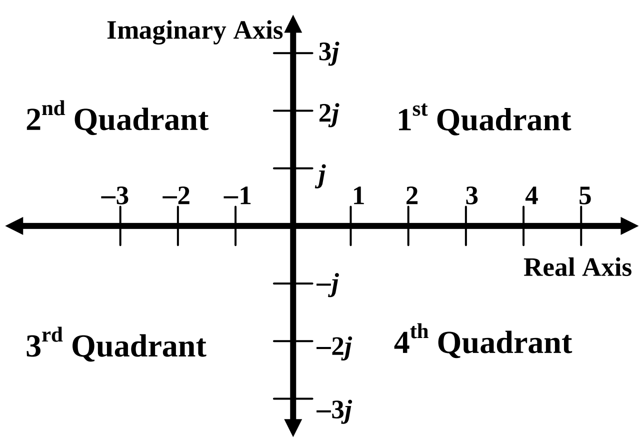


Fig. 1. Representation of Complex Plane

3.1.4. Transforms: Transforms allow moving a problem from a coordinate system or domain where it is difficult to solve to one where it is easier to solve (Fig. 2).

3.1.5. Simpler equations in the transform domain make the problem easier to solve than in the original domain. We can transfer sinusoidal, single-frequency AC circuit problems to a domain where we can use algebra to solve them rather than calculus. Solving problems in algebra is always easier than calculus!

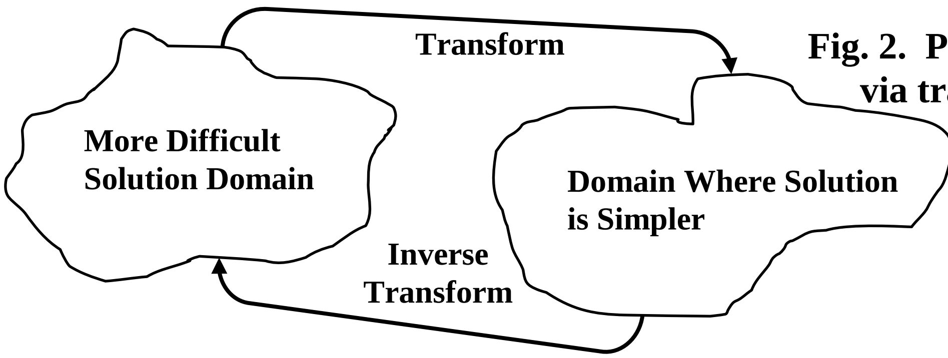


Fig. 2. Problem solving via transforms.

3.1.6. The catch: We need transforms (formulas) to the new domain. Then "inverse transforms" are required to return the solution to the time domain, where it is useful.

3.2. A New Domain: In the phasor or frequency (ω) domain, sinusoidal AC circuit problems are easier to solve. Note: $\omega = 2\pi f$, f the frequency in Hertz (thus ω is in radians/sec).

3.2.1. Transforms: Skipping the derivation (it will come later), we simply list frequency domain transforms. Note that AC voltage is usually expressed as $v(t) = V_p \cos(\omega t)$, ($V_p$ = peak voltage).

3.2.2. Euler's formula: $e^{\pm jx} = \cos x \pm j \sin x$. Cosine is the real part of the function and sine is the imaginary part. Thus, $\cos x = \text{Re}\{e^{jx}\}$ and $\sin x = \text{Im}\{e^{jx}\}$, where Re = "real part," and Im = "imaginary part." Using the cosine function to represent sinusoidal AC voltage, then: $v(t) = V_p \cos(\omega t) = V_p \text{Re}\{e^{j(\omega t)}\}$.

3.3. Transforms to the Frequency Domain: Some circuit elements have a different representation in the frequency or ω domain.

Element	Time Domain	ω Domain Transform
Sinusoidal AC Voltage	$V_p \text{Re}\{e^{j(\omega t)}\}$	V_p
Resistance	R	R
Inductance	L	$j\omega L$
Capacitance	C	$1/j\omega C$

3.4. Comments:

3.4.1. Resistors transform dirctly to the frequency domain.

3.4.2. Inductance → <u>inductive impedance</u> in the *ω* domain (that is, $Z_L = j\omega L = jX_L$). $|Z_L|$ is the <u>inductive reactance</u> (symbolized as X_L), or $|Z_L| = X_L = \omega L$.

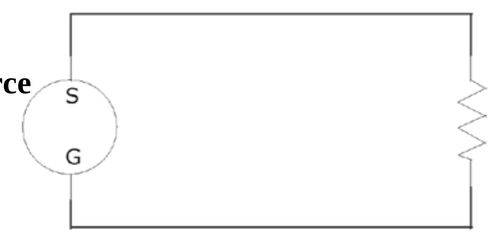
3.4.3. Capacitance → <u>capacitive impedance</u> in the frequency domain as $Z_C = (1/j)X_C = 1/j\omega C = -j/\omega C$, ($X_C = |Z_C| = 1/\omega C$).

3.4.4. There is no frequency information in the <u>voltage transform</u>: $v(t) = V_p \cos(\omega t) \to V_p$. Note that inductive and capacitive impedance <u>do include frequency information</u>.

3.5. Solving For Currents in the New Domain: In most EE problems, we know voltages and component values (*R*, *L*, *C*). We generally solve for <u>circuit currents</u>, which is <u>easier in the frequency domain</u>. Note: *V* = *I·R* <u>in the time domain</u>; *V* = *I·Z* <u>in the *ω* domain</u>, where *Z* = *R* ± *jX*. In both domains, voltage is still in Volts and current in Amperes. Dimensions of resistance and reactance (both) is <u>Ohms</u>.

3.5.1. Resistor in AC circuit – solution in the phasor domain:

AC Voltage Source =10 cos (1000*t*)



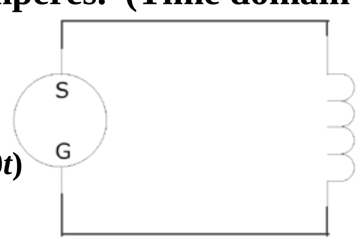
100 Ω

Fig. 3. AC Resistor Circuit.

In Fig. 3, the *ω* domain voltage transform = 10, and resistance = 100. Since *V*=*I·Z* in the *ω* domain, then $I = V/Z = V/R = 10/100$ =0.1 Amperes. <u>This answer is converted to the time domain below</u>.

3.5.2. Inductor in AC circuit (Fig. 4): In the *ω* domain, 10 mH → *jωL* = *j*1000(0.1) = *j*10; *V* = 10 Volts. Then *I* = *V/Z* = *V/ jωL* = 10/*j*10 = −*j*1 Amperes. (Time domain answer below.)

AC Voltage Source =10 cos (1000*t*)



10 mH

Fig. 4. AC Inductor Circuit.

3.5.3. Capacitor AC circuit (Fig. 5): The 100 µF capacitor → 1/ *jωC* = −*j*/1000(100)(10)$^{-6}$ = −*j*/0.1 = −*j*10 in the *ω* domain. Then *I* = *V/Z* = 10/(−*j*10) = *j*1 Amperes. Again, this is the <u>frequency domain</u> answer.

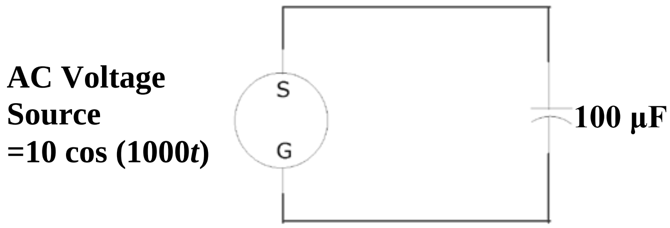


Fig. 5. AC Capacitor Circuit.

3.5.4. Resistor and inductor circuit: In Fig.6, R and L transform to 10 and $j\omega L = j(1000)(10)(10)^{-3} = j10$. Then $I = V/Z = 10/(10 + j10) =$ (rationalizing) $\dfrac{10(10 - j10)}{100 + 100} = \dfrac{100 - j100}{200} = 0.5 - j0.5$.

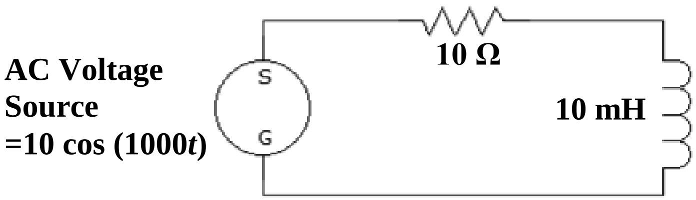


Fig. 6. AC Inductor-Resistor Circuit.

3.6. Inverse Transforms: To make solutions from the ω domain useful, we must do the reverse (or inverse) transform to the time domain.

3.6.1. The answers above are in Cartesian coordinates ($X \pm jY$). These $X \pm jY$ results would be more useful in polar coordinates ($R\angle\theta$).

3.6.2. From 3.2.2, Euler's formula: $e^{\pm jx} = \cos x \pm j\sin x$. Or, $\text{Re}^{\pm jx} = R\cos x \pm Rj\sin x$, R a real constant. Let $A\cos x = X$ and $Aj\sin x = jY$. Then $Re^{\pm jx} = X \pm jY$. But $Re^{\pm jx}$ is $R\angle\theta$ in the Complex plane. Then ($X \pm jY$) = $Re^{\pm j(\arctan[Y/X])}$. The time-domain expression for current (as for voltage) is the real part of $Ie^{\pm j\omega t} = I\cos\omega t$. To convert ω-domain current to the time domain:

- **Convert ω-domain current I from $X \pm jY$ to polar format: $R\angle\theta = Re^{\pm j(\arctan[Y/X])}$, $R = \sqrt{X^2 + Y^2}$, $\angle\theta = \arctan[\pm Y/X]$.**
- **Multiply the result by $e^{j\omega t}$.**
- **Take the real (cosine) part as the time-domain current.**

3.7. Inverse transforms of solutions in 3.6:

3.7.1. In 3.5.1, phasor current (I_P) = 0.1. Following rules of 3.6.2, $i(t) = \text{Re}\{0.1e^{j\omega t}\} = 0.1\text{Re}\{\cos(\omega t) + j\sin(\omega t)\} = 0.1\cos 1000t$ Amperes. Thus, voltage and current are cosine functions They rise and fall together, or are "in phase" (Fig. 7).

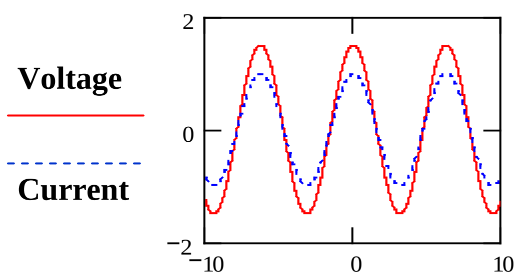


Fig. 7. AC Voltage and Current in a Resistor. Note *V* and *I* in-phase.

3.7.2. From 3.5.2, $I_P = -j1 = 1\angle -90 = (1)e^{-j90}$. From 3.6.2, multiply by $e^{j\omega t}$ and take the real part (remembering that $e^x e^y = e^{x+y}$)
$i(t) = \text{Re}\{-j1e^{j\omega t}\} = 1\text{Re}\{e^{-j90}e^{j\omega t}\} = \text{Re}\{e^{j(\omega t-90)}\} = \cos(1000t-90)$ A.

3.7.3. The result is sinusoidal current with a peak value of 1 ampere, with an associated <u>phase angle</u>. That is, it oscillates at the same radian frequency of 1000 rad/sec, but is <u>not</u> in lock-step with the voltage. Its oscillation is 90 degrees behind the voltage, as shown in the graph below (Fig. 8). This "phase angle" is <u>constant</u>. Current is always <u>exactly</u> 90 degrees behind the voltage, <u>a significant characteristic of inductors in sinusoidal AC circuits</u>.

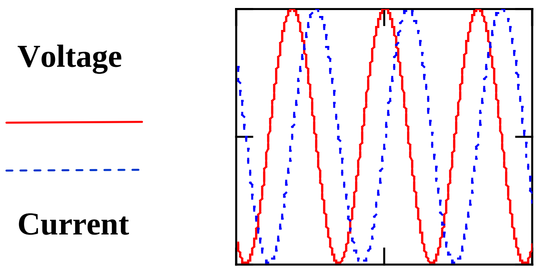


Fig. 8. AC Voltage and Current in an Inductor. Note that I Lags V by 90°.

3.7.4. In the formula for the current, $10\cos(1000t-90)$, <u>the 1000*t* term is in radians</u>; <u>the phase angle is in degrees</u>. <u>This odd mismatch of different angular expressions</u> is simply the "way it has been done" for a long time by EE's, so get used to it. To calculate the value of the cosine at some time *t*, you must either convert the <u>first term to degrees</u>, or the <u>last to radians</u>!

3.7.5. From 3.5.3, $I_P = j1$. Using the inverse transform formula,
$i(t) = \text{Re}\{j1e^{j\omega t}\} = 1\text{Re}\{e^{j90}e^{j\omega t}\} = 1\text{Re}\{e^{j(\omega t+90)}\} = \cos(1000t+90)$ A.
Here, the sinusoidal current <u>leads</u> the voltage by 90 degrees (Fig. 9), reaching a maximum or minimum 90 degrees before the voltage, a <u>significant characteristic of capacitors in AC circuits</u>.

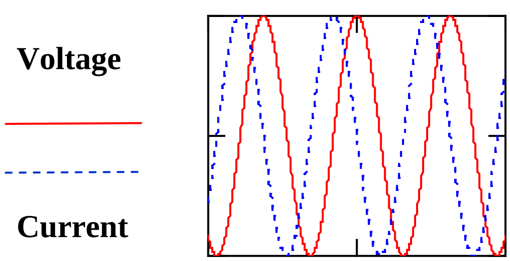


Fig. 9. AC Voltage and Current in a Capacitor. Note that I leads V by 90°.

3.7.6. From 3.5.4 (Fig. 6), converting the Cartesian result $I = 0.5 - j0.5$:
$I = 0.5 - j0.5 = \sqrt{0.5}\angle -45° = 0.707\angle -45° = 0.707e^{-j45°}$. Converting

to the time domain: $i(t) = \mathrm{Re}\{e^{j\omega t}(0.707e^{-45°})\} = 0.707\,\mathrm{Re}(e^{j(\omega t - 45°)})$ $= 0.707\cos(1000t - 45°)$. Here, the phase angle θ = – 45°; current lags voltage inductively, <u>but less than without the resistor</u>. In an *RC* circuit, current would lead voltage, but <u>by less than 90°</u>.

3.7.7. Note: In the ω domain, <u>Kirchhoff's voltage and current laws still hold</u>, and series or parallel impedances behave <u>just as series and parallel resistors in AC or DC circuits</u>. <u>The only difference is that impedances can be complex numbers</u>.

3.8. Summary: The steps in solving a steady-state AC circuit problem:

- Translate circuit parameters into the ω or frequency domain.
- Solve the problem, achieving a solution of the form $X \pm jY$.
- Convert this solution to an $R\angle\theta$ format and thence to $Re^{\pm j\theta}$ form.
- Multiply by $e^{j\omega t}$.
- The real (cosine) part of the result is the time-domain current, *i(t)*.

4. <u>Pre-Work</u>: Prior to the lab, study this outline and <u>complete the worksheet</u>.
5. <u>Experimental Procedure</u>: Make sure that you have all the parts required.

5.1. *V-I* Relationship in an AC *RL* Circuit: We first study an *RL* circuit.

5.1.1. Construct a series *RL* circuit as in Fig. 6 above, using a 10 mH inductor and a 16 Ω resistor. <u>Measure and record</u> the resistor and inductor values using the DMM and the LC meter. Connect oscilloscope channel one across both elements, and channel two across the resistor (which should be closer to the black leads [Fig. 10]). Connect signal generator and DMM across both resistor and inductor (Fig. 10), set to 5 Vp-p, 1000 Hz, checking with oscilloscope. Use DMM as a check of the RMS voltage of the signal generator.

5.1.2. Channel 1 is the "reference," as it shows the overall circuit voltage.

5.1.3. Make sure traces are about the same size. Use "Autoscale," then adjust manually as needed. Overlap the traces, which will help the measurements below. Since the resistor value is <u>real</u> (no reactance), <u>voltage across it is a direct measure of the current – both magnitude and phase</u> (see oscilloscope traces in Fig. 11).

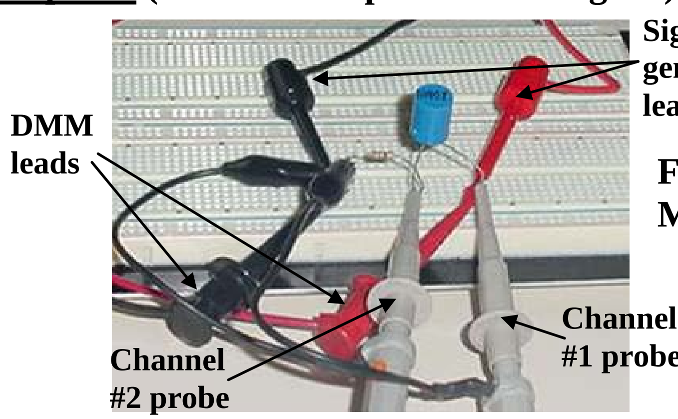


Fig. 10. AC RL Circuit Measurement Set-up.

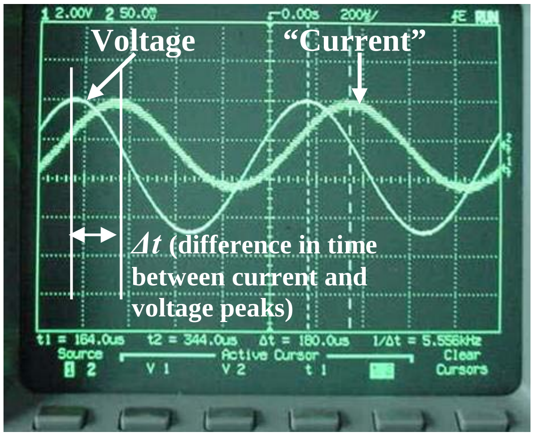


Fig. 11. AC RL Circuit Oscilloscope Traces. The “current” trace is really resistor voltage, but it is in phase with the current.

5.1.4. Measure peak-to-peak voltage (V_{pp}) across the resistor, using the cursors. Then $V_p = V_{pp}/2$; peak current = $I_p = V_p/R = V_{pp}/2R$.

5.1.5. Using time cursors, measure time difference between I_p and V_p (see above). <u>This “Δt” will determine the phase angle</u>.

5.2. Examining V-I Relationship in an AC RC Circuit: Replace inductor with a 10 μF capacitor to create an RC circuit (measure the exact value and record). Leave 16 Ω resistor in place (see Fig. 12).

5.2.1. Make sure both traces are still visible (adjust channel 2 sensitivity if necessary; continue to let traces overlap). Channel 1 (input voltage) is the reference; channel 2 shows resistor voltage. Again, resistor voltage shows <u>current magnitude and phase</u> (Fig 13).

5.2.2. Using the cursors, measure V_{pp} on both channels. Peak-to-peak resistor voltage can be converted to peak current as above.

5.2.3. Using vertical cursors, measure “Δt” between current and voltage peaks. <u>“Δt” will be used to calculate the current phase angle</u>.

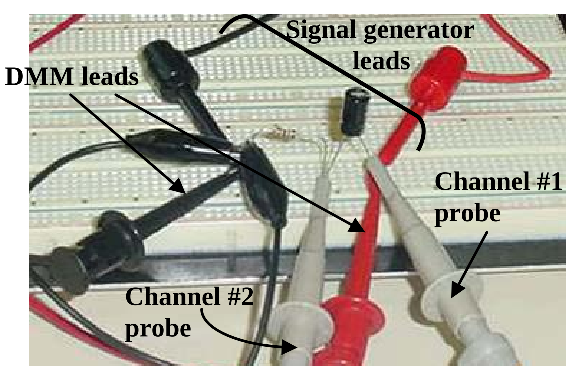


Fig. 12. AC RC Circuit Measurement Set-up.

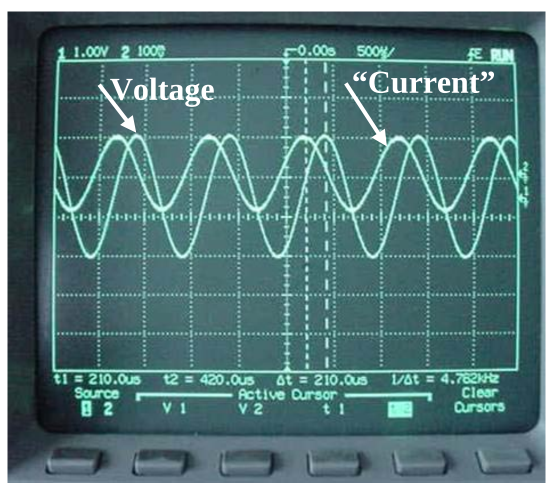


Fig. 13. AC RC Circuit Oscilloscope Traces. The “current” trace is really resistor voltage, but in phase with the current.

6. **<u>Laboratory Area Cleanup</u>: Return parts kit to the cabinet. <u>Make sure work area is clean</u>.**
7. **<u>Writing the Laboratory Report</u>: In your report, do the following:**
   - **7.1.Since $i(t) = I_p \cos(\omega t \pm \theta)$ from 3.8, construct expressions for *i(t)* in the *RL* and *RC* circuits of 5.1 and 5.2, using the given circuit values.**
   - **7.2.You developed an equation in Worksheet #5 for *i(t)* using the experimental data gathered in 5.1 and 5.2. From your measurements, write an expression for *i(t)* in each case.**
   - **7.3.Compare the *i(t)* expressions developed in 7.1 and 7.2. Discuss any discrepancies.**
   - **7.4.Discovery Exercise #1 (space for answers is on Experiment #5 Data Sheet):**
     - **7.4.1. Given an inductor *L*, the ω-domain impedance is $Z_L = j\omega L$. If <u>frequency domain series impedances add directly</u>, develop a formula for the equivalent inductance of two series inductors. Remember that if $Z_L = j\omega L$, then $L = Z_L / j\omega$.**
     - **7.4.2. From this result, generalize a formula for total inductance of *n* series inductors.**
   - **7.5.Discovery Exercise #2 (space for answers is on Experiment #5 Data Sheet):**
     - **7.5.1. For a capacitor *C*, the ω-domain impedance is $Z_C = 1 / j\omega C$. Since frequency domain series impedances add directly, develop a formula for the capacitance of two series capacitors. Remember that if $Z_C = 1 / j\omega C$, then $C = 1 / j\omega Z_C$.**
     - **7.5.2. From this result, generalize a formula for total capacitance of *n* series capacitors What is surprising about this answer?**
   - **7.6.Based on 7.4 and 7.5 above, what is your "best guess" about the equivalent inductance of <u>parallel inductors</u> and <u>parallel capacitors</u>?**
   - **7.7.Given a series *RLC* circuit (inductor, capacitor, and resistor in series) with impedances as follows: $Z_C = -j100\Omega,\ Z_L = j100\Omega,\ R = 100\Omega$. What is the total impedance of the circuit? What is the current phase angle? What is surprising about this?**

Experiment #5 Data Sheet

1. **Measured value of 16Ω resistor: _________ 10 mH inductor: __________**

2. **RL Circuit measurements:**
   2.1. Measured peak-peak (p-p) voltage on 16Ω resistor: __________

   2.2. P-p resistor current (A): ________ and peak current (A): __________

   2.3. Time delta (μsec) between current and voltage peaks: __________

   2.4. Time-domain expression for *i(t)*, based on measures above: __________

   2.5. Calculated expression for *i(t)*, based on measured *R* and *L*: __________

   2.6. List your ideas for discrepancies, if any: __________________________

   ____

   ____

3. **Measured value of 10 μF capacitor: __________**

4. **RC Circuit measurements:**
   4.1. Measured peak-peak voltage on 16Ω resistor: __________

   4.2. Peak-to-peak current (A): _______ and peak current (A): __________

   4.3. Time delta (μsec) between current and voltage peaks: __________

   4.4. Time-domain expression for *i(t)*, based on measures above: __________

   4.5. Calculated expression for *i(t)*, based on measured *R* and *C*: __________

   4.6. List your ideas for discrepancies, if any: __________________________

   ____

   ____

Experiment #5 Data Sheet, Page 2

5. **Discovery Exercise #1:**
   5.1. Impedance of two series inductors ($Z_{LTotal} = j\omega L_1 + j\omega L_2$): ___________

   5.2. Calculated equivalent inductance ($L_{Total} = Z_{LTotal} / j\omega$): ___________

   5.3. Equivalent value of two inductors of value L in series: ___________

   5.4. From 5.2 and 5.3, general formula for equivalent value of any number of series inductances:

   ____

   ____

6. **Discovery Exercise #2:**
   6.1. Impedance of two series capacitors ($Z_{CTotal} = [1 / j\omega C_1] + [1 / j\omega C_2]$):

   ____

   6.2. Calculated equivalent capacitance ($C_{Total} = 1 / j\omega Z_{CTotal}$): ___________

   6.3. Equivalent value of two capacitances of value C in series: ___________

   6.4. From 6.2 and 6.3, general formula for equivalent value of any number of series capacitances:

   ____

   ____

Experiment #5 Worksheet

Note: Experiment #5 is generally the most challenging exercise in EE 1202. Please read Experiment #5 carefully at least twice and then take your time on the exercises below to make sure that you understand the theoretical material.

1. **In what quadrant of the complex plane are these numbers located?**

   −12+*j*7 ___________________ **−10−*j*50** ___________________

   8−*j*2 ___________________ **1+*j*100** ___________________

2. **Rationalize the complex numbers below (answer in the space provided):**

   26/(6−*j*4)___________________ **(8−*j*8)/(2+*j*2)** ___________________

3. **Inductor and capacitor impedances are given as: $Z_L = j\omega L = jX_L$ and $Z_C = 1/j\omega C = -j/\omega C = -jX_C$. Assume you have a 10µF capacitor and a 10mH inductor. Calculate the reactances of these components at the following frequencies and list in the space provided:**

   1 MHz (1,000,000 Hz): X_L________________**Ω** X_C________________**Ω**

   50KHz (50,000 Hz): X_L________________**Ω** X_C________________**Ω**

   0Hz: X_L ________________**Ω** X_C________________**Ω**

4. **Different items in the time domain transform in different ways to the ω domain: $R \to R$, $L \to j\omega L$, $C \to 1/j\omega C$, ($\omega = 2\pi f$, $v(t) = V\cos\omega t \to V$). Given a circuit with $v(t) = 10\cos 1000t$, *R*=100Ω, *L*=10mH, and *C*=10µF, calculate the values in the ω domain of:**

   voltage __________ **resistance** __________

   inductive impedance __________ **capacitive impedance** __________

5. **After transforming voltage and circuit to the ω domain, find the current by dividing voltage by impedance. This usually results in a complex number. To convert back to the time-domain, which is the answer sought, do four things:**
   - **Rationalize the complex number; the result is an $X \pm jY$ representation.**
   - **Convert complex number to polar-coordinates, then power-of-*e* format..**
   - **Multiply the current in power-of-*e* form by $e^{j\omega t}$.**
   - **Take the real part to get the time-domain current representation.**

Experiment #5 Worksheet (Page 2)

6. Based on the procedure in 5 above, convert the following ω domain currents back into the time domain (assume ω = 1000):

I = 10+j10 ____________ ***I = −8+j4*** ____________

7. If $v(t) = 10\cos 10000t,\ R = 100\Omega,\ C = 100\mu F$, find $i(t)$.

____

8. In 5.1 and 5.2 you will take measurements of the voltage across a resistor (which is a measure of the current in the resistor) and the phase angle between voltage and current, measured in µsec. With these measurements you can determine the current as a function of time, that is $i(t) = I_P\cos(\omega t + \theta),\ I_P$ the peak AC current and θ the phase angel in degrees. I_P and θ are found as follows:

8.1. Peak current = $I_P = \dfrac{V_{RP-P}/2}{R}$, where V_{RP-P} is the peak-to-peak voltage across the resistor that you measure, and *R* is the resistor in the circuit.

8.2. The phase angle θ is defined by the following equation:

$\theta = \text{phase}\angle$ in degrees $= \dfrac{(t2 - t1)\mu\sec}{1000\mu\sec}(360°) = \dfrac{\Delta t}{1000}(360°)$. In the phase angle equation, the time difference $\Delta t (= t2 - t1)$ is your measured time between the voltage and current peaks, 1000 *µsec* is the period of the 1000Hz AC sinusoidal signal, and 360° is the period of a cosine. You will use this equation to calculate θ in section 8.2 of your experiment.

9. As an exercise to become familiar with the calculations in 8 above, compute the current in an AC capacitor circuit if the peak-to-peak voltage across a 100Ω resistor is 20V, the time measurements are $t2 = 300\mu\sec$ and $t1 = 100\mu\sec$, and the frequency of the AC sinusoidal voltage is 1000 Hz. Remember: $\omega = 2\pi f$.

$i(t) =$

____